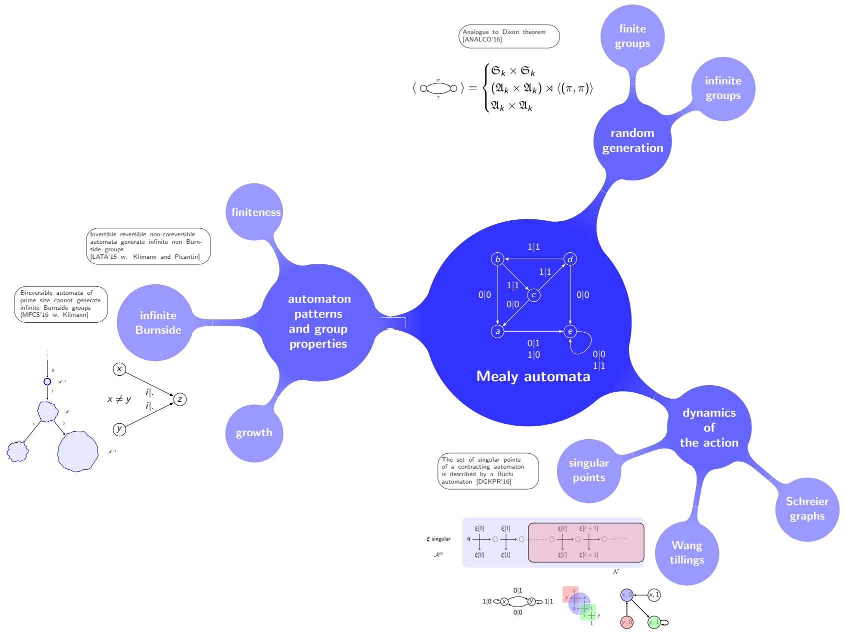
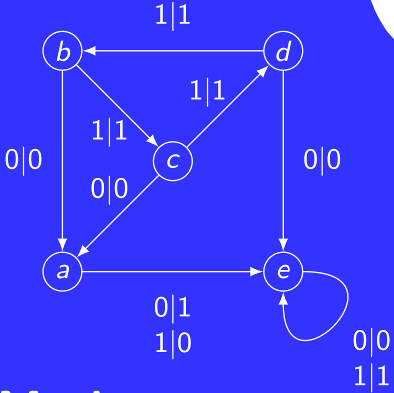
Mealy machines, automaton (semi)groups, decision problems, and random generation

Thibault Godin

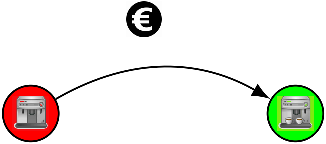
PhD defense, July 13, 2017

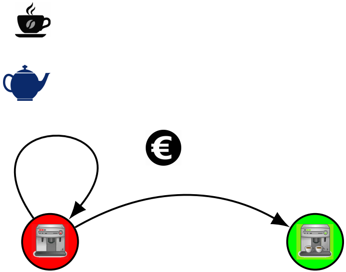
ANR JCJC 12 JS02 012 01

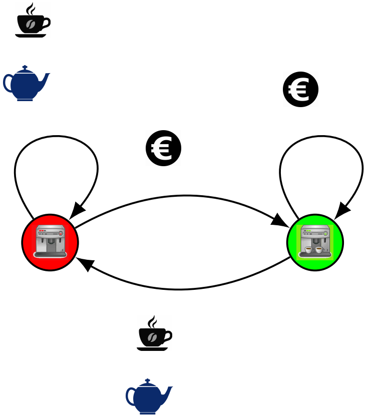

Mealy automata
1|1
0|0
0|1
1|0
random generation
finite groups
infinite groups
Analogue to Dixon theorem [ANALCO'16]
automaton patterns and group properties
finiteness
infinite Burnside
growth
Invertible reversible non-coreversible automata generate infinite non Burnside groups [LATA'15 w. Klimann and Picantin]
Bireversible automata of prime size cannot generate infinite Burnside groups [MFCS'16 w. Klimann]
$x \neq y$
dynamics of the action
singular points
Wang tillings
Schreier graphs
The set of singular points of a contracting automaton is described by a Büchi automaton [DGKPR'16]
ξ singular

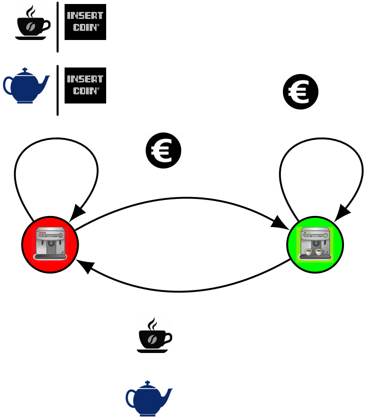


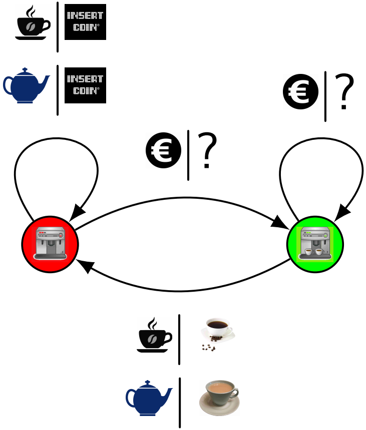
Mealy automata

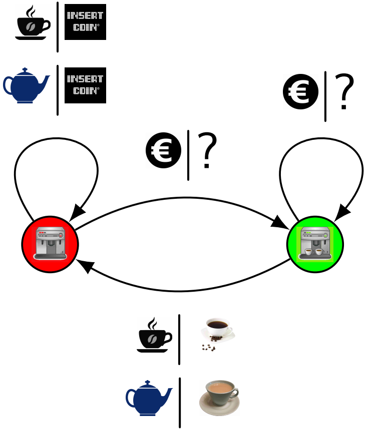


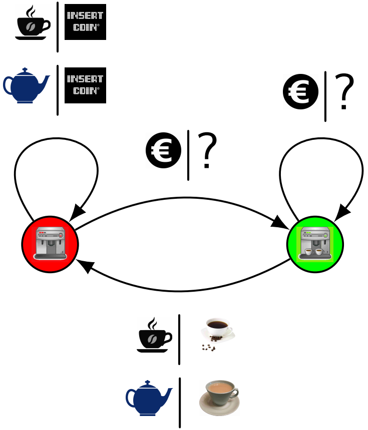




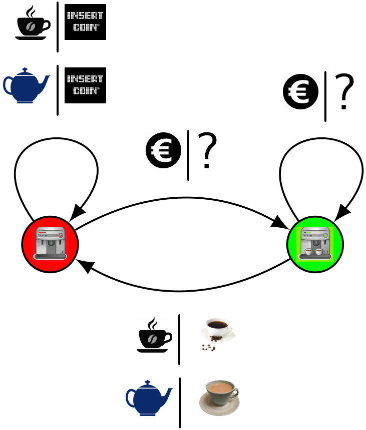

INSERT COIN
INSERT COIN

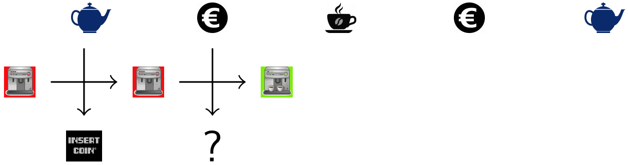

INSERT COIN
INSERT COIN
€ | ?
€ | ?

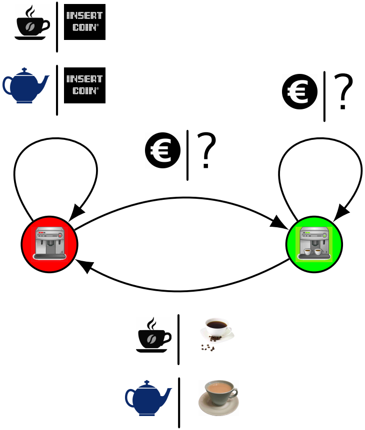

INSERT COIN
INSERT COIN
?
?

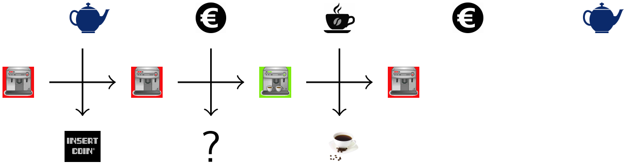

INSERT COIN
INSERT COIN
?
?

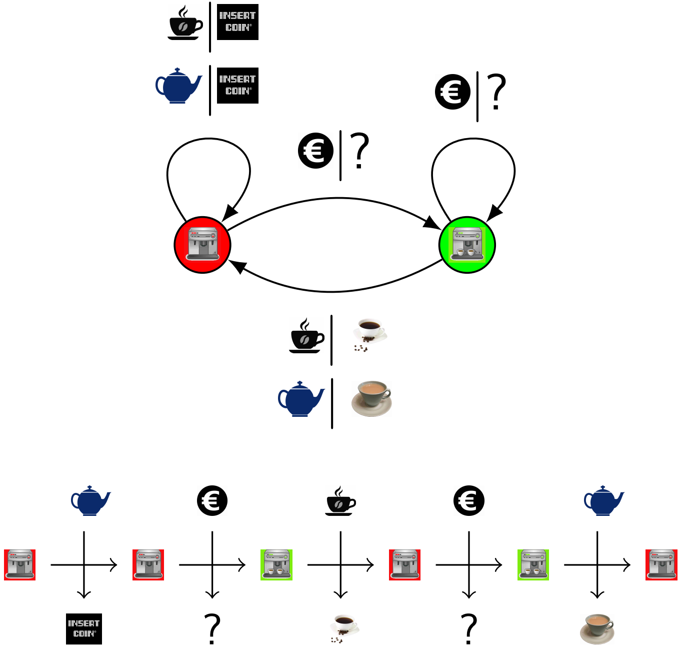

INSERT COIN

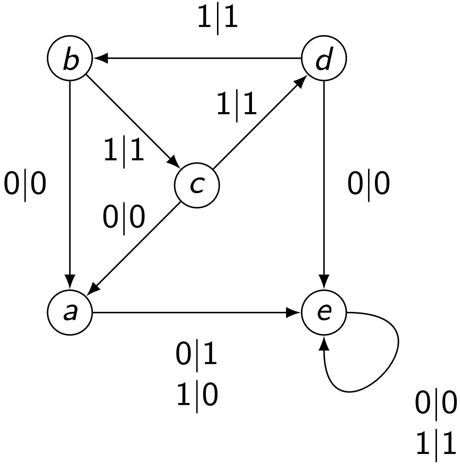

INSERT COIN
INSERT COIN
?
?


INSERT COIN
?


INSERT COIN
INSERT COIN
?
?

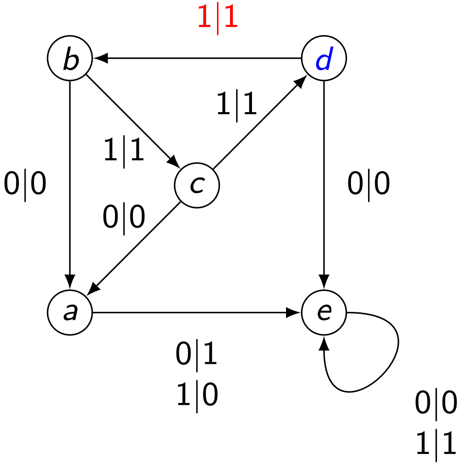

INSERT COIN
?


INSERT COIN
INSERT COIN
?
?
INSERT COIN
?
?



