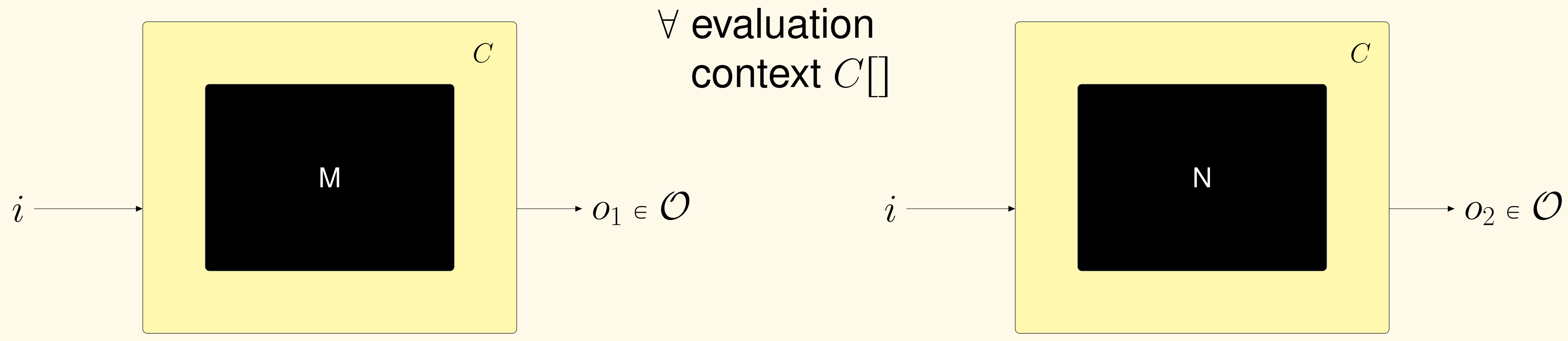
A Refutation of Sallé's Longstanding Conjecture

The Lambda Calculus... why?

At the basis of all functional languages. Simple syntax but Turing complete. The lattice of λ-theories is a mathematically rich structure.
Many links with mathematical logic : intuitionistic logic, linear logic, universal algebra, category theory, recursion theory...

Observational Equivalences $\mathcal{H}^+$ and $\mathcal{H}^*$

When are two programs M and N observationally equivalent?



The answer depends on the $\mathcal{O}$bservables

The λ-theory

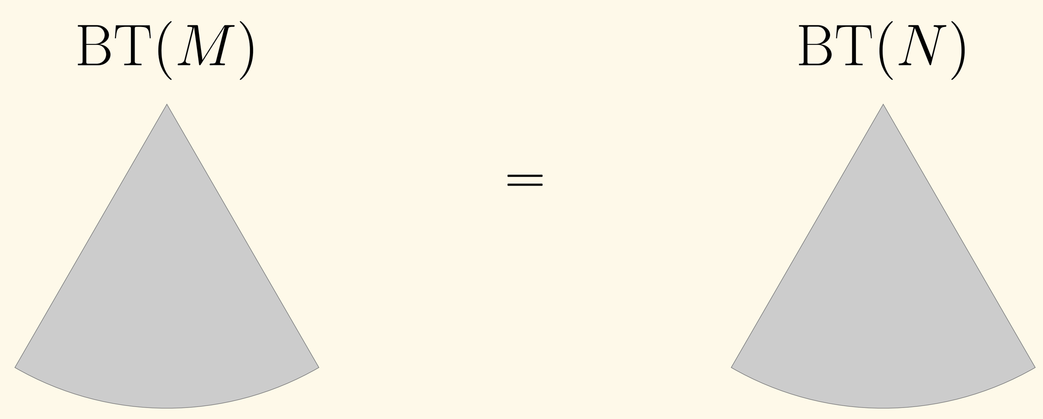
- $\mathcal{H}^*$ corresponds to $\mathcal{O} :=$ solvable λ-terms
- $\mathcal{H}^+$ corresponds to $\mathcal{O} :=$ β-normalizable λ-terms

