A HIERARCHY OF LOCAL DECISION

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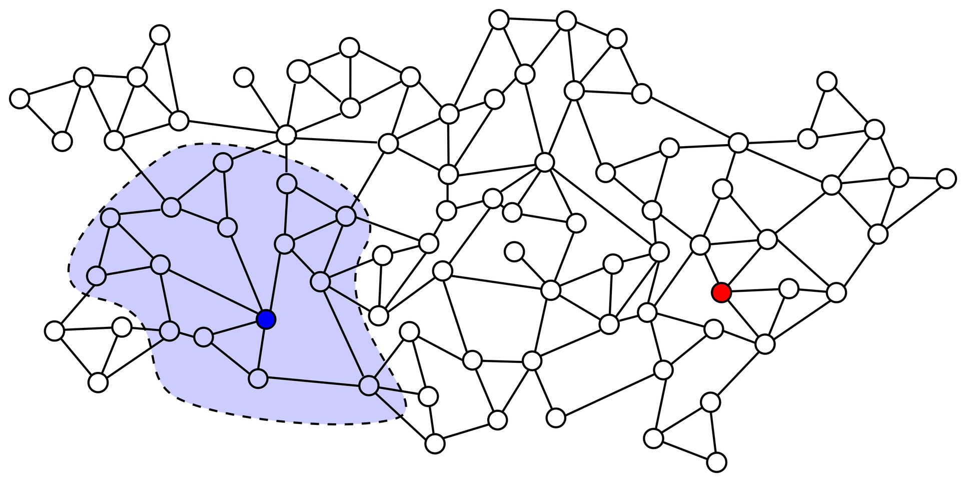
DISTRIBUTED GRAPH ALGORITHMS

A distributed graph algorithm is an algorithm that every node of a network will run to compute a solution to a graph problem. The nodes have the knowledge of a small neighbourhood only which, in this poster, is a constant radius ball.



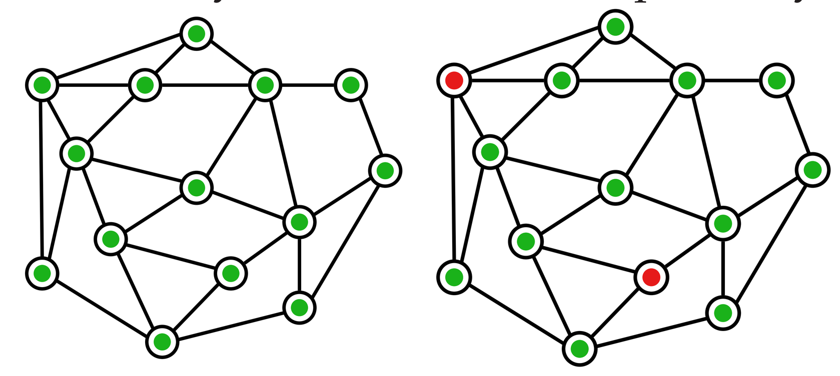
LOCAL DECISION

Most of the literature → computing something, e.g. a colouring

Local decision → deciding properties, e.g. acyclicity of the graph.

Goal: study complexity classes of decision problems (see *Survey of distributed decision* by F& F).

Decision mecanism: the graph is accepted if and only if all the nodes accept locally.