Mealy automaton $\mathcal{G}$

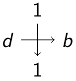
$\mathcal{A} = (Q, \Sigma, \delta, \rho)$

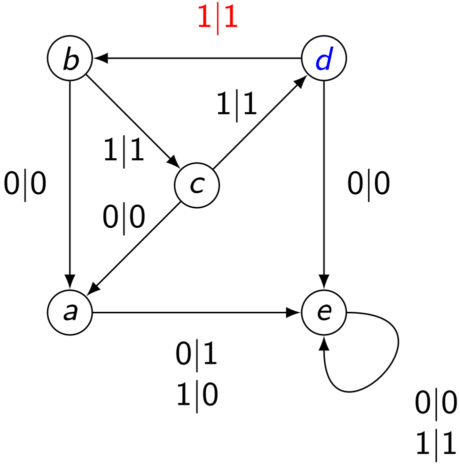


Mealy automaton $\mathcal{G}$

$$\mathcal{A} = (Q, \Sigma, \delta, \rho)$$

$$\rho_q : \Sigma \ \to \Sigma \ , \ q \in Q$$



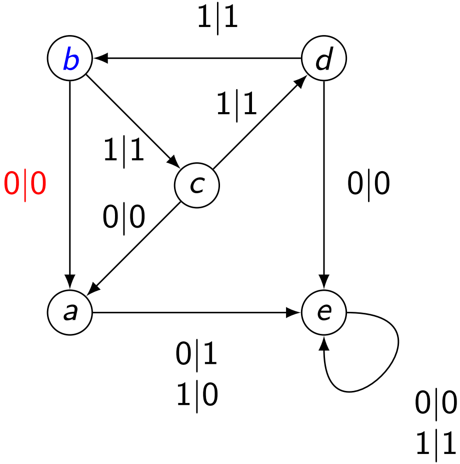


Mealy automaton $\mathcal{G}$

$\mathcal{A} = (Q, \Sigma, \delta, \rho)$

$\rho_q : \Sigma^* \to \Sigma^*, \; q \in Q$

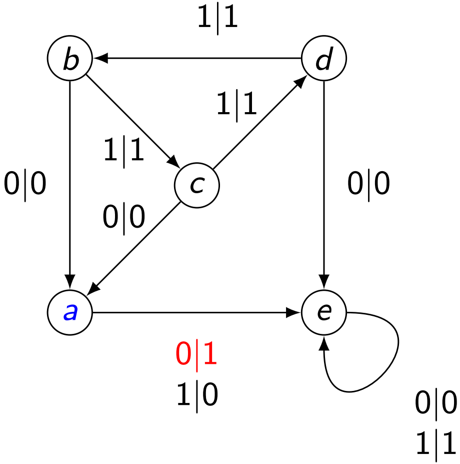
	1	0	0	0	1
d					



Mealy automaton $\mathcal{G}$

$$\mathcal{A} = (Q, \Sigma, \delta, \rho)$$

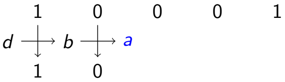
$$\rho_q : \Sigma^* \to \Sigma^*,\ q \in Q$$

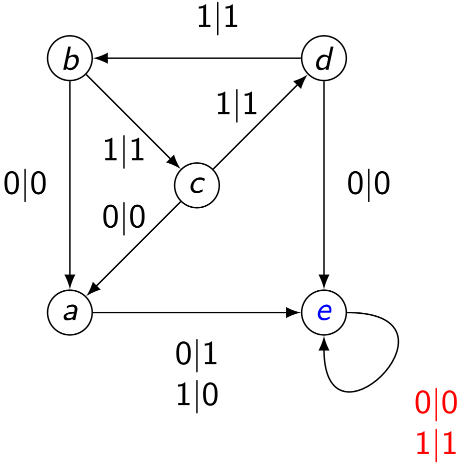


Mealy automaton $\mathcal{G}$

$$\mathcal{A} = (Q, \Sigma, \delta, \rho)$$

$$\rho_q : \Sigma^* \to \Sigma^*, \ q \in Q$$

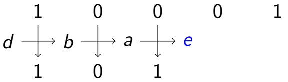


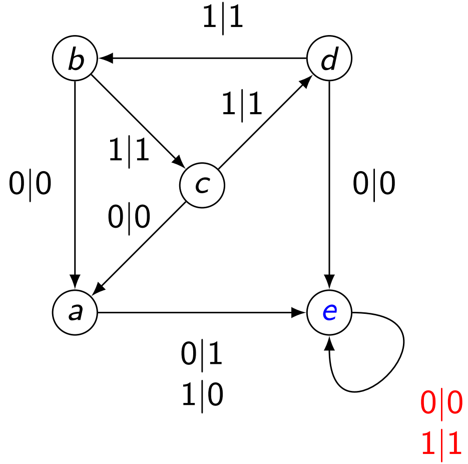


Mealy automaton $\mathcal{G}$

$$\mathcal{A} = (Q, \Sigma, \delta, \rho)$$

$$\rho_q : \Sigma^* \to \Sigma^*,\ q \in Q$$

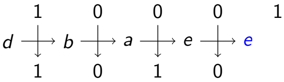


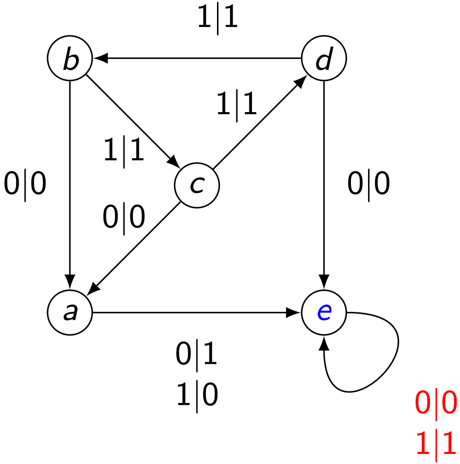


Mealy automaton $\mathcal{G}$

$$\mathcal{A} = (Q, \Sigma, \delta, \rho)$$

$$\rho_q : \Sigma^* \to \Sigma^*, \; q \in Q$$

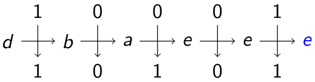


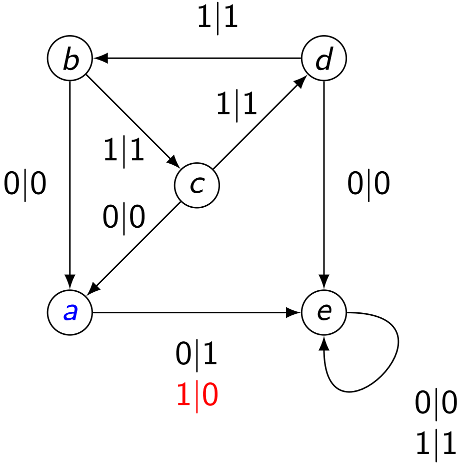


Mealy automaton $\mathcal{G}$

$\mathcal{A} = (Q, \Sigma, \delta, \rho)$

$\rho_q : \Sigma^* \to \Sigma^*,\ q \in Q$

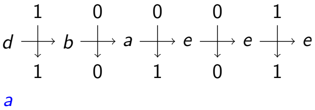




Mealy automaton $\mathcal{G}$

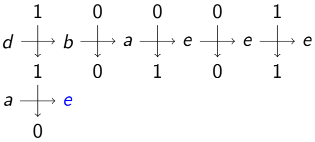
$$\mathcal{A} = (Q, \Sigma, \delta, \rho)$$

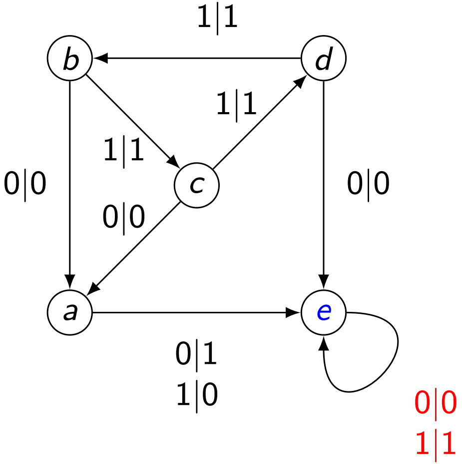
$$\rho_q : \Sigma^* \to \Sigma^*, \ q \in Q^*$$



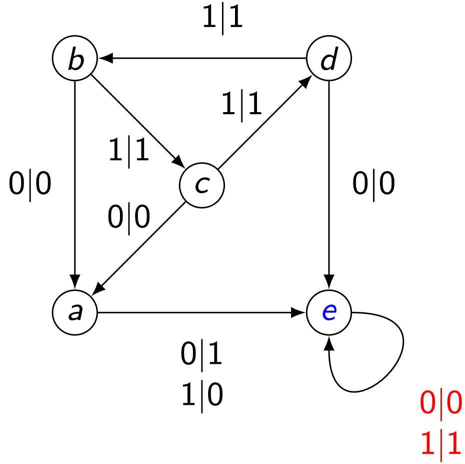
$\mathcal{A} = (Q, \Sigma, \delta, \rho)$

$\rho_q : \Sigma^* \to \Sigma^*, \; q \in Q^*$





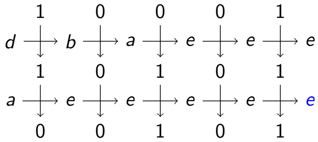
Mealy automaton $\mathcal{G}$

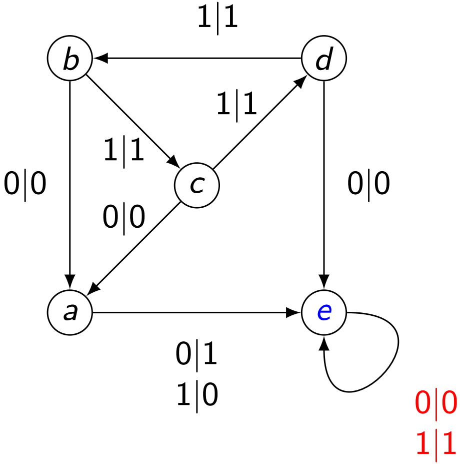


Mealy automaton $\mathcal{G}$

$$\mathcal{A} = (Q, \Sigma, \delta, \rho)$$

$$\rho_q : \Sigma^* \to \Sigma^*,\ q \in Q^*$$

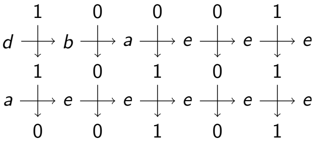




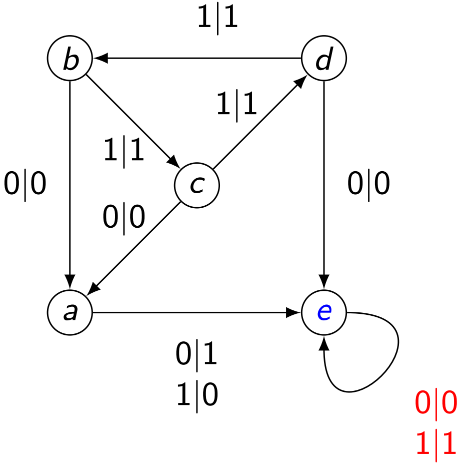
Mealy automaton $\mathcal{G}$

$$\mathcal{A} = (Q, \Sigma, \delta, \rho)$$

$$\rho_q : \Sigma^* \to \Sigma^*, \ q \in Q^*$$



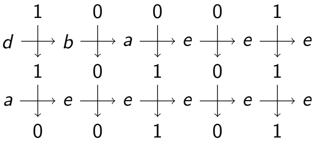
$$\rho_{da}(10001) = \rho_a(\rho_d(10001))$$



Mealy automaton $\mathcal{G}$

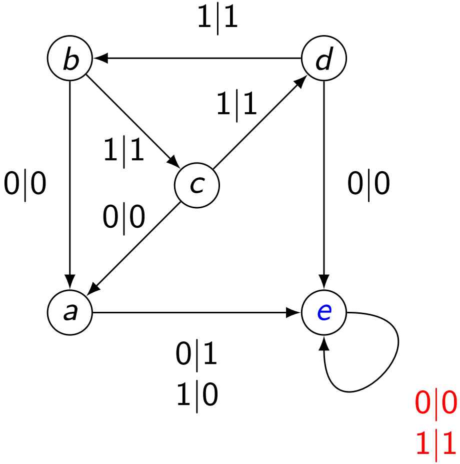
$$\mathcal{A} = (Q, \Sigma, \delta, \rho)$$

$$\rho_q : \Sigma^* \to \Sigma^*,\ q \in Q^*$$



$$\rho_{da}(10001) = \rho_a(\rho_d(10001))$$

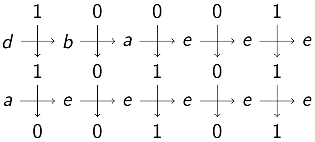
$$\langle \mathcal{A} \rangle := \langle \rho_q \mid q \in Q^* \rangle$$



Mealy automaton $\mathcal{G}$

$$\mathcal{A} = (Q, \Sigma, \delta, \rho)$$

$$\rho_q : \Sigma^* \to \Sigma^*, \; q \in Q^*$$



$$\rho_{da}(10001) = \rho_a(\rho_d(10001))$$

$$\langle \mathcal{A} \rangle := \langle \rho_q \mid q \in Q^* \rangle$$

da is a state of $\mathcal{G}^2$

Order

Order of an element

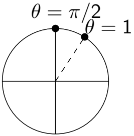
$x \in G$ has finite order if $\exists n \geq 1, x^n = \mathrm{e}$

Order

Order of an element

$x \in G$ has finite order if $\exists n \geq 1, x^n = e$

- $\mathbb{Z}/n\mathbb{Z}$: every element has finite order
- $\mathbb{Z}$: 0 is the only element of finite order
- On the circle $\mathbb{R}/2\pi\mathbb{Z}$: $\pi/2$ has finite order, but 1 has infinite order



The Burnside problem

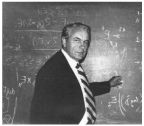
Burnside (1902):

Can a finitely generated group have all elements of finite order and be infinite?

The Burnside problem

Burnside (1902):

Can a finitely generated group have all elements of finite order and be infinite?

