Reversible Mealy Automata and the Burnside Problem

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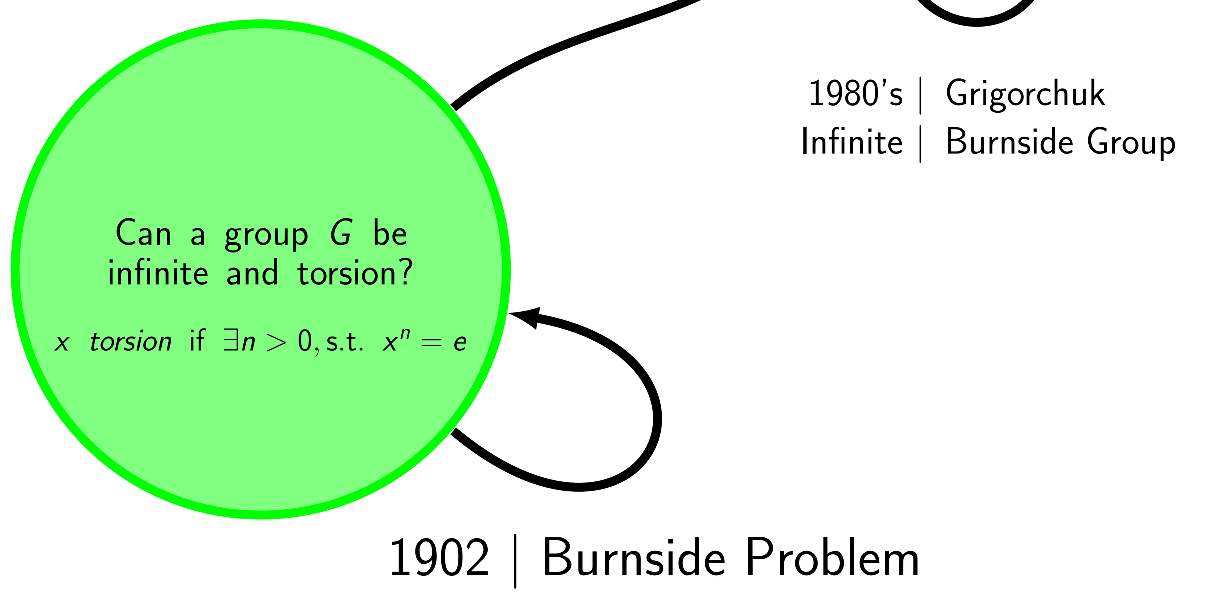
Mealy Automaton

A Mealy automaton $\mathcal{A}$ is a 4-tuple $(Q, \Sigma, \delta, \rho)$ where:
Q is a finite set, the state set
Σ is a finite set, the alphabet
$\delta = (\delta_i)_{i\in\Sigma}$ with $\delta_i : Q \to Q$ a function, the transition function
$\rho = (\rho_q)_{q\in Q}$ with $\rho_q : \Sigma \to \Sigma$ a function, the production function

An automaton is said to be:
Invertible when ρ_q is a permutation $\forall q \in Q$
Reversible when δ_x is a permutation $\forall x \in \Sigma$
Coreversible when $\tilde{\delta}_x$ associate to the output letter x is a permutation $\forall x \in \Sigma$
Bireversible when both invertible, reversible and coreversible

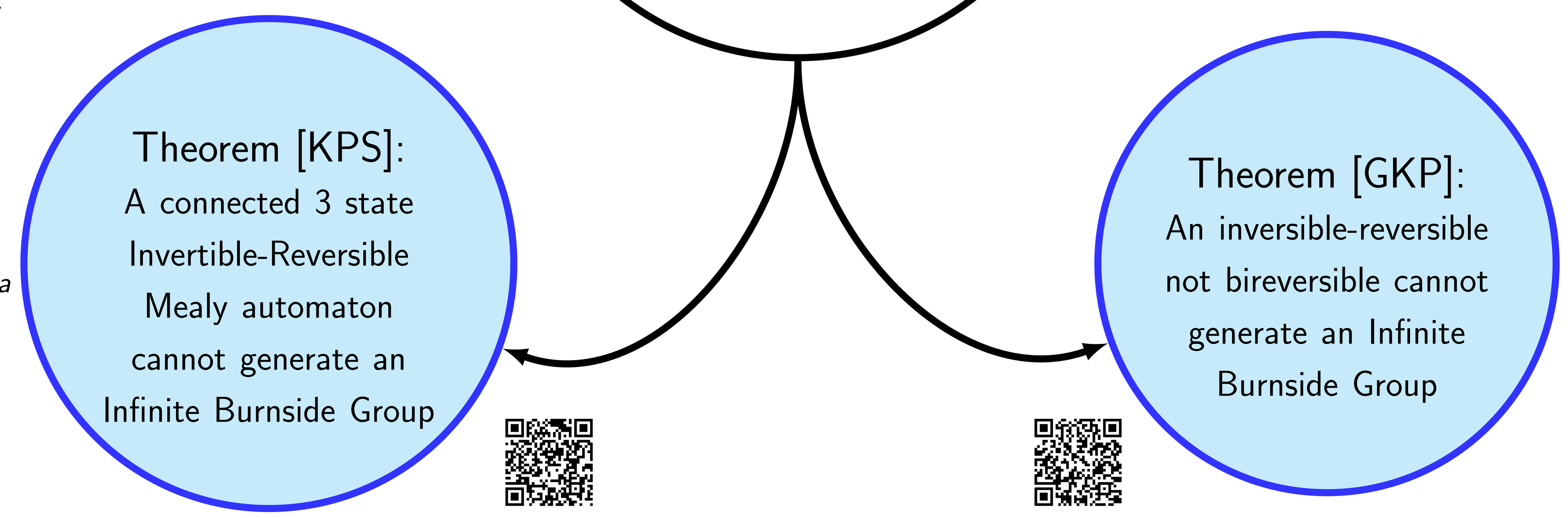


Size of Connected Components

More generally if $u \in Q^*$ then ρ_u is torsion iff the sequence of the sizes of the connected components containing u^i is bounded.

Size Ratio between Connected Components

If $\mathcal{A}$ is Reversible then the size-ratio between a connected component and its ancestors is always an integer



Automaton Group

The group generated by an automaton $\mathcal{A}$ is $\langle \mathcal{A} \rangle = \langle \rho_q \mid q \in Q \rangle = \{\rho_u \mid u \in Q^*\}$. Any finite group can by generated by a Mealy automaton. Moreover automaton groups have been useful in several group theoretical problems, such as Day, Gromov or Atiyah.

We focused on the well-known Burnside problem (1902), consisting to know whether a finitely generated group can be both infinite and torsion. It was solved in 1964 by Golod and Safarevich, but a much simpler example arises from automaton groups: the Grigorchuk group (discovered in 1980).

An interesting issue is to predict the properties of the group generated by a Mealy automaton. It is often a hard question, for instance even the finiteness problem was proved to be undecidable by Gillibert for semigroups (and the situation is still unknown for the group case). One can ask how the properties of the automaton impact the ones of the generated group.

Up to now every infinite Burnside automaton group is generated by an Invertible non-Reversible Mealy automaton, which leads to ask wheter a reversible automaton can generate such a group. Our work gives partial answers to this question.

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