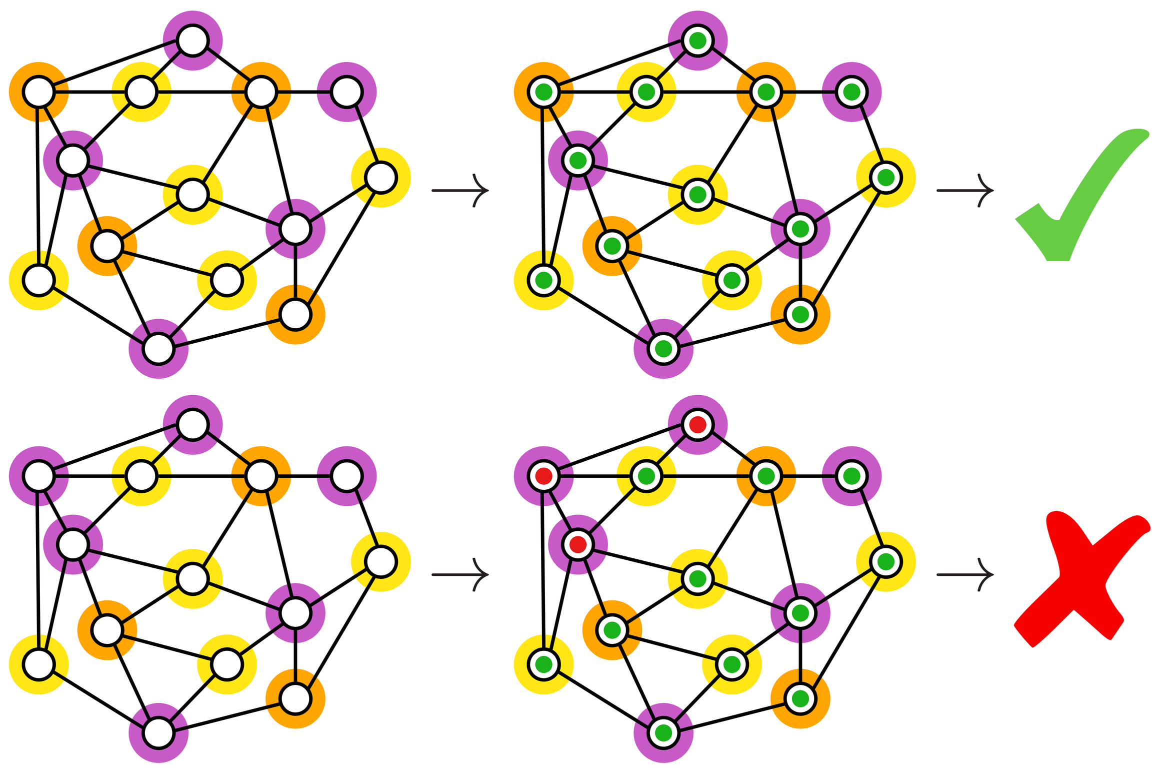
LEVEL 0: LOCALLY DECIDABLE

We consider languages, that are sets of graphs (G), with inputs on the nodes (x_v, $v \in V(G)$). For example, let $\mathcal{L}_{\text{3-colored}}$ be the language of the coloured graphs such that the colouring is a proper 3-colouring.

A language is locally decidable if the nodes can decide locally if the network belongs to it, using the above mechanism.

EXAMPLE

The language $\mathcal{L}_{\text{3-colored}}$ is locally decidable.



DEFINITION LD

We define the basic class of complexity, LD:

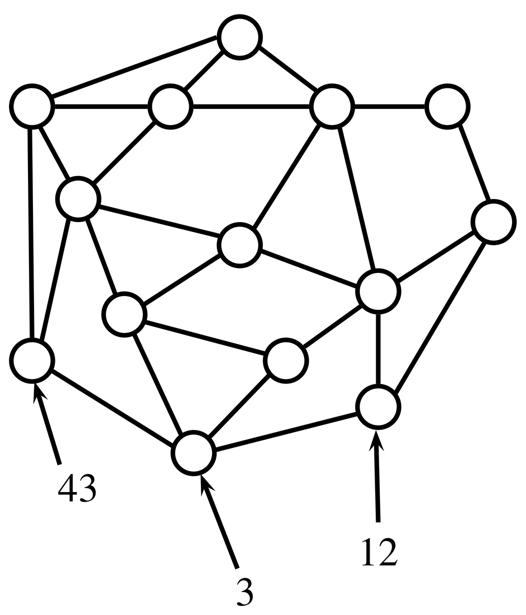
$\mathcal{L} \in LD$ if and only if
$\exists A \in$ Cst-dist s.t. $\forall G, x, (G,x) \in \mathcal{L} \Leftrightarrow A(G,x) = 1$
where $A(G,x) = 1$ means $\forall v, A(G, x_v, v) = 1$

The analogue in the sequential setting is P:

$\mathcal{L} \in P$ if and only if
$\exists A \in$ Polytime such that $\forall x, x \in \mathcal{L} \Leftrightarrow A(x) = 1$

REMARKS

- The nodes have distinct IDs. They can use them during the computation. The language should not depend on the IDs of the graph.
- The degree is not bounded: constant size neighbourhoods can be big.
- Local model: no limit on bandwidth or local computation.