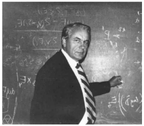


Golod and Shafarevich: yes! (1964)

The Burnside problem

Burnside (1902):

Can a finitely generated group have all elements of finite order and be infinite?



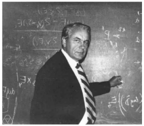
Golod and Shafarevich: yes! (1964)

Aleshin+Grigorchuk:
an example
generated by a
Mealy automaton
(1972+1980)

The Burnside problem

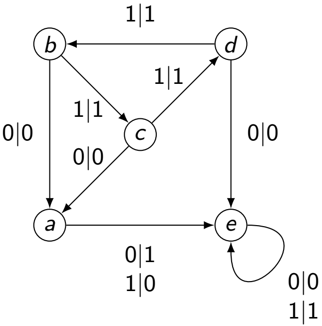
Burnside (1902):

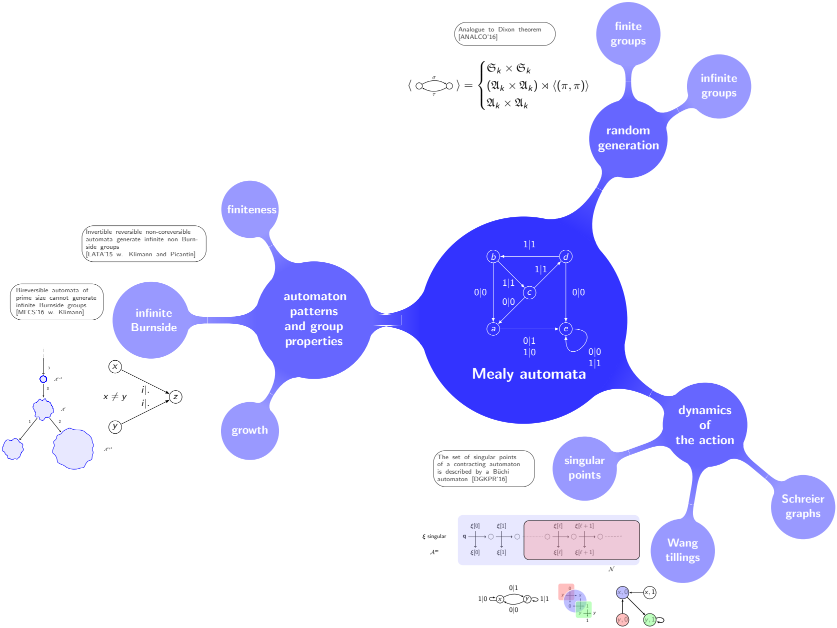
Can a finitely generated group have all elements of finite order and be infinite?



Golod and Shafarevich: yes! (1964)

Aleshin+Grigorchuk: an example generated by a Mealy automaton (1972+1980)




Analogue to Dixon theorem [ANALCO'16]
finite groups
infinite groups
random generation
Invertible reversible non-coreversible automata generate infinite non Burnside groups [LATA'15 w. Klimann and Picantin]
finiteness
Bireversible automata of prime size cannot generate infinite Burnside groups [MFCS'16 w. Klimann]
infinite Burnside
automaton patterns and group properties
growth
Mealy automata
dynamics of the action
singular points
Schreier graphs
Wang tillings
The set of singular points of a contracting automaton is described by a Büchi automaton [DGKPR'16]
ξ singular
x ≠ y

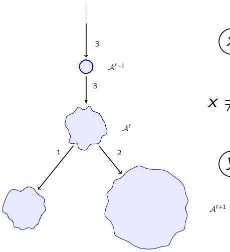
finiteness

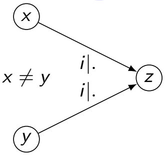
Invertible reversible non-coreversible automata generate infinite non Burnside groups
[LATA'15 w. Klimann and Picantin]

Bireversible automata of prime size cannot generate infinite Burnside groups
[MFCS'16 w. Klimann]

infinite Burnside

automaton patterns and group properties





growth

The set o
of a contr
is describe
automaton

Mealy automata

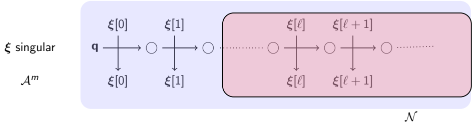
dynamics of the action

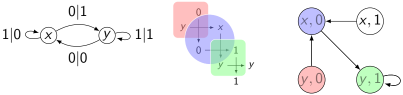
singular points

Wang tillings

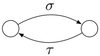
Schreier graphs

The set of singular points of a contracting automaton is described by a Büchi automaton [DGKPR'16]

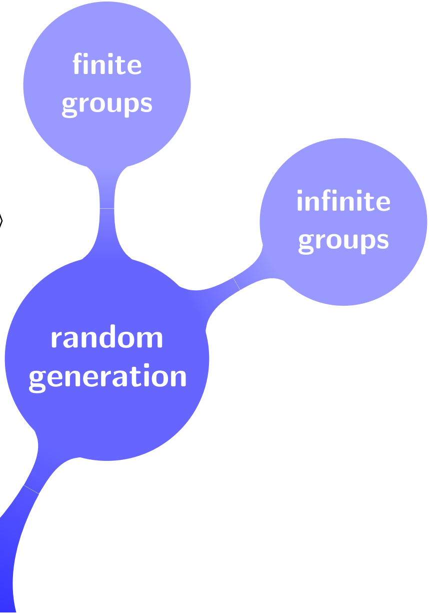




Analogue to Dixon theorem [ANALCO'16]

$\langle$  $\rangle$

$$= \begin{cases} \mathfrak{S}_k \times \mathfrak{S}_k \\ (\mathfrak{A}_k \times \mathfrak{A}_k) \rtimes \langle (\pi, \pi) \rangle \\ \mathfrak{A}_k \times \mathfrak{A}_k \end{cases}$$



Finite random groups

Theorem

Any finite group G is a subgroup of $\mathfrak{S}_{|G|}$.

Finite random groups

Theorem

Any finite group G is a subgroup of $\mathfrak{S}_{|G|}$.

First idea

Pick up some permutations $\sigma_1, \ldots, \sigma_n$ of $\{1, \ldots, k\}$, look at $\langle \sigma_1, \ldots, \sigma_n \rangle$.

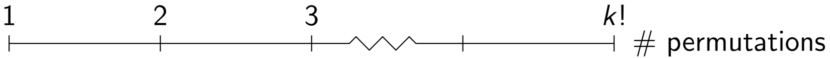
Finite random groups

Theorem

Any finite group G is a subgroup of $\mathfrak{S}_{|G|}$.

First idea

Pick up some permutations $\sigma_1, \ldots, \sigma_n$ of $\{1, \ldots, k\}$, look at $\langle \sigma_1, \ldots, \sigma_n \rangle$.



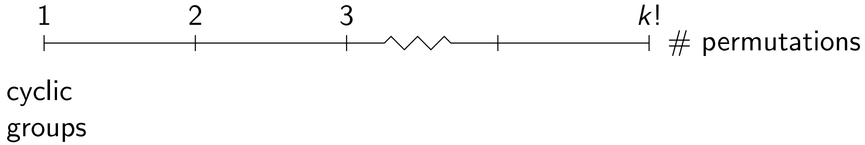
Finite random groups

Theorem

Any finite group G is a subgroup of $\mathfrak{S}_{|G|}$.

First idea

Pick up some permutations $\sigma_1, \ldots, \sigma_n$ of $\{1, \ldots, k\}$, look at $\langle \sigma_1, \ldots, \sigma_n \rangle$.



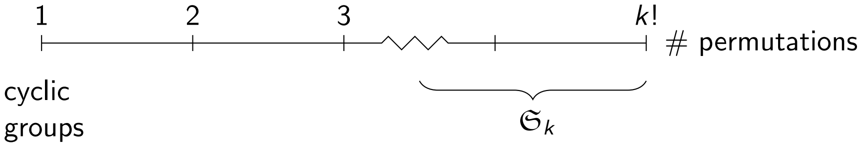
Finite random groups

Theorem

Any finite group G is a subgroup of $\mathfrak{S}_{|G|}$.

First idea

Pick up some permutations $\sigma_1, \ldots, \sigma_n$ of $\{1, \ldots, k\}$, look at $\langle \sigma_1, \ldots, \sigma_n \rangle$.



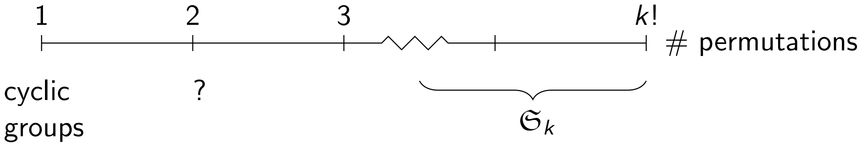
Finite random groups

Theorem

Any finite group G is a subgroup of $\mathfrak{S}_{|G|}$.

First idea

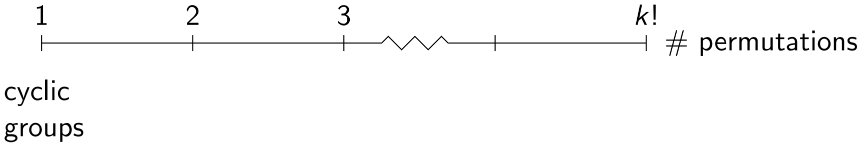
Pick up some permutations $\sigma_1, \ldots, \sigma_n$ of $\{1, \ldots, k\}$, look at $\langle \sigma_1, \ldots, \sigma_n \rangle$.



Finite random groups

Theorem (Dixon, 1969)

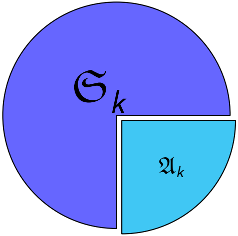
$$\text{w.g.p. } \langle \sigma, \tau \rangle = \begin{cases} \mathfrak{S}_k \\ \mathfrak{A}_k \end{cases}$$

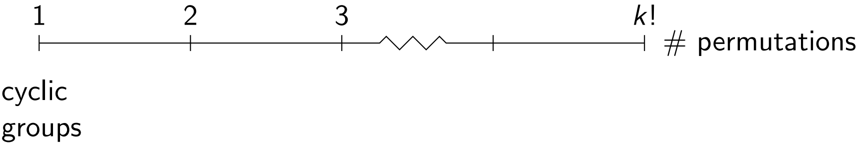


Finite random groups

Theorem (Dixon, 1969)

$$\text{w.g.p. } \langle \sigma, \tau \rangle = \begin{cases} \mathfrak{S}_k \\ \mathfrak{A}_k \end{cases}$$

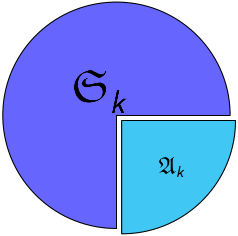


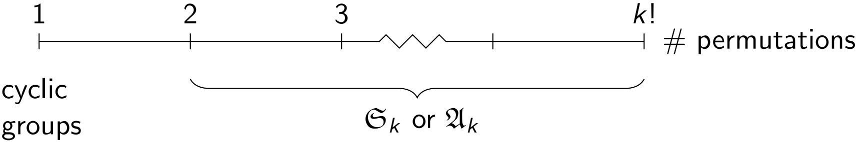


Finite random groups

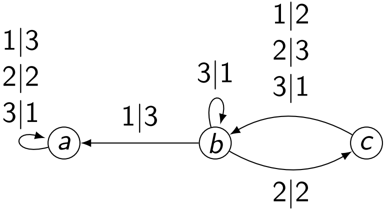
Theorem (Dixon, 1969)

$$\text{w.g.p. } \langle \sigma, \tau \rangle = \begin{cases} \mathfrak{S}_k \\ \mathfrak{A}_k \end{cases}$$



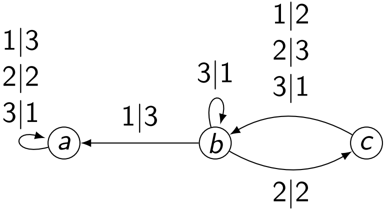


Random automata



Is the generated group finite?

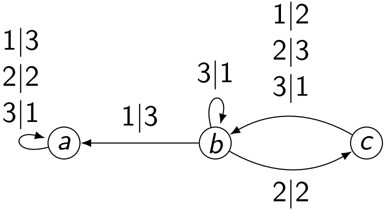
Random automata



Is the generated group finite?

Yes, size $2^{64} \cdot 3^4$.

