Computation Tree Logic

CTL

Formules de CTL

$\varphi,\psi ::= P \mid \neg\varphi \mid \varphi\vee\psi \mid \mathbf{EX}\varphi \mid \mathbf{AX}\varphi \mid \mathbf{E}\varphi\mathbf{U}\psi \mid \mathbf{A}\varphi\mathbf{U}\psi$

avec $P \in AP$

+ Abbréviations :

$\top, \bot, \wedge, \Rightarrow$

$\mathbf{F}\varphi = \top\ \mathbf{U}\ \varphi$: “eventually“, → $\mathbf{EF}\varphi$ $\mathbf{AF}\varphi$

$\mathbf{G}\ \varphi = \neg\ \mathbf{F}\ \neg\varphi$: “always“ → $\mathbf{EG}\varphi$ $\mathbf{AG}\varphi$

$\varphi\mathbf{W}\ \psi = \varphi\mathbf{U}\ \psi \vee \mathbf{G}\ \varphi$: “weak until” → $\mathbf{E}\varphi\mathbf{W}\psi$ $\mathbf{A}\varphi\mathbf{W}\psi$

CTL - sémantique

$\mathbf{S} = (Q, \rightarrow, q_{init}, \ell)$

Exec(q) = ens. des exécutions infinies partant de q.

$\rho \in$ Exec(q): $\rho = q_0 q_1 q_2 q_3 q_4 \ldots$ avec $q_0=q$ et $q_i \rightarrow q_{i+1}$

Notation: $\rho(i) = q_i \quad \forall\ i\geq 0$

On interprète les formules de CTL sur des états de $\mathbf{S}$.

$q \models P$ iff $P \in \ell(q)$

$q \models \mathbf{EX}\varphi$ iff $\exists\ q\rightarrow q'$ t.q. $q' \models \varphi$

$q \models \mathbf{AX}\ \varphi$ iff $\forall\ q\rightarrow q'$, on a: $q' \models \varphi$

$q \models \mathbf{E}\varphi\mathbf{U}\psi$ iff $\exists\rho\in$Exec(q) t.q. $\exists\ i \geq 0$ t.q. ($\rho(i) \models \psi$ et
($\forall\ 0 \leq j<i$: $\rho(j) \models \varphi$))

$q \models \mathbf{A}\varphi\mathbf{U}\psi$ iff $\forall\ \rho \in$ Exec(q), $\exists\ i \geq 0$ t.q. ($\rho(i) \models \psi$ et
($\forall\ 0 \leq j<i$: $\rho(j) \models \varphi$))

CTL

Définition alternative (équivalente !!):

Formules d'état:

$\varphi,\psi ::= P \mid \neg\varphi \mid \varphi\vee\psi \mid \mathbf{E}\ \varphi_p \mid \mathbf{A}\ \varphi_p$

$P \in AP$

Formules de chemin:

$\varphi_p,\psi_p ::= \mathbf{X}\ \varphi \mid \varphi\ \mathbf{U}\ \psi$

$\mathbf{E}\ \varphi_p$ = « il existe un chemin vérifiant φ_p »

$\mathbf{A}\ \varphi_p$ = « tous les chemins vérifient φ_p »

CTL - sémantique

Définition alternative (équivalente !!):

$q \models P$ iff $P \in \ell(q)$

$q \models \mathbf{E}\,\varphi_p$ iff $\exists\, \rho \in Exec(q)$ t.q. $\rho \models \varphi_p$

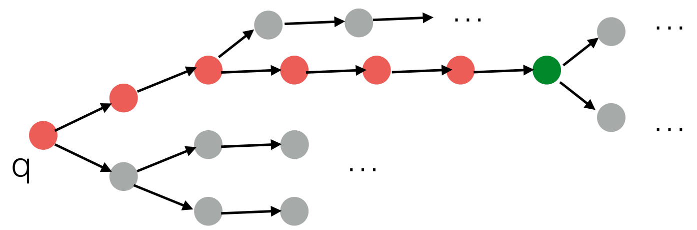
$q \models \mathbf{A}\,\varphi_p$ iff $\forall\, \rho \in Exec(q)$, $\rho \models \varphi_p$