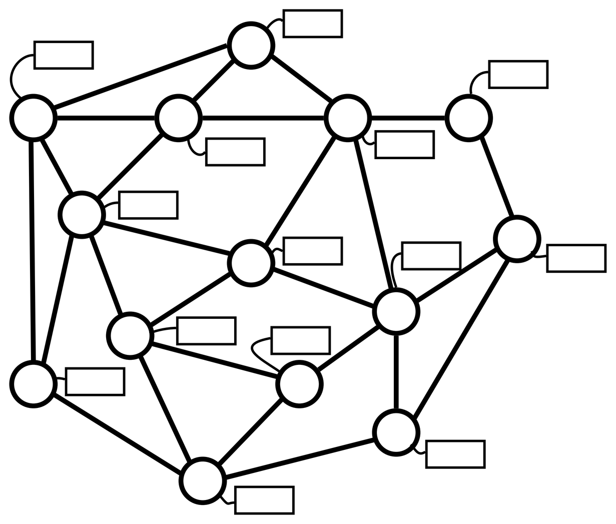
IF IT RINGS A BELL

You may know the paper *What can be computed locally?* by Naor and Stockmeyer (1995), where a very similar class, called LCL, is defined.

LEVEL 1: LOCALLY VERIFIABLE

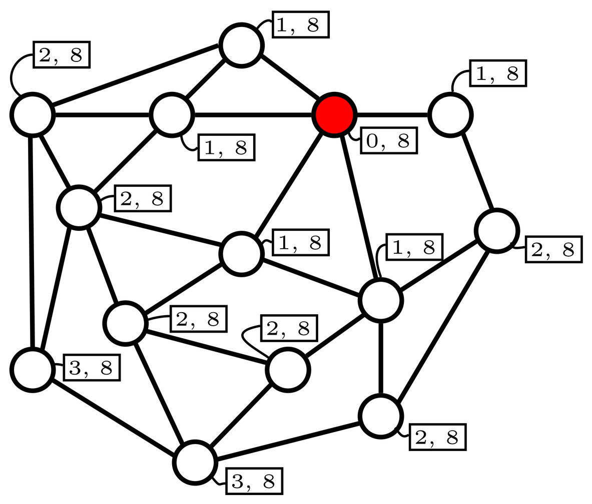
Locally checkable proof = a function that assigns to each node of the graph a label.

As the certificates in sequential non-determinism, it is a proof that the instance is correct. It must be verifiable locally.



EXAMPLE: LEADER

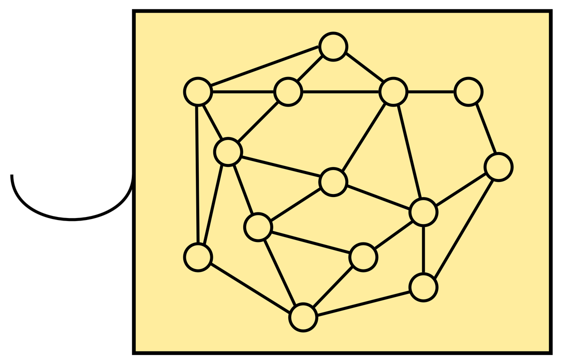
Language: graphs with exactly one leader (e.g. a node with a special flag). A locally checkable proof is shown below.



SIZE OF THE PROOFS

The labels can depend on IDs
↪ a $O(n^2)$ label encodes the graph
↪ any property can be decided.

Challenge: having small labels.



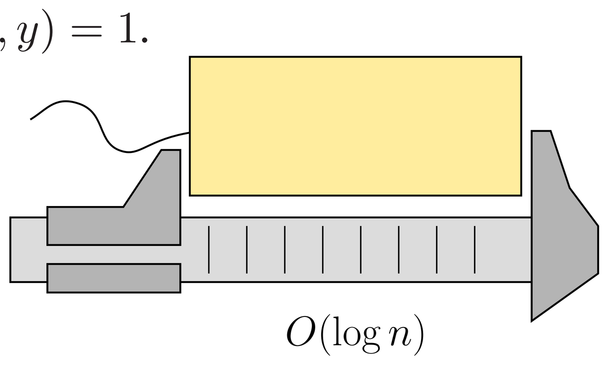
LOGLCP

We consider the class LogLCP, an analogue of NP:

$\mathcal{L} \in$ LogLCP if and only if
$\exists A \in$ Cst-dist such that for all $G, x, (G,x) \in \mathcal{L} \Leftrightarrow$
$\exists y$, with $|y| \in O(\log(n)), A(G,x,y) = 1$

$\mathcal{L} \in NP$ if and only if
$\exists A \in$ Polytime such that for all $x, x \in \mathcal{L} \Leftrightarrow$
$\exists y$, with $|y| \in O(\text{poly}(n)), A(x,y) = 1$.

LogLCP contains many languages: leader, acyclicity, colourability, and all the complement of languages in LD.