Random automata

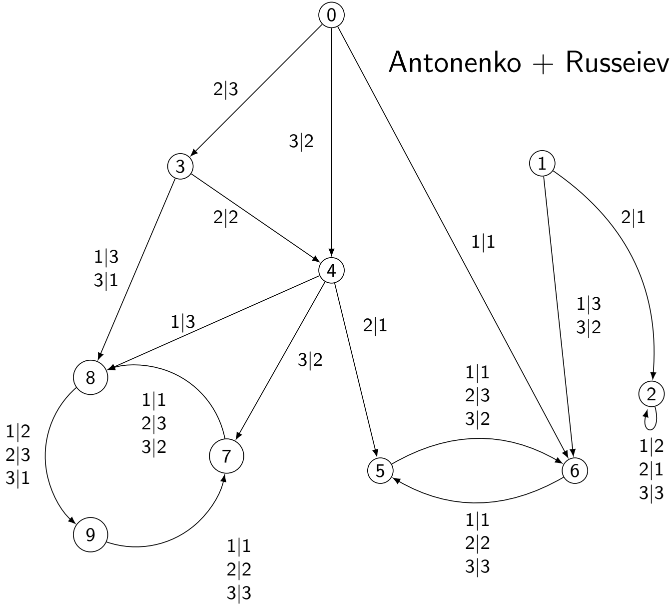


Is the generated group finite?

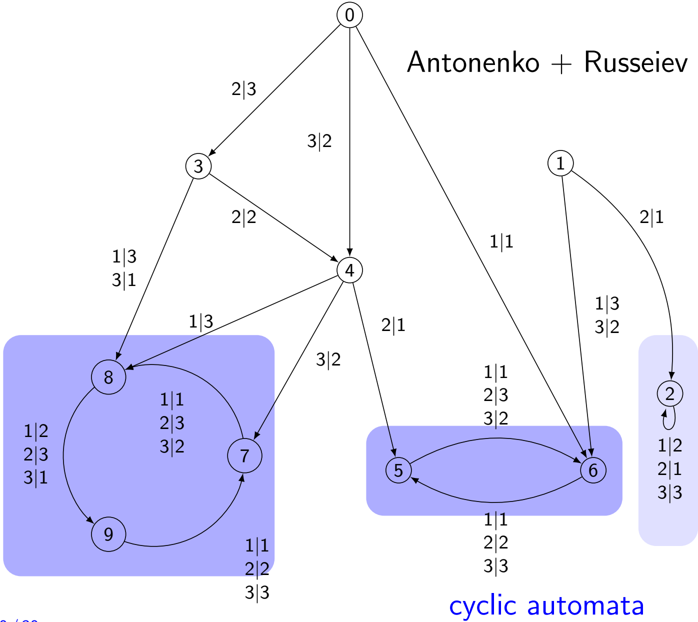
Yes, size $2^{64} \cdot 3^4$.

Difficult problem + unefficient rejection sampling.

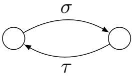
Random automata



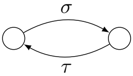
Random automata



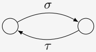
Random 2-state cyclic automata

$\langle$  $\rangle = \langle(\sigma, \tau), (\tau, \sigma)\rangle$

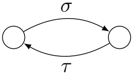
Random 2-state cyclic automata

$\langle$  $\rangle = \langle (\sigma, \tau), (\tau, \sigma) \rangle$

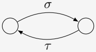
Contribution

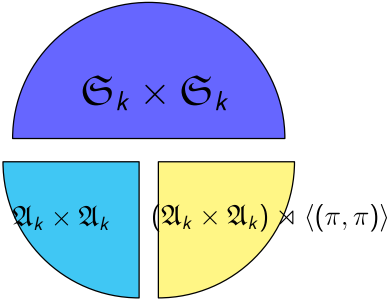
$\langle$  $\rangle = \begin{cases} \mathfrak{S}_k \times \mathfrak{S}_k \\ (\mathfrak{A}_k \times \mathfrak{A}_k) \rtimes \langle (\pi, \pi) \rangle \\ \mathfrak{A}_k \times \mathfrak{A}_k \end{cases}$

Random 2-state cyclic automata

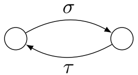
$\langle$  $\rangle = \langle(\sigma,\tau),(\tau,\sigma)\rangle$

Contribution

$\langle$  $\rangle = \begin{cases} \mathfrak{S}_k \times \mathfrak{S}_k \\ (\mathfrak{A}_k \times \mathfrak{A}_k) \rtimes \langle(\pi,\pi)\rangle \\ \mathfrak{A}_k \times \mathfrak{A}_k \end{cases}$

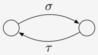


Random 2-state cyclic automata

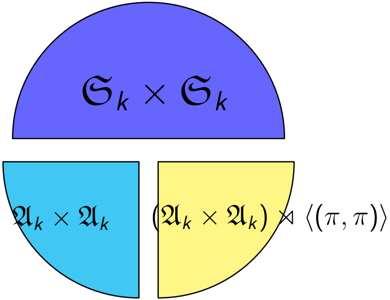


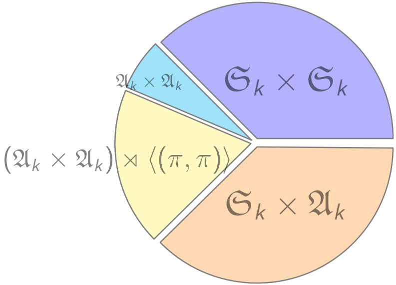
$$\langle \quad \rangle = \langle (\sigma, \tau), (\tau, \sigma) \rangle$$

Contribution

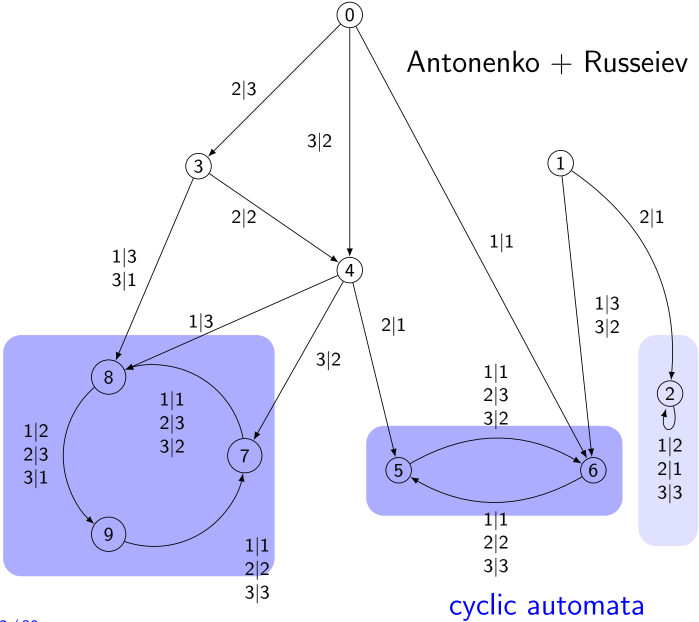


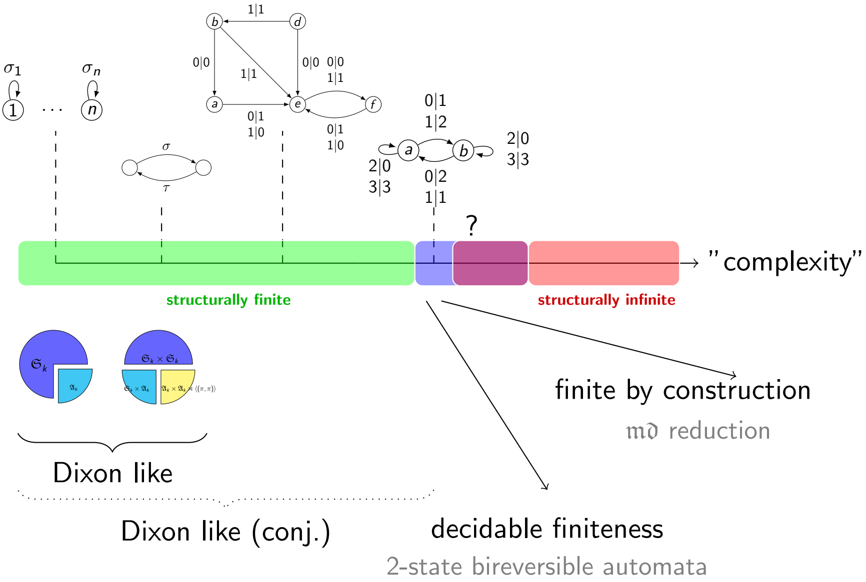
$$\langle \quad \rangle = \begin{cases} \mathfrak{S}_k \times \mathfrak{S}_k \\ (\mathfrak{A}_k \times \mathfrak{A}_k) \rtimes \langle (\pi, \pi) \rangle \\ \mathfrak{A}_k \times \mathfrak{A}_k \end{cases}$$





Random automata





1|1
0|0
1|1
0|0 0|0
1|1
0|1
1|0
0|1
1|0
σ1
σn
1
n
σ
τ
0|1
1|2
2|0
3|3
2|0
3|3
0|2
1|1
?
"complexity"
structurally finite
structurally infinite
Dixon like
Dixon like (conj.)
finite by construction
𝔪𝔡 reduction
decidable finiteness
2-state bireversible automata

1|0 0|0 1|1

Mealy automata

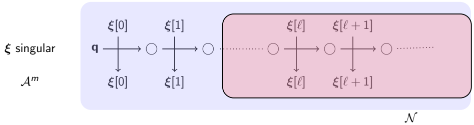
dynamics of the action

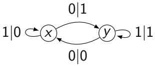
singular points

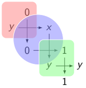
Wang tillings

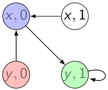
Schreier graphs

The set of singular points of a contracting automaton is described by a Büchi automaton [DGKPR'16]







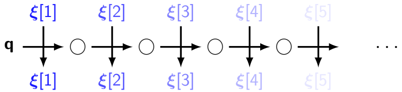


Stabilisers and singular points

The stabilisers of an infinite point ξ is $\mathsf{Stab}_{\langle\mathcal{A}\rangle}(\xi) = \{g \in \langle\mathcal{A}\rangle \mid g(\xi) = \xi\}$

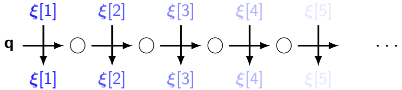
Stabilisers and singular points

The stabilisers of an infinite point ξ is $\mathrm{Stab}_{\langle\mathcal{A}\rangle}(\xi) = \{g \in \langle\mathcal{A}\rangle \mid g(\xi) = \xi\}$

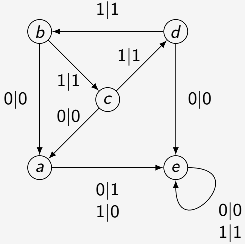


Stabilisers and singular points

The stabilisers of an infinite point ξ is $\text{Stab}_{\langle\mathcal{A}\rangle}(\xi) = \{g \in \langle\mathcal{A}\rangle \mid g(\xi) = \xi\}$



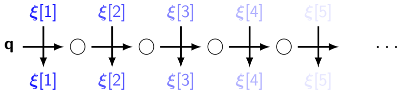
Example



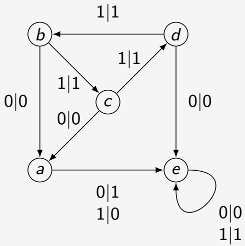
$\rho_e, \rho_b, \rho_c, \rho_d \in \text{Stab}_{\langle\mathcal{G}\rangle}(1^\omega)$
studied by Y. Vorobets

Stabilisers and singular points

The stabilisers of an infinite point ξ is $\mathrm{Stab}_{\langle\mathcal{A}\rangle}(\xi) = \{g \in \langle\mathcal{A}\rangle \mid g(\xi) = \xi\}$

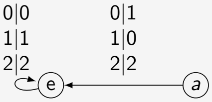


Example



$\rho_e, \rho_b, \rho_c, \rho_d \in \mathrm{Stab}_{\langle\mathcal{G}\rangle}(1^\omega)$
studied by Y. Vorobets

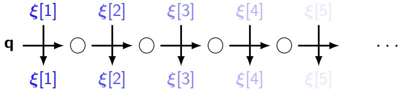
Interesting elements



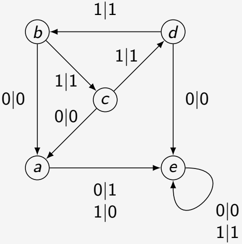
2^ω is stabilised by ρ_a

Stabilisers and singular points

The stabilisers of an infinite point ξ is $\mathrm{Stab}_{\langle\mathcal{A}\rangle}(\xi) = \{g \in \langle\mathcal{A}\rangle \mid g(\xi) = \xi\}$

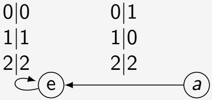


Example



$\rho_e, \rho_b, \rho_c, \rho_d \in \mathrm{Stab}_{\langle\mathcal{G}\rangle}(1^\omega)$
studied by Y. Vorobets

