CALMOC: Categorical and Algebraic Models of Computation

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Project Information

- CALMOC: Categorical and Algebraic Models of Computations.
  NWO Project 612.000.936.
  http://www.cs.ru.nl/calmoc

NWO Netherlands Organisation for Scientific Research

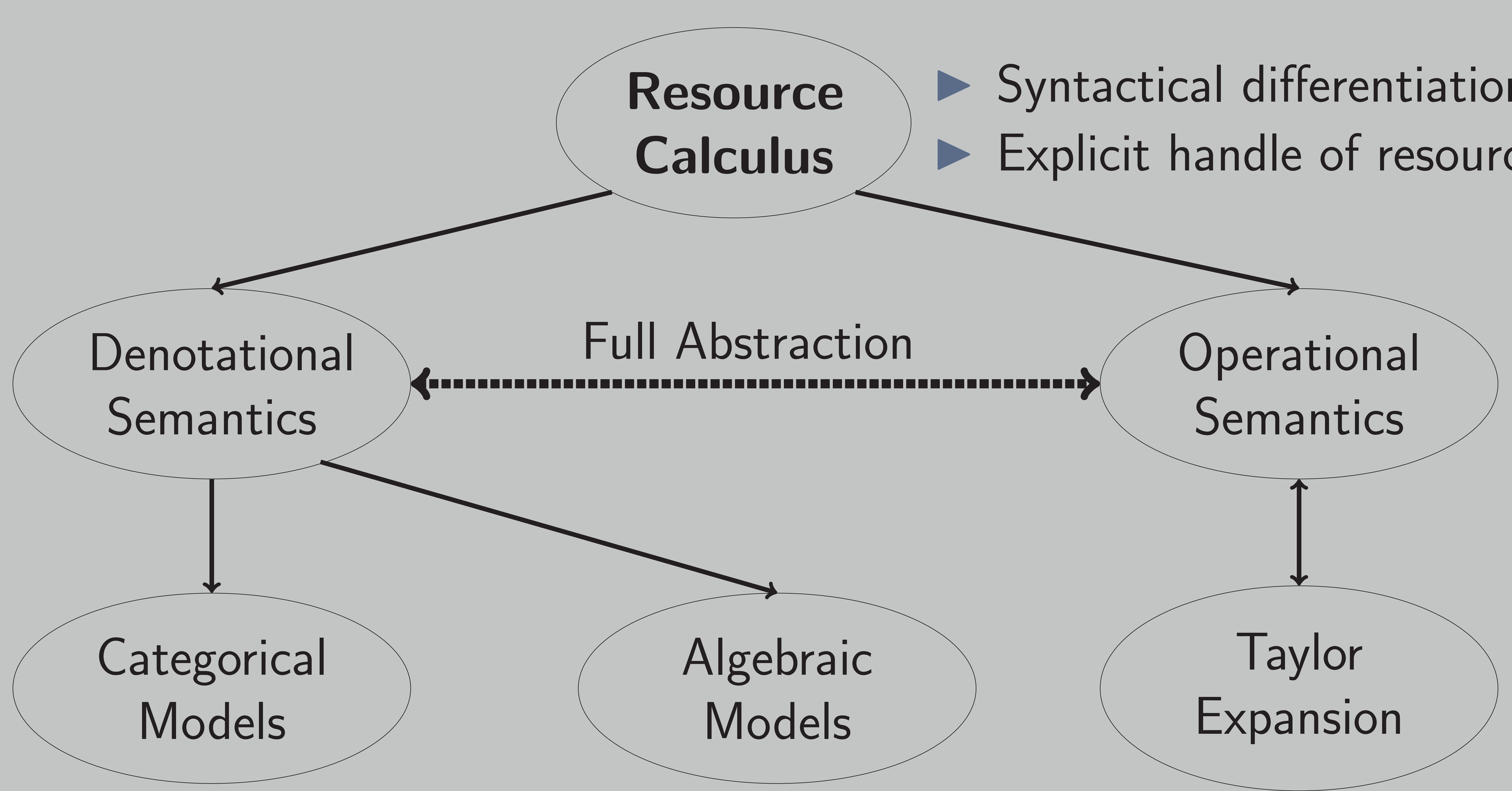
The Problem

- Non-determinism: understand and exploit **true concurrency**
  - extensive study of process calculi, trace semantics, relaxed memory models *did not* provide definitive answers.
- Program robustness: static check of dynamic properties
  - control at compile-time on the amount of resources needed by a program at running-time
- Optimization via program rewriting:
  - syntactic and semantic characterization of operational equivalence

Methodology

- **Object of study**: the **resource calculus** (Ehrhard-Regnier 2003)
  - functional programming language based on λ-calculus,
  - explicit handle on the resources used by a program during its execution (unlike Java or C).
- Abstract mathematical description of models of the resource calculus
  - definition of model based on category-theory / universal algebra.
- Mathematical tools for studying the resource calculus:
  - program decomposition through Taylor expansion (link with analysis),
  - natural duality theory for algebraic models (link with topology)
- Study of definability/adequacy/full abstraction on concrete models:
  - relational semantics: from functional to relational interpretations,
  - game semantics: from static to interactive denotations.