$\rho \models \mathbf{X}\,\varphi$ iff $\rho(1) \models \varphi$

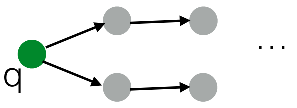
$\rho \models \varphi\,\mathbf{U}\,\psi$ iff $\exists\, i \geq 0$ ($\rho(i) \models \psi$ et ($\forall\, 0 \leq j < i$: $\rho(j) \models \varphi$))

CTL - semantics

$q \models$ **E** **red** **U** **green**

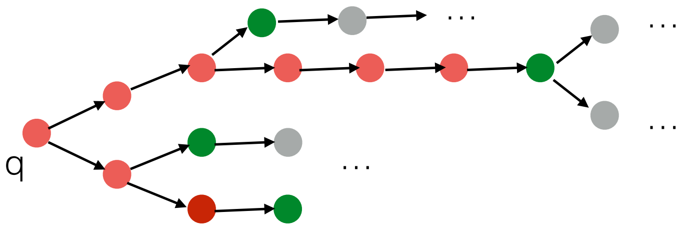


or:

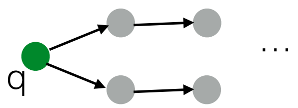


CTL - semantics

$q \models$ **A** **red** **U** **green**

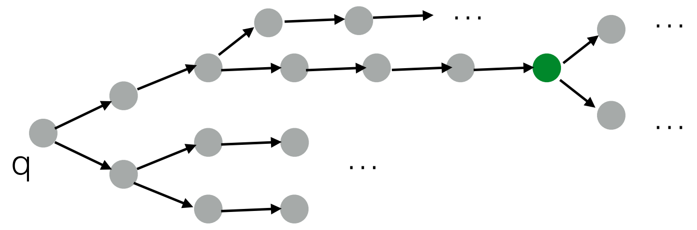


ou:



CTL - semantics

$q \models$ **EF** **green**



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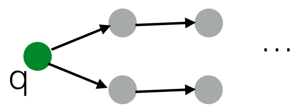
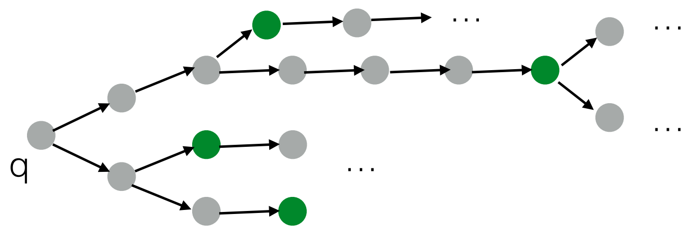
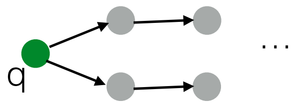


CTL - semantics

q ⊨ **AF** green

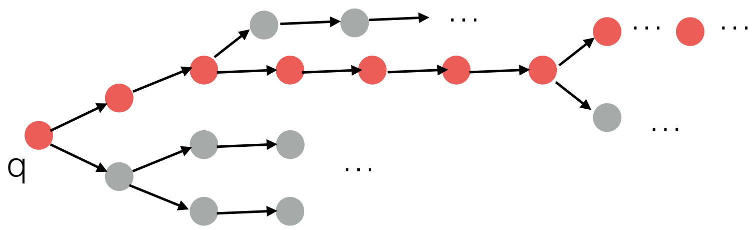


ou:



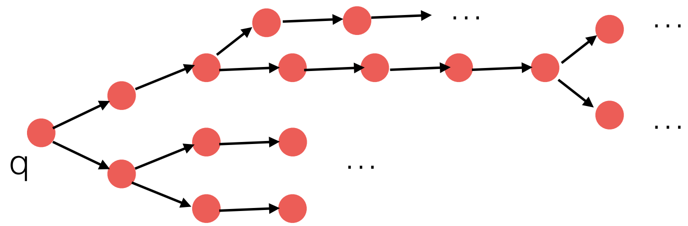
CTL - semantics

q ⊨ **EG** red



CTL - semantics

q ⊨ **AG** red



Every reachable states is red !

Exemples de formules

A G (**E F** accueil)

A F A G ok

AG (request ⇒ **AF** service)

AG AF (a ∧ b) implique **AG AF** a ∧ **AG AF** b

AG AF a ∧ **AG AF** b n'implique pas **AG AF** (a ∧ b)