Interesting elements



2^ω is stabilised by ρ_a

Singular points

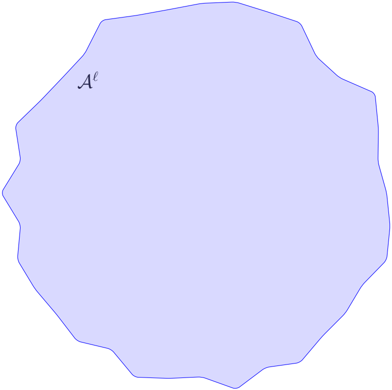
ξ singular if $\exists g$ stabilizing ξ and avoiding ending in e

Contracting automata

$\mathcal{A}$ contracting $\iff \exists$ finite $\mathcal{N}$, $\forall \mathbf{q}, \forall \boldsymbol{\xi}, \exists n,\ \delta_{\boldsymbol{\xi}[:n]}(\mathbf{q}) \in \mathcal{N}$

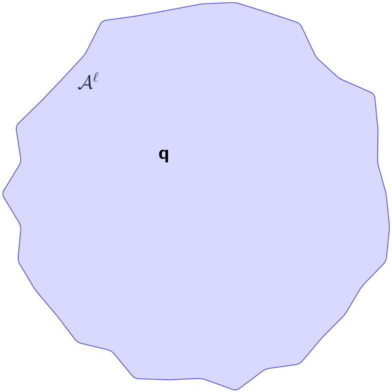
Contracting automata

$\mathcal{A}$ contracting $\iff \exists$ finite $\mathcal{N}$, $\forall \mathbf{q}, \forall \boldsymbol{\xi}, \exists n,\ \delta_{\boldsymbol{\xi}[:n]}(\mathbf{q}) \in \mathcal{N}$



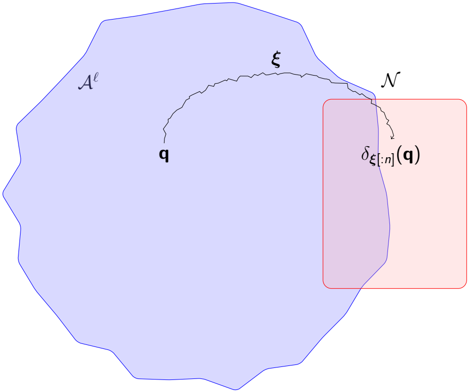
Contracting automata

$\mathcal{A}$ contracting $\iff \exists$ finite $\mathcal{N}$, $\forall \mathbf{q}, \forall \boldsymbol{\xi}, \exists n,\ \delta_{\boldsymbol{\xi}[:n]}(\mathbf{q}) \in \mathcal{N}$



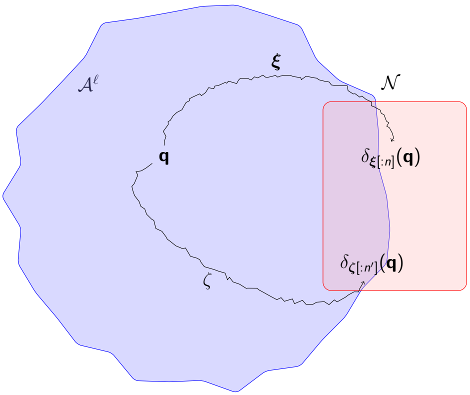
Contracting automata

$\mathcal{A}$ contracting $\iff \exists$ finite $\mathcal{N}$, $\forall \mathbf{q}, \forall \boldsymbol{\xi}, \exists n,\ \delta_{\boldsymbol{\xi}[:n]}(\mathbf{q}) \in \mathcal{N}$



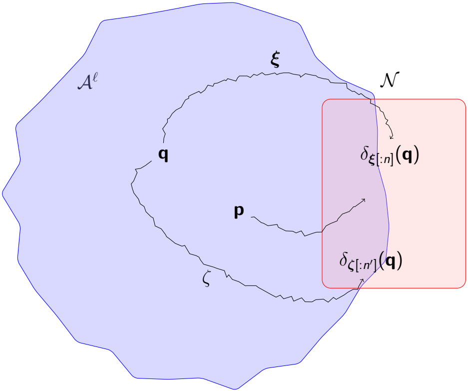
Contracting automata

$\mathcal{A}$ contracting $\iff \exists$ finite $\mathcal{N}$, $\forall \mathbf{q}, \forall \boldsymbol{\xi}, \exists n,\ \delta_{\boldsymbol{\xi}[:n]}(\mathbf{q}) \in \mathcal{N}$



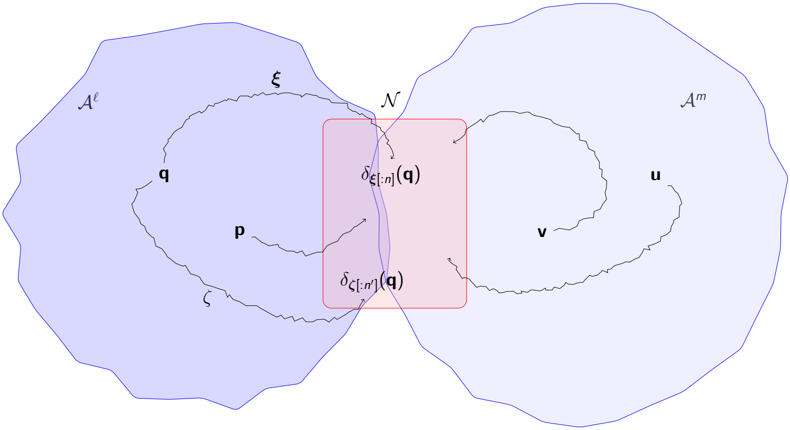
Contracting automata

$\mathcal{A}$ contracting $\iff \exists$ finite $\mathcal{N}$, $\forall \mathbf{q}, \forall \boldsymbol{\xi}, \exists n,\ \delta_{\boldsymbol{\xi}[:n]}(\mathbf{q}) \in \mathcal{N}$



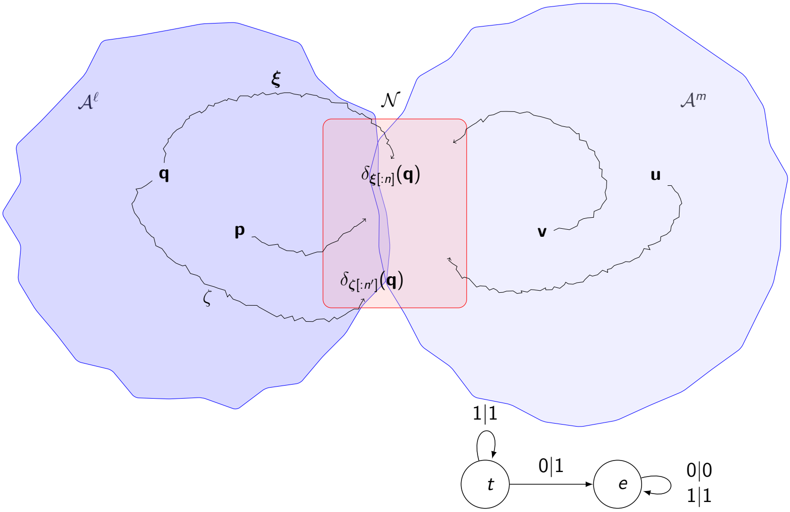
Contracting automata

$\mathcal{A}$ contracting $\iff \exists$ finite $\mathcal{N}$, $\forall \mathbf{q}, \forall \boldsymbol{\xi}, \exists n,\ \delta_{\boldsymbol{\xi}[:n]}(\mathbf{q}) \in \mathcal{N}$



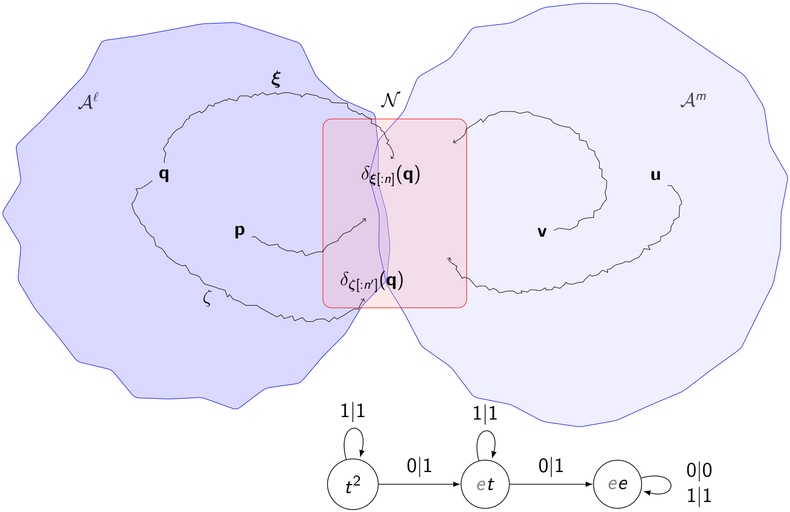
Contracting automata

$\mathcal{A}$ contracting $\iff \exists$ finite $\mathcal{N}$, $\forall \mathbf{q}, \forall \boldsymbol{\xi}, \exists n,\ \delta_{\boldsymbol{\xi}[:n]}(\mathbf{q}) \in \mathcal{N}$



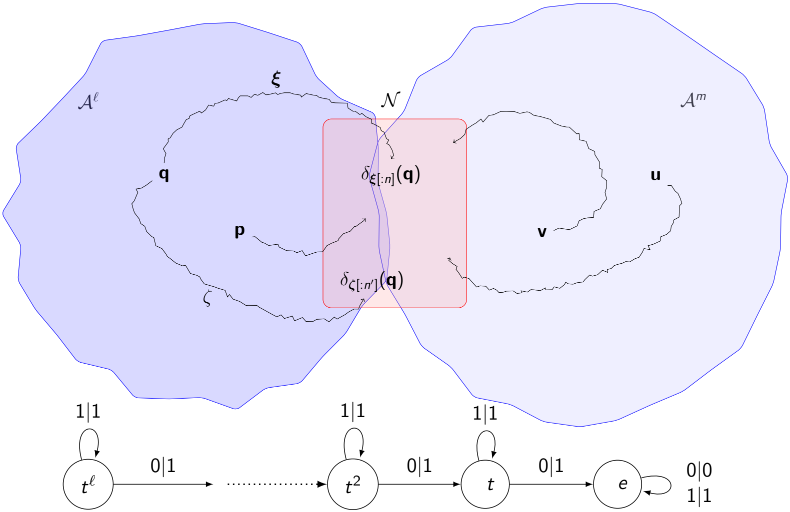
Contracting automata

$\mathcal{A}$ contracting $\iff \exists$ finite $\mathcal{N}$, $\forall \mathbf{q}, \forall \boldsymbol{\xi}, \exists n,\ \delta_{\boldsymbol{\xi}[:n]}(\mathbf{q}) \in \mathcal{N}$

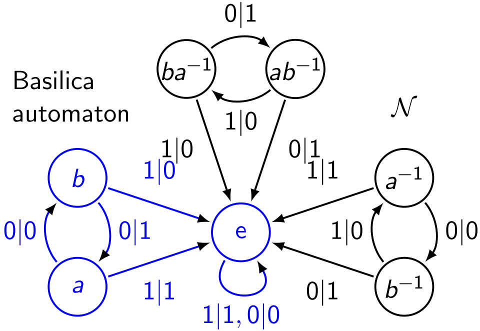


Contracting automata

$\mathcal{A}$ contracting $\iff \exists$ finite $\mathcal{N}$, $\forall \mathbf{q}, \forall \boldsymbol{\xi}, \exists n,\ \delta_{\boldsymbol{\xi}[:n]}(\mathbf{q}) \in \mathcal{N}$



Contracting automata and singular points



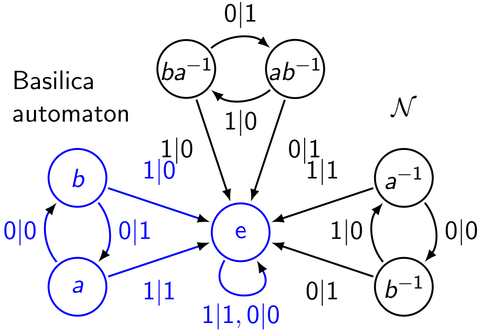
Contracting automata and singular points