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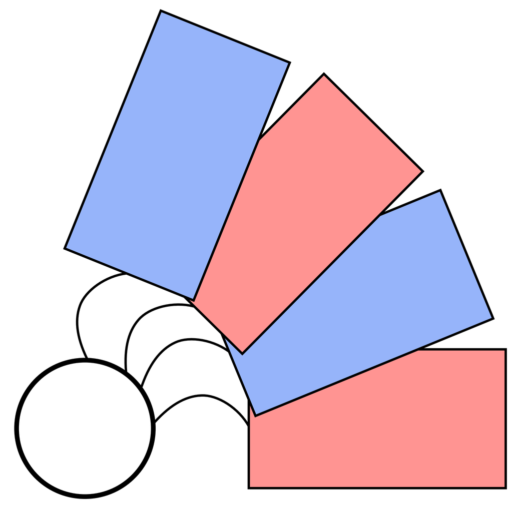
Göös and Suomela defined locally checkable proofs in an eponymous paper. The basic concept is older, is called *proof labelling scheme* and has been studied by Korman, Kutten, Peleg and Masuzawa, among others.

LEVEL k: LOCAL HIERARCHY

- Before: analogue of P and NP.
- This work: analogue of the polynomial hierarchy
- Idea: a prover and a disprover give labels to the nodes one after the other, to convince them respectively to accept and to reject the graph.



CLASSES Σ_p^L AND Π_p^L

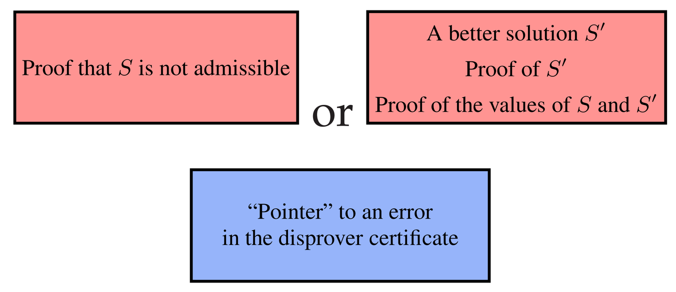
We define the classes Σ_p^L and Π_p^L. All the labels y have logarithmic size.

$\mathcal{L} \in \Sigma_p^L$ if and only if
$\exists A \in$ Cst-dist such that for all G, x,
$(G,x) \in \mathcal{L} \Leftrightarrow \exists y_1, \forall y_2, ..., Q_p y_p, A(G,x,y) = 1$

$\mathcal{L} \in \Pi_p^L$ if and only if
$\exists A \in$ Cst-dist such that for all G, x,
$(G,x) \in \mathcal{L} \Leftrightarrow \forall y_1, \exists y_2, ..., Q_p y_p, A(G,x,y) = 1$

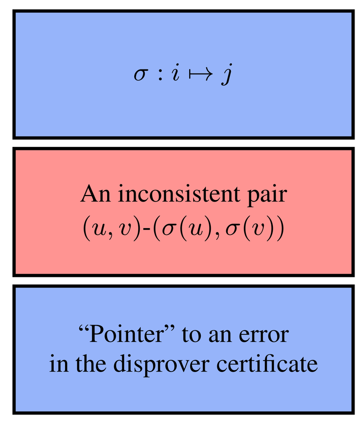
EXAMPLE: OPTIMAL SOLUTIONS

For many combinatorial problems, the set of optimal solutions is in Π_2^L. The protocol is on the right, with the disprover in red, and prover in blue.



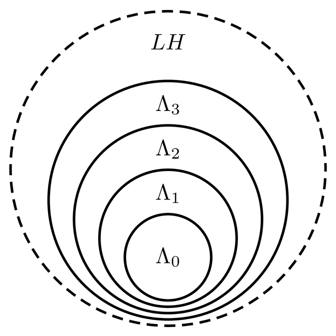
EXAMPLE: NON TRIVIAL AUTOMORPHISM

Language: the graphs that have a non-trivial automorphism. This language has a protocol in Σ_3^L, described on the right.



STRUCTURAL RESULTS

- Collapses: $\Sigma_{2i} = \Sigma_{2i-1}$ and $\Pi_{2i+1} = \Pi_{2i}$. We rename the classes as Λ_i.
- There are interesting co-classes.
- The levels 0, 1 and 2 are separated.
- There are languages outside the hierarchy.



OPEN PROBLEM

Are level 2 and 3 separated ?

IF IT RINGS A BELL

Reiter proved a connexion between a similar hierarchy and MSO logic. The same hierarchy but with no labels in the certificate has a poster at WoLA!