$\mathcal{B}\omega$: Böhm Trees + Strong Extensionality

Böhm Trees

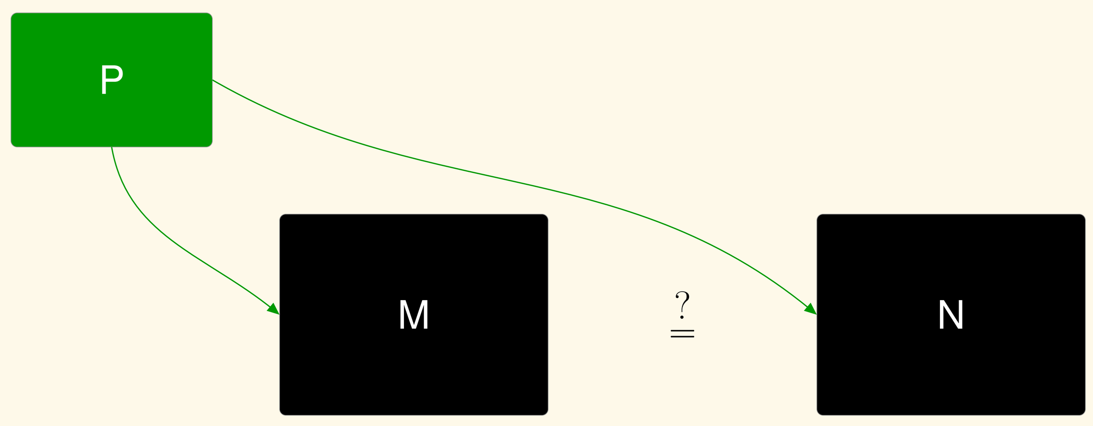
$$\mathcal{B} \vdash M = N$$

$$\Longleftrightarrow$$



The ω-rule

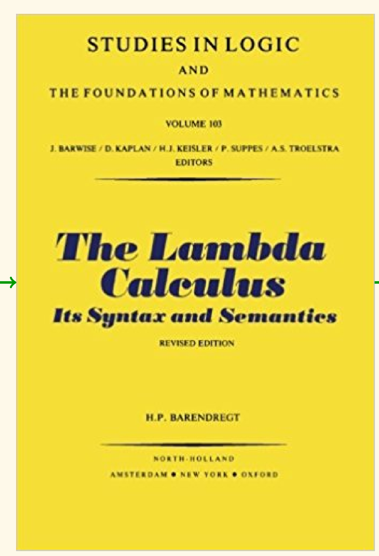
$$\forall P \in \Lambda^o \,.\, MP = NP \Longrightarrow M = N$$



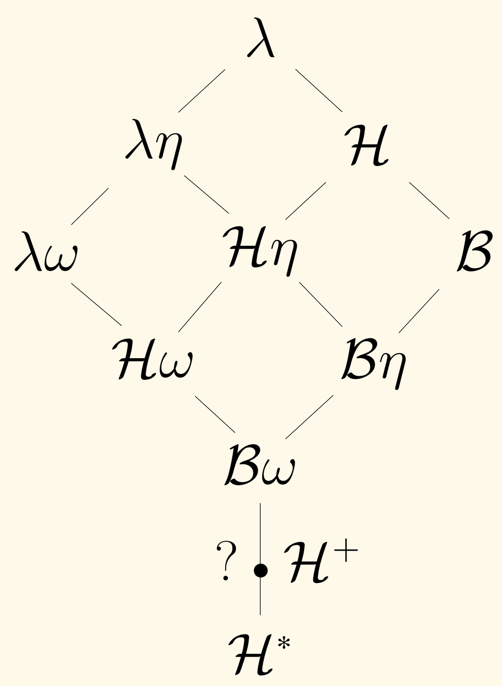
Sallé's Conjecture — 1979

Lambda Calculus Conference in Swansea 1979

Theorem 17.4.16



The Kite of λ-Theories



The Refutation — 2017

The Recipe

Ingredients

$\mathcal{B}\omega \subseteq \mathcal{H}^+$:

- The characterisations of $\mathcal{H}^+, \mathcal{H}^*$ in terms of extensional Böhm trees,
- a weak semi-separation theorem,
- Böhm-out technique (q.b.).

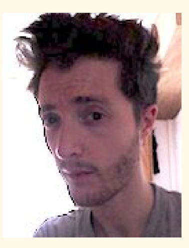
$\mathcal{H}^+ \subseteq \mathcal{B}\omega$:

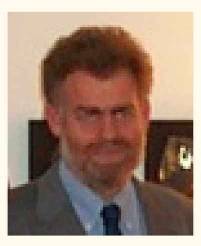
- Self-interpreters,
- a pinch of recursion theory,
- a key property of solvable terms.

An International Collaboration


Breuvart
LIPN

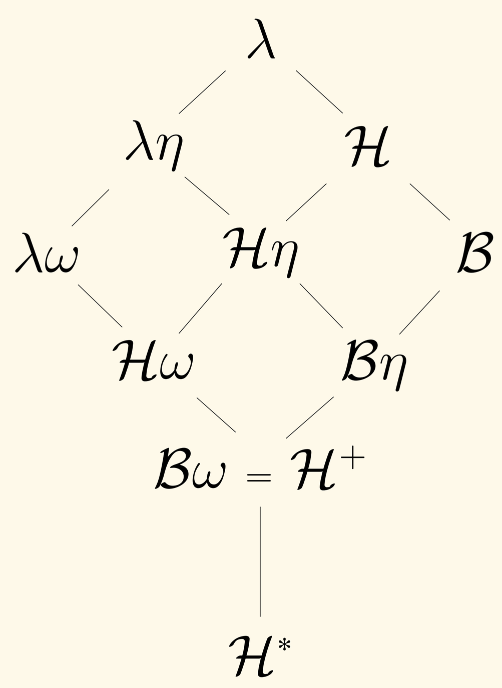

Manzonetto
LIPN


Ruoppolo
LIPN


Intrigila
Univ. Tor Vergata


Polonsky
North Carolina

The New Kite



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