The Big Picture



- Syntactical differentiation
- Explicit handle of resources

- Relational Models
- Game-based Models

- Stone-duality
- Topology

- New Theory of Program Approximation

Design of New Non-Deterministic and Resource Sensitive Programming Languages

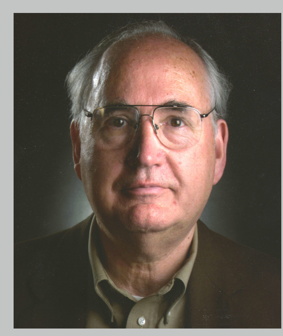
New Ingredient: Taylor Expansion of Programs

- Program differentiation:
  - add a syntactic derivative operator $\mathbf{D}(\cdot)$ computing the *best linear approximation* of a program,
  - excellent candidate to increase control over programs executed in environments with bounded resources.
- Taylor Expansion: replace the usual application $\mathbf{P} \cdot \mathbf{x}$ of a program $\mathbf{P}$ to an input $\mathbf{x}$ by a series of *linear applications*

$$\mathbf{P} \cdot \mathbf{x} = \sum_{n=0}^{\infty} \frac{1}{n!} (\mathbf{D}^n \mathbf{P} \cdot \mathbf{x}^n) 0$$

- **Breakthrough**: transfer results
  - from the linear fragment of the resource calculus (*simple!*)
  - to classic programming languages/full resource calculus (*complex!*).

Expected Results: Stone Duality for Algebraic Models



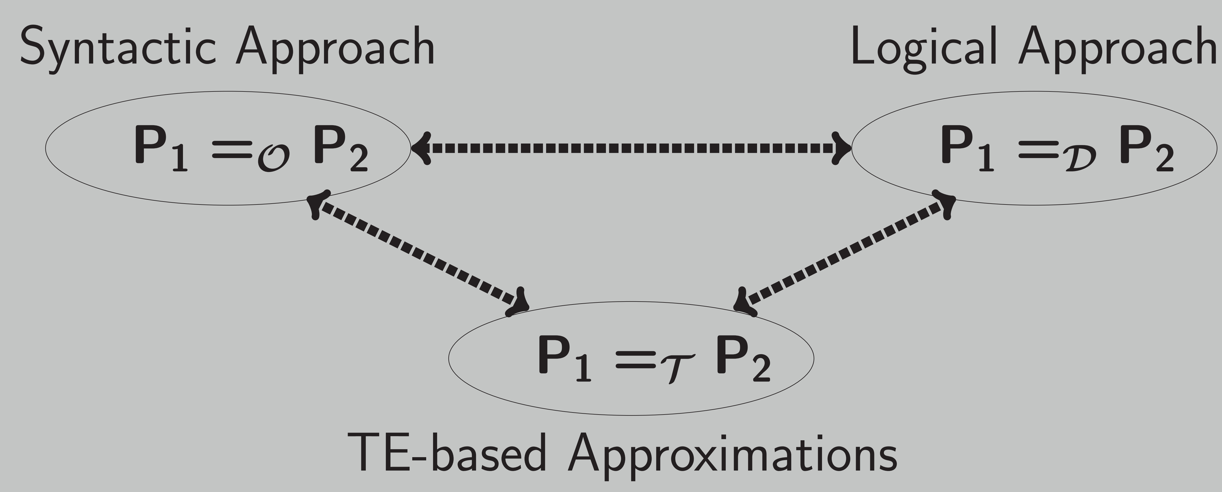
Scott

- Scott domains vs Stone spaces
- Stone duality builds a bridge between
  - Algebraic models of computation
  - Topological spaces

Stone

Expected Results: Characterization of Behavioural Equivalence

- *Behavioural equivalence* of two programs $\mathbf{P}_1$ and $\mathbf{P}_2$:
  - Syntactic approach $\mathbf{P}_1 =_{\mathcal{O}} \mathbf{P}_2$: two programs are equivalent if they have the same behaviour in every context,
  - Logical approach $\mathbf{P}_1 =_{\mathcal{D}} \mathbf{P}_2$: two programs are equivalent if they have the same interpretation in a model $\mathcal{D}$,
  - New approach $\mathbf{P}_1 =_{\mathcal{T}} \mathbf{P}_2$: two programs are equivalent if they have "similar" Taylor expansions.
- Proof of equivalence of these approaches:



- **Ambitious task**: replace the traditional theory of program approximations based on Böhm trees with a mathematical model of *resource consumption*.

Expected Results: Resource Game Semantics

- Resource sensitive models based on game theory,
  - from static to dynamic and interactive interpretations of programs,
- Main ingredients:
  - 2-player games: player vs opponent,
  - alternating games,
  - plays satisfying well-bracketing,
  - non-deterministic strategies,
- Build and study a fully abstract model.

Player vs Opponent

Applications

- Communications: study of programs running in environments with bounded resources (smartphones, PDA's, etc.),
- Security: prevent run-time failures caused by memory limitations in critical fragments of code,
- Programming: design of new programming languages inspired from semantics,
- Fancy computer science: handle data that cannot be duplicated for physical reasons like *q-bits* in quantum programming.

Research Team

- Coordinators:
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