Error-sensitive proof-labeling schemes

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Locality
Distributed decision
Building vs. deciding

Yes/No
Distributed languages

Context:
- Communication graph $G$
- Node inputs, $x : v \mapsto x(v)$

A language is
a set of configurations $(G, x)$

s.t. $\forall G, \exists x, (G, x) \in \mathcal{L}$
Properly-colored graphs

\[
\mathcal{L} = \{(G, x) \text{ s.t. } x \text{ is a proper coloring of } G\}
\]
Spanning forest

\[ L = \{ (G, x) \text{ s.t. } x \text{ describes a spanning forest of } G \} \]
\[ \mathcal{L} = \{(G, x) \text{ s.t. } x \text{ describes a spanning forest of } G\} \]
Every node:
- gathers its 1-neighbourhood
- outputs a local decision accept or reject.
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- gathers its 1-neighbourhood
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Decision mechanism

$$(G, x)$$ is accepted if all nodes accept.
Decision mechanism

$(G, x)$ is rejected if at least one node rejects.
Properly-colored graphs
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Spanning forest
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Proof-labeling schemes

Distributed non-determinism
Proof-labeling schemes
Proof-labeling schemes
Proof-labeling schemes
Proof-labeling schemes

Given a proof-labeling scheme for $\mathcal{L}$:

For all ($G, x$):

- If ($G, x$) $\in \mathcal{L}$:
  $$\exists c \text{ s.t. } (G, x, c) \text{ is accepted.}$$

- If ($G, x$) $\notin \mathcal{L}$:
  $$\forall c, (G, x, c) \text{ is rejected.}$$
Proof-labeling schemes
Proof-labeling schemes
Proof-labeling schemes
Proof-labeling schemes
Error-sensitivity of proof-labeling schemes
One node to reject
More nodes to reject
Distance

$$d(((G, x_1), (G, x_2)) = \# \{ v : x_1(v) \neq x_2(v) \}$$
Distance

\[ d((G, x_1), (G, x_2)) = \#\{v : x_1(v) \neq x_2(v)\} \]
Distance

\[ d((G, x), \mathcal{L}) = \min_{(G', x') \in \mathcal{L}} d((G, x), (G', x')) \]
A PLS is error-sensitive if the number of rejecting nodes grows linearly with the distance.
Error-sensitivity

with a formula

A PLS is error-sensitive if there exists $\alpha > 0$ s.t., for all $(G, x)$, for all certificate:

$$\#\{\text{Rejecting nodes}\} \geq \alpha \cdot d((G, x), \mathcal{L})$$
Examples
### Acyclicity Problems

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<tr>
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<th>Adjacency lists</th>
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<td><strong>Spanning Forest</strong></td>
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Spanning forest with pointers
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Spanning forest with pointers
Spanning forest with pointers has an error-sensitive PLS.
## Acyclicity problems

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Spanning tree with pointers
Spanning tree with pointers
Spanning tree with pointers
Spanning tree
with pointers
Spanning tree with pointers
Spanning tree with pointers
Spanning tree with pointers

has no error-sensitive PLS (for any certificate size).
Structural characterization
Theorem

A language $\mathcal{L}$ admits an error-sensitive PLS

$\iff$

$\mathcal{L}$ is locally stable
Local stability

Hybridization

\[ \in \mathcal{L} \]
Local stability

Boundary nodes
Local stability

$L$ is locally stable if:

$$\exists \beta, \forall G, \forall \text{hybridization},$$

$$d(G, L) \leq \beta \cdot \#\{\text{Boundary nodes}\}$$

$$d(\text{hybrid}, L) \leq \beta \cdot \#\{\text{Boundary nodes}\}$$
Spanning tree with pointers is not locally stable
Spanning tree with adjacency lists is locally stable
## Acyclicity problems

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Spanning tree with adjacency lists
Spanning tree with adjacency lists
Spanning tree with adjacency lists
Spanning tree with adjacency lists
With adjacency lists

Thm: With adjacency lists, spanning tree and minimum spanning tree, are locally stable.

⇒ they have error-sensitive PLS.
Compact schemes
Compact PLS

**Theorem** (Korman et al.) :
- ST has a $O(\log n)$-PLS;
- MST has a $O(\log^2 n)$-PLS.
Compact PLS

New Theorem:

- ST has a $O(\log n)$-ESPLS;
- MST has a $O(\log^2 n)$-ESPLS.
Open problem

Does error-sensitivity always come for free (when achievable)?