

A LAMBDA CALCULUS SATELLITE

Henk Barendregt & Giulio Manzonetto

Contributors

Henk Barendregt

Institute of Computing & Information Science
Radboud University Nijmegen

All parts, except
Chapters 5, 7, §1.2

Stefano Guerrini

Northern Paris Computer Science Laboratory
CNRS & LIPN, University Sorbonne Paris Nord

Chapter 5, §1.2

Giulio Manzonetto

Northern Paris Computer Science Laboratory
CNRS & LIPN, University Sorbonne Paris Nord

All parts, except
Chapters 5, 7, §1.2

Vincent Padovani

Research Institute on the Foundations of Computer Science
CNRS & IRIF, University Paris Cité

Chapter 7

Contents in short

Acknowledgements	xv
About λ -calculus	1
About this book	13
General notations	23

I Preliminaries

1 The λ -calculus in a nutshell	29
2 Böhm trees and variations	57
3 Theories and models of λ -calculus	79

II Reduction

4 Leaving a β -reduction plane	107
5 Optimal lambda reduction	117
6 Infinitary lambda calculus	163
7 Starlings	199

III Conversion

8 Perpendicular Lines Property	239
9 Bijectivity and invertibility in $\lambda\eta$	251

IV Theories

10 Sensible theories	293
11 The kite	311

V Models

12 Ordered models and theories	347
13 Filter models	367
14 Relational models	399
15 Church algebras for λ -calculus	435

VI Open Problems

16 Open Problems	463
------------------------	-----

VII Appendix

A Mathematical background	489
References	550
Indices	551

Contents

Acknowledgements	xv	5.1 Families of redexes	120
About λ-calculus	1	5.2 Extraction	125
About this book	13	5.3 The labeled λ -calculus	140
General notations	23	5.4 Optimal reductions	153
I Preliminaries		5.5 Sharing graphs	161
1 The λ-calculus in a nutshell	29	6 Infinitary lambda calculus	163
1.1 The λ -calculus — Its syntax	29	6.1 The infinitary λ -calculus	163
1.2 Properties of reduction	37	6.2 Relative computability	175
1.3 RuS and consequences	51	6.3 Restoring confluence	180
2 Böhm trees and variations	57	6.4 Extensional infinitary λ -calculi	189
2.1 About coinduction	57	7 Starlings	199
2.2 Numerical sequences and trees	63	7.1 The S -fragment of CL	201
2.3 Böhm(-like) trees	64	7.2 Normalization	206
2.4 Variations of Böhm trees	70	7.3 Infinite reductions	211
3 Theories and models of λ-calculus	79	7.4 Head-normalization is decidable	220
3.1 The lattice of λ -theories	79	7.5 The word problem for S	223
3.2 Denotational models	86	7.6 Non-normalizing patterns	228
II Reduction		7.7 Translating S -terms into λ -calculus	231
4 Leaving a β-reduction plane	107	III Conversion	
4.1 Planes and cyclic reductions	109	8 Perpendicular Lines Property	239
4.2 Recurrent terms and the Plane Property	112	8.1 Validity of PLP in $\mathcal{M}(\lambda)$	241
5 Optimal lambda reduction	117	8.2 $\mathcal{M}(\mathcal{B}), \mathcal{M}^\circ(\mathcal{B}) \models \text{PLP}$	242
		8.3 Invalidity of PLP in $\mathcal{M}^\circ(\lambda)$	243
		9 Bijectivity and invertibility in $\lambda\eta$	251
		9.1 Equi-unsolvability	255
		9.2 Invertibility in $\mathcal{M}^\circ(\lambda\eta)$	266
		9.3 Partial characterizations of L/R-invertibility	271

IV Theories

10 Sensible theories	293
10.1 The range property fails in \mathcal{H}	295
10.2 The FPP fails in sensible theories	305
11 The kite	311
11.1 Degrees of extensionality for Böhm trees	313
11.2 \mathcal{H}^+ satisfies the ω -rule	318
11.3 Characterizing \mathcal{H}^+	327
11.4 A characterization of $\mathcal{B}\eta$	335

V Models

12 Ordered models and theories	347
12.1 Inequational theories and ordered models	348
12.2 Extensional orders on Böhm trees	352
13 Filter models	367
13.1 Intersection type systems	370
13.2 Filter models in logical form	379
13.3 Filter models: some case-studies	389
14 Relational models	399
14.1 The class of relational graph models	401
14.2 Tensor type assignment systems	407
14.3 $\lambda_{\otimes}^{\text{HNPR}}$ — A case study	418
14.4 Rgms in logical form	427
14.5 Relational graph theories	429

15 Church algebras for λ-calculus	435
15.1 Algebras and Boolean products	438
15.2 Church algebras	441
15.3 Easiness in universal algebra	447
15.4 Applications to λ -calculus models and theories	450
15.5 The main semantics are hugely incomplete	456

VI Open Problems

16 Open Problems	463
16.1 Reduction and conversion	463
16.2 Models and theories	470
16.3 $\lambda(\mathcal{H})\omega$ in the analytical hierarchy	476
16.4 Illative Combinatory Logic	481

VII Appendix

A Mathematical background	489
A.1 The lean notation for λ -terms	491
A.2 A summary of category theory	495
A.3 A summary of domain theory	501
A.4 A summary of universal algebra	509

References	550
-------------------	------------

Indices	551
Index of definitions	552
Index of names	565
Index of symbols	572