0 0

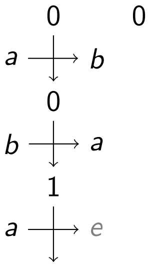
a

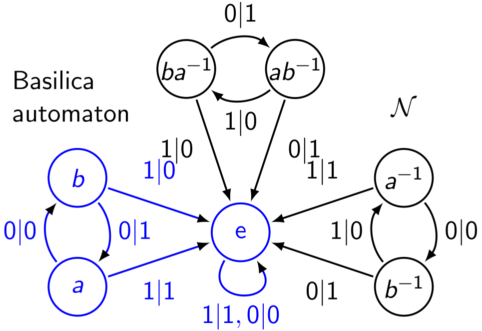
b

a

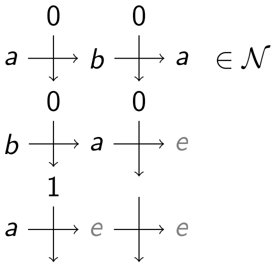


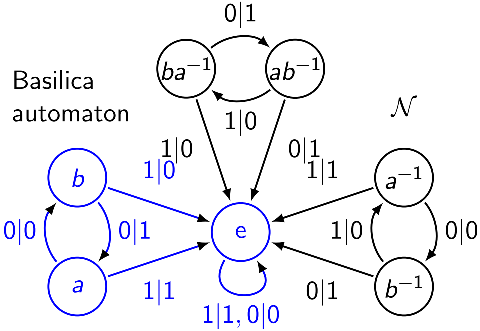
Contracting automata and singular points





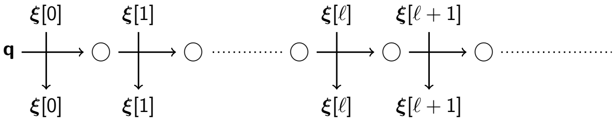
Contracting automata and singular points

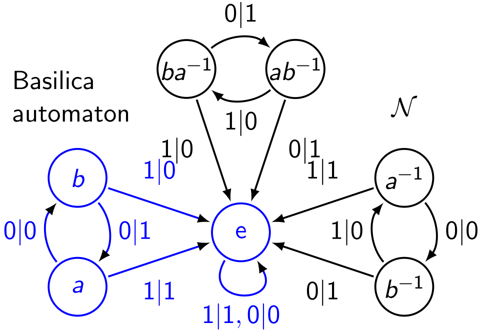




Contracting automata and singular points

$\boldsymbol{\xi}$ singular

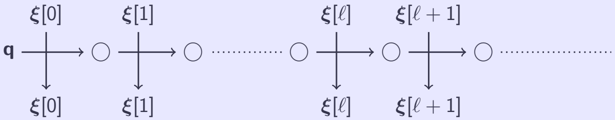


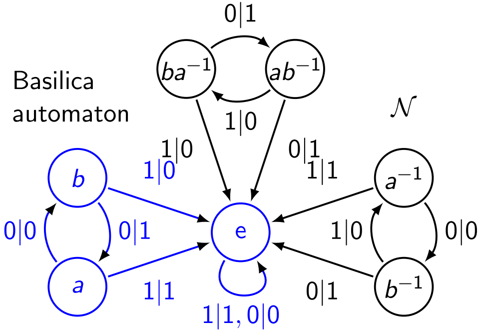


Contracting automata and singular points

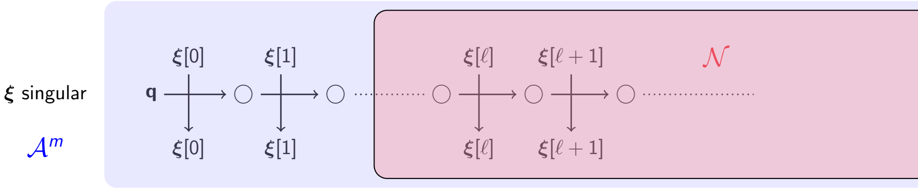
ξ singular

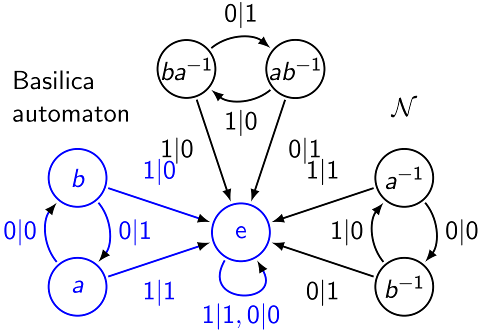
$\mathcal{A}^m$



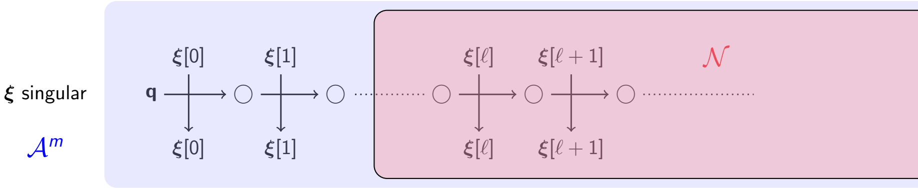


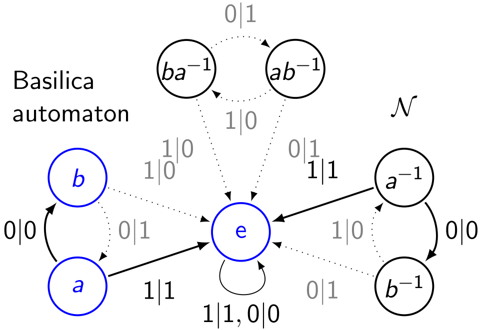
Contracting automata and singular points



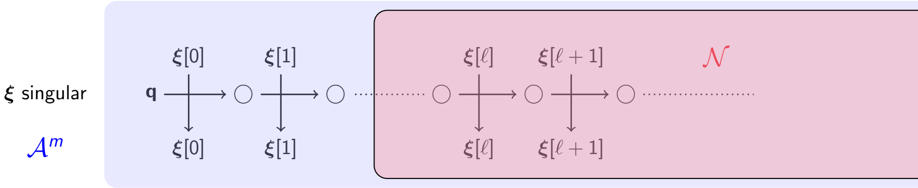


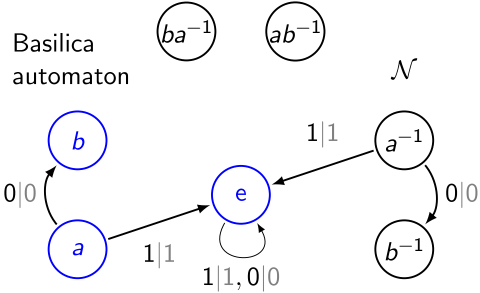
Contracting automata and singular points





Contracting automata and singular points

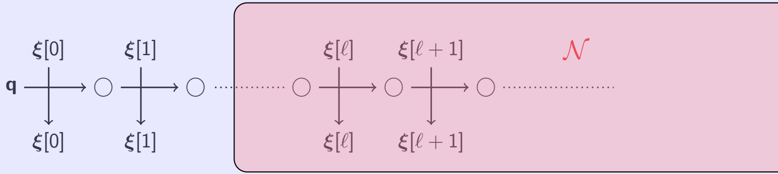




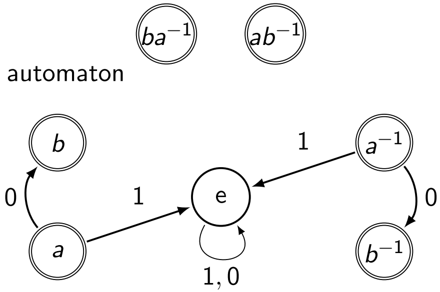
Contracting automata and singular points

ξ singular

$\mathcal{A}^m$



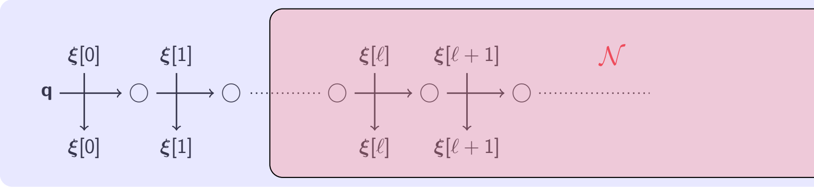
Büchi automaton



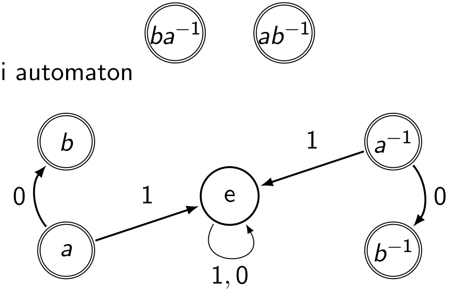
Contracting automata and singular points

ξ singular

$\mathcal{A}^m$



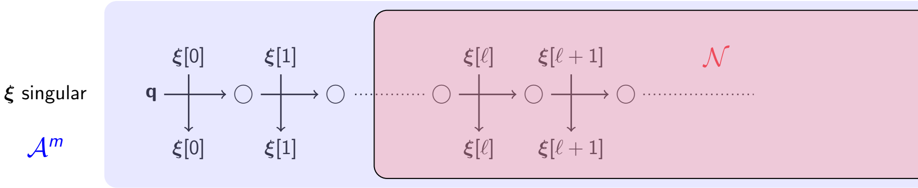
Büchi automaton

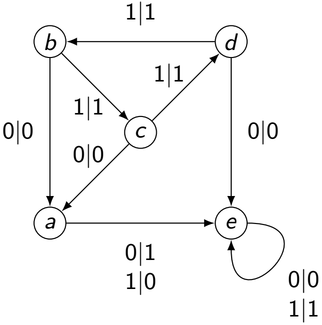


Contribution

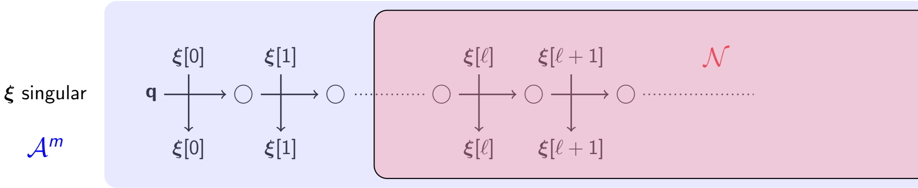
$\mathrm{Sing}(\mathcal{B}) = \emptyset$.

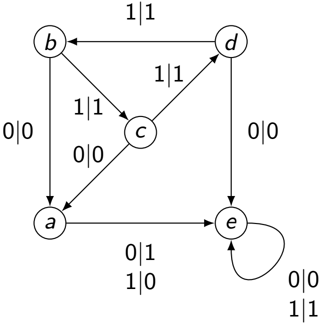
Contracting automata and singular points

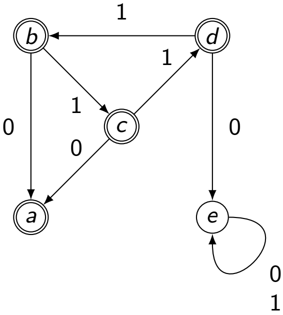




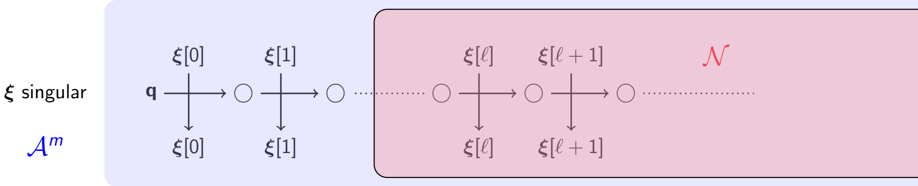
Contracting automata and singular points

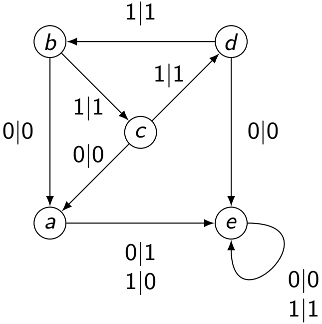


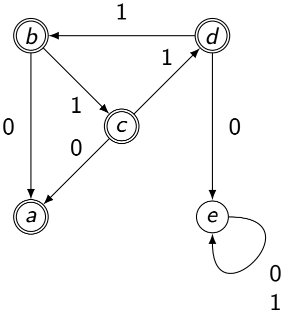




Contracting automata and singular points







Proposition [Vorobets, DGKPR]

$\mathrm{Sing}(\mathcal{G}) = (0+1)^*1^\omega$.

automaton patterns and group properties

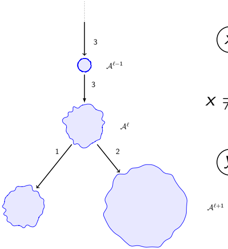
finiteness

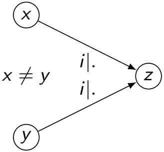
infinite Burnside

growth

Invertible reversible non-coreversible automata generate infinite non Burnside groups
[LATA'15 w. Klimann and Picantin]

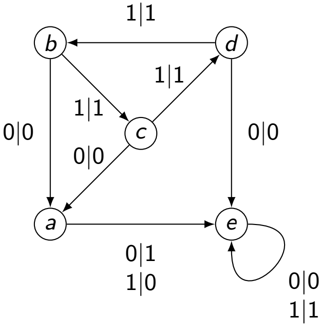
Bireversible automata of prime size cannot generate infinite Burnside groups
[MFCS'16 w. Klimann]



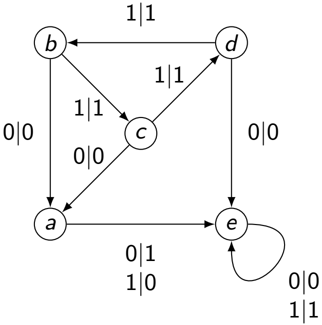


The set o
of a contr
is describe
automaton

About the Grigorchuk automaton



About the Grigorchuk automaton



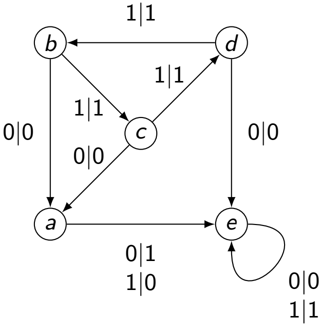
Actions of the states on the letters:

$$\rho_a : 0 \mapsto 1 \mapsto 0$$

$$\rho_b, \rho_c, \rho_d, \rho_e : 0 \mapsto 0; \quad 1 \mapsto 1$$

$\rightarrow$permutations

About the Grigorchuk automaton



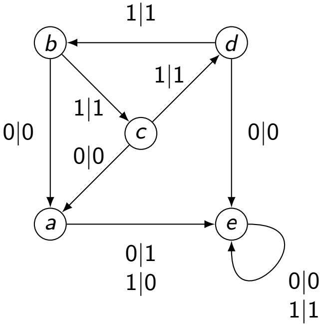
Actions of the states on the letters:

$$\rho_a : 0 \mapsto 1 \mapsto 0$$

$$\rho_b, \rho_c, \rho_d, \rho_e : 0 \mapsto 0; \quad 1 \mapsto 1$$

→permutations
→**invertible**

About the Grigorchuk automaton



Actions of the states on the letters:

$$\rho_a : 0 \mapsto 1 \mapsto 0$$

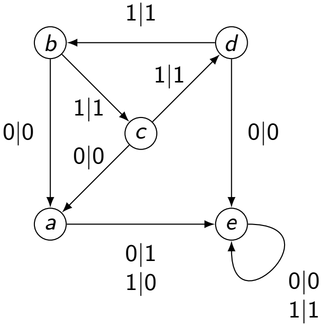
$$\rho_b, \rho_c, \rho_d, \rho_e : 0 \mapsto 0; \quad 1 \mapsto 1$$

→permutations
→**invertible**

Action of a letter on the states:
$\delta_0 : a, d, e \mapsto e; \quad b, c \mapsto a$

→not a permutation

About the Grigorchuk automaton



Actions of the states on the letters:

$$\rho_a : 0 \mapsto 1 \mapsto 0$$

$$\rho_b, \rho_c, \rho_d, \rho_e : 0 \mapsto 0; \quad 1 \mapsto 1$$

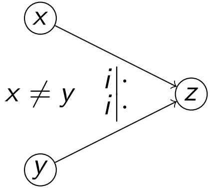
→permutations
→**invertible**

Action of a letter on the states:
$\delta_0 : a, d, e \mapsto e; \quad b, c \mapsto a$

→not a permutation
→**non-reversible**

About the Grigorchuk automaton

Reversibility:
Each input letter permutes the stateset.



Actions of the states on the letters:

$$\rho_a : 0 \mapsto 1 \mapsto 0$$
$$\rho_b, \rho_c, \rho_d, \rho_e : 0 \mapsto 0; \quad 1 \mapsto 1$$

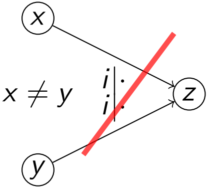
→permutations
→**invertible**

Action of a letter on the states:
$\delta_0 : a, d, e \mapsto e; \quad b, c \mapsto a$

→not a permutation
→**non-reversible**

About the Grigorchuk automaton

Reversibility:
Each input letter permutes the stateset.



Actions of the states on the letters:

$$\rho_a : 0 \mapsto 1 \mapsto 0$$
$$\rho_b, \rho_c, \rho_d, \rho_e : 0 \mapsto 0; \quad 1 \mapsto 1$$

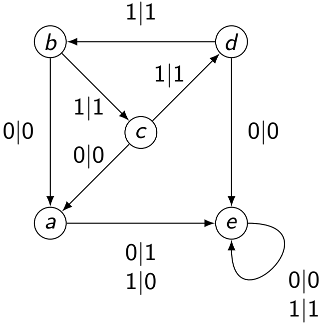
→permutations
→**invertible**

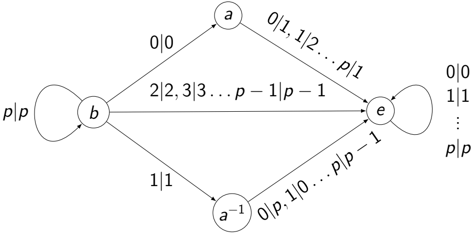
Action of a letter on the states:
$\delta_0 : a, d, e \mapsto e; \quad b, c \mapsto a$

→not a permutation
→**non-reversible**

Observation

Every known automaton generating an infinite Burnside group happens to be non-reversible.





Question

Can a reversible automaton generate an infinite Burnside group?

Theorem(s)

An invertible and reversible automata which is:

cannot generate an infinite Burnside group.

Theorem(s)

An invertible and reversible automata which is:

2-state

[Klimann]

STACS'13

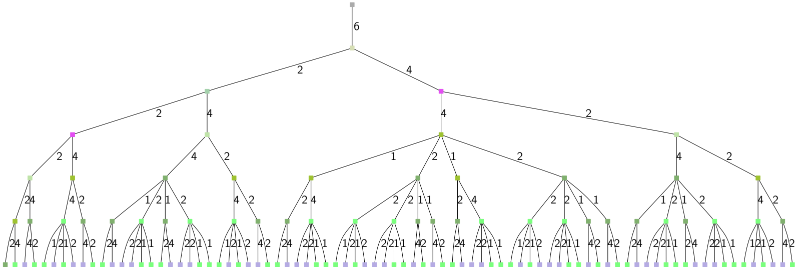
cannot generate an infinite Burnside group.

Theorem(s)

An invertible and reversible automata which is:

2-state	connected 3-state
[Klimann]	[Klimann, Picantin, and Savchuk]
STACS'13	DLT'15

cannot generate an infinite Burnside group.

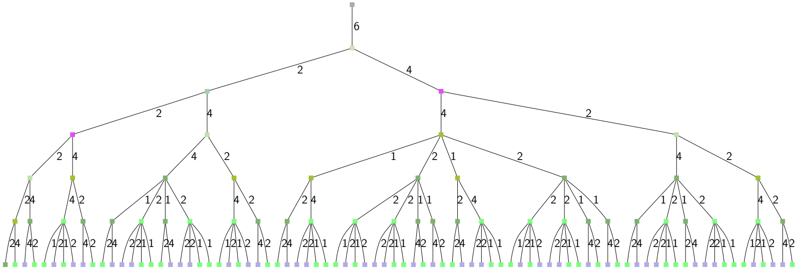


Theorem(s)

An invertible and reversible automata which is:

2-state	connected 3-state	non coreversible
[Klimann]	[Klimann, Picantin, and Savchuk]	[G., Klimann, and Picantin]
STACS'13	DLT'15	LATA'15

cannot generate an infinite Burnside group.

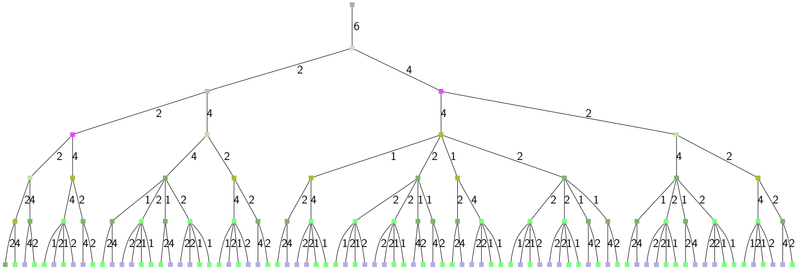


Theorem(s)

An invertible and reversible automata which is:

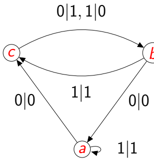
2-state	connected 3-state	non coreversible	connected with prime size
[Klimann]	[Klimann, Picantin, and Savchuk]	[G., Klimann, and Picantin]	[G. and Klimann]
STACS'13	DLT'15	LATA'15	MFCS'16

cannot generate an infinite Burnside group.



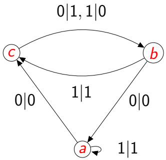
Schreier tree

$\mathcal{A}$

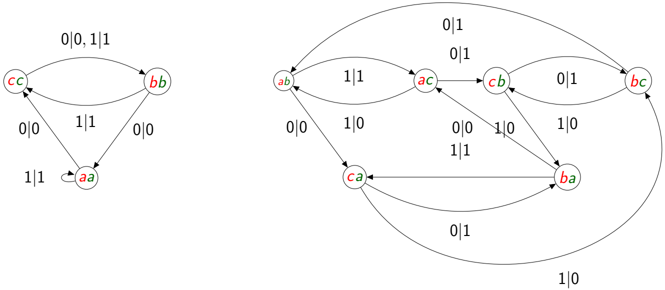


Schreier tree

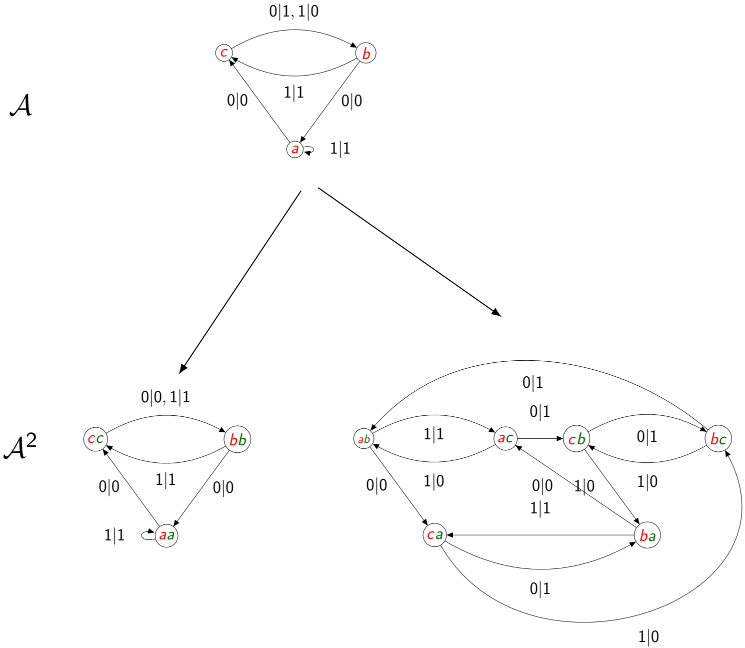
$\mathcal{A}$



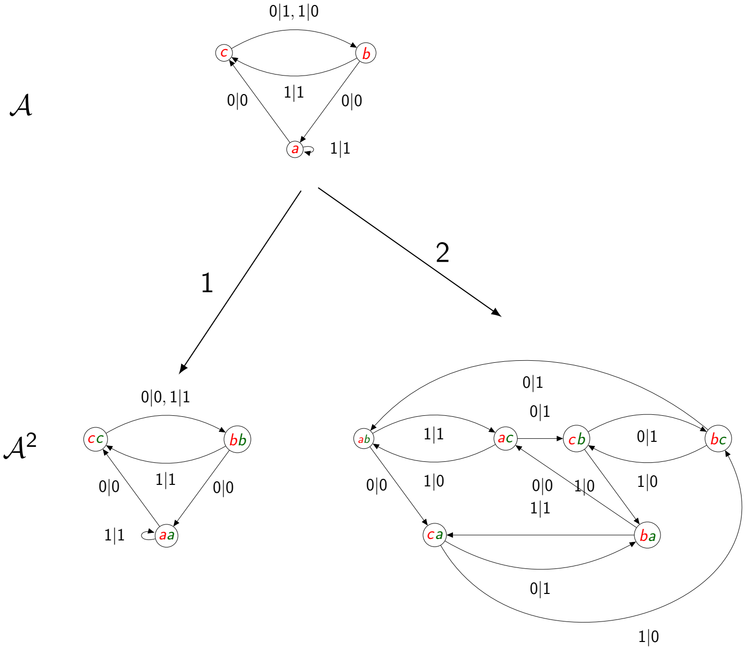
$\mathcal{A}^2$



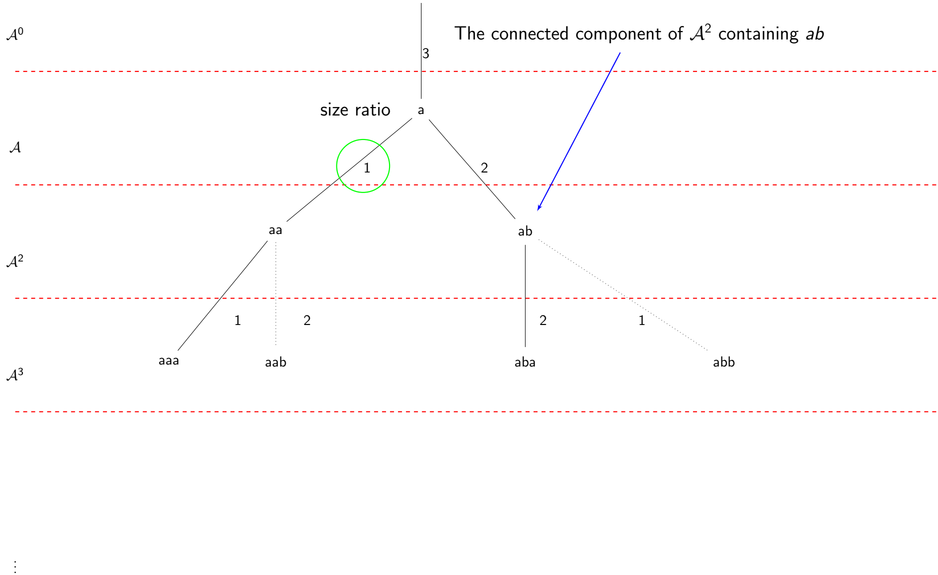
Schreier tree



Schreier tree



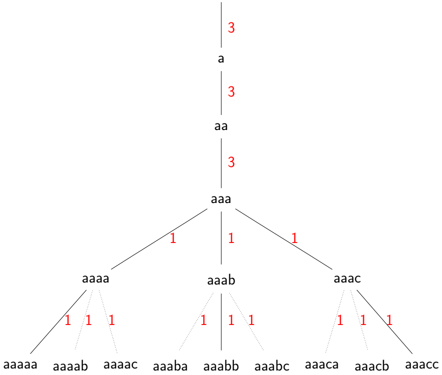
Schreier tree



Boundedness

Proposition

$\langle \mathcal{A} \rangle$ is finite iff the labels of the cc of $(\mathcal{A}^n)_n$ are ultimately 1.



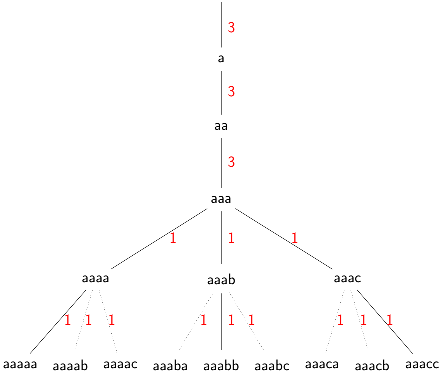
Boundedness

Proposition

$\langle \mathcal{A} \rangle$ is finite iff the labels of the cc of $(\mathcal{A}^n)_n$ are ultimately 1.

Proposition

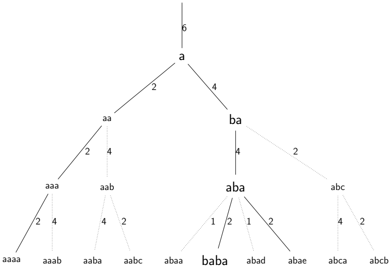
ρ_q has finite order iff the labels of the cc containing q^n are ultimately 1.

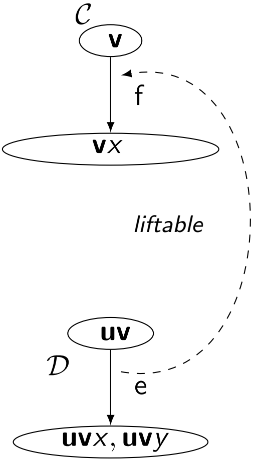


Liftable

Proposition

e liftable to $f \Rightarrow \text{label}(e) \leq \text{label}(f)$.

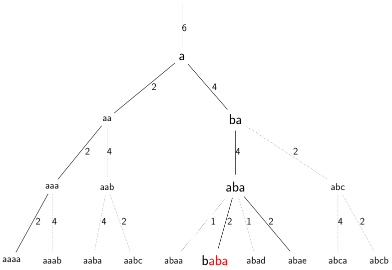


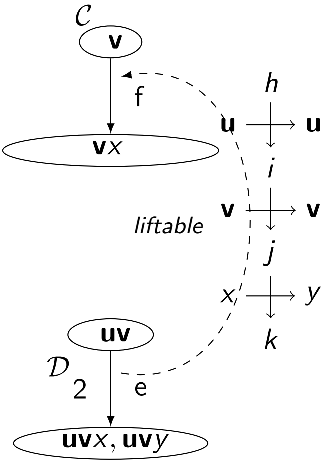


Liftable

Proposition

e liftable to $f \Rightarrow \text{label}(e) \leq \text{label}(f)$.

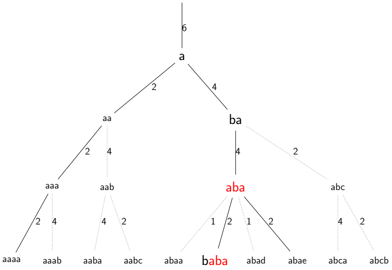


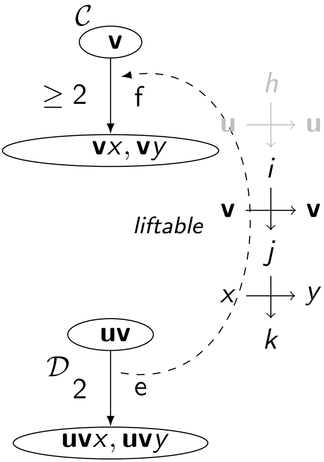


Liftable

Proposition

e liftable to $f \Rightarrow \text{label}(e) \leq \text{label}(f)$.

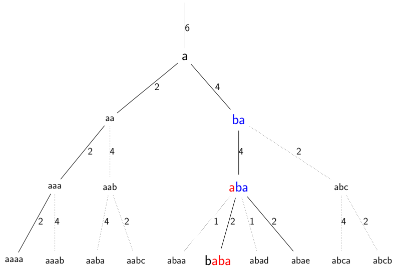


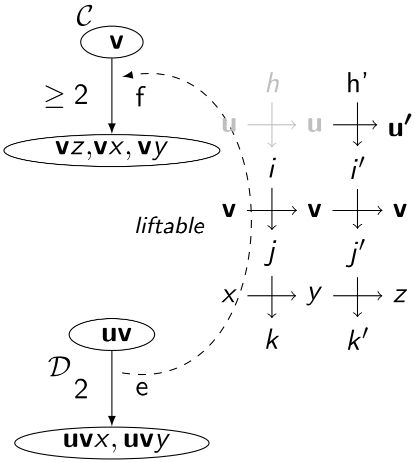


Liftable

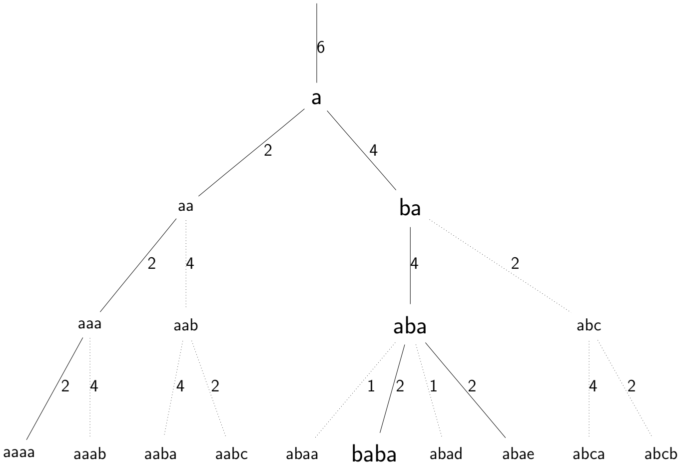
Proposition

e liftable to $f \Rightarrow \text{label}(e) \leq \text{label}(f)$.

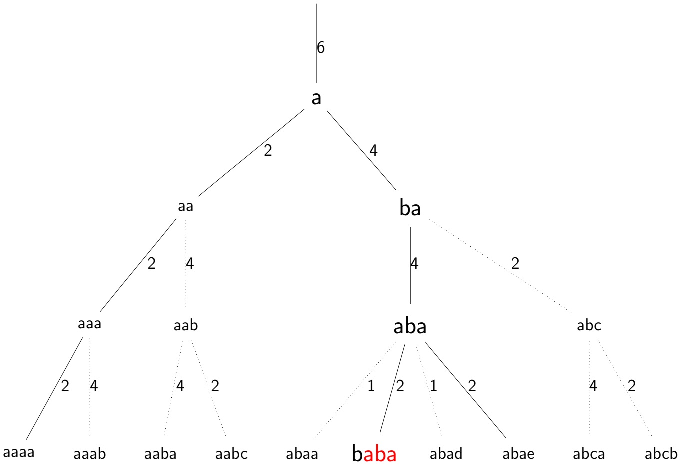




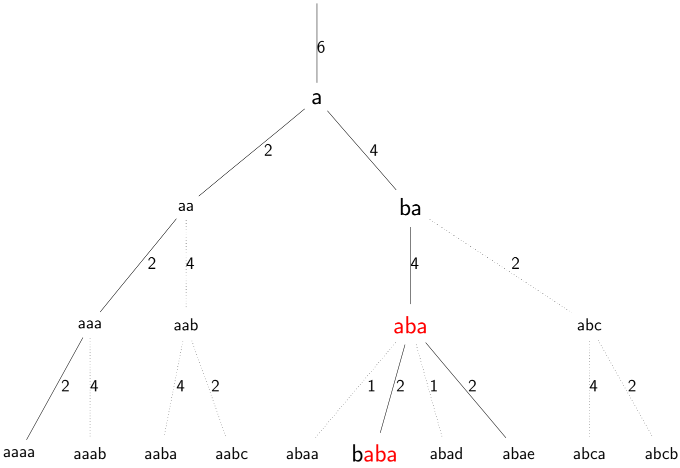
Liftable paths



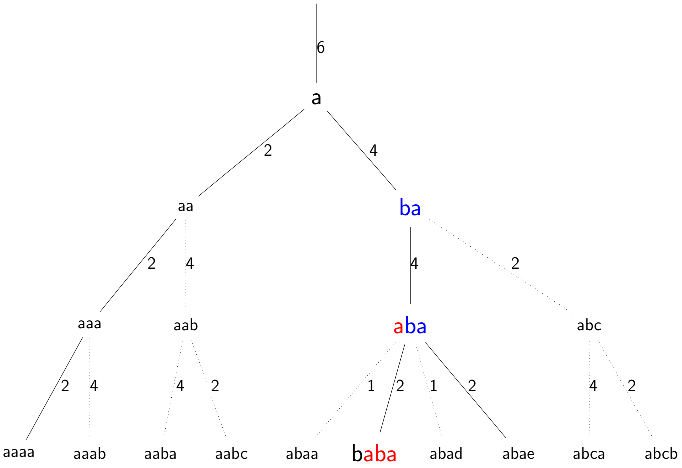
Liftable paths



Liftable paths



Liftable paths



Jungle tree

active $\equiv$ labels not ending with 1^ω.

If active liftable path ✓: not Burnside.

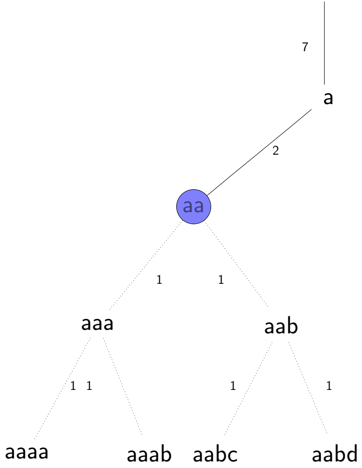
Otherwise :

Jungle tree

active $\equiv$ labels not ending with 1^ω.

If active liftable path ✓: not Burnside.

Otherwise :



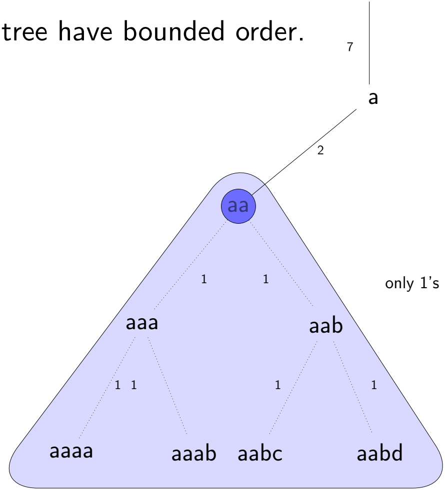
Jungle tree

active $\equiv$ labels not ending with 1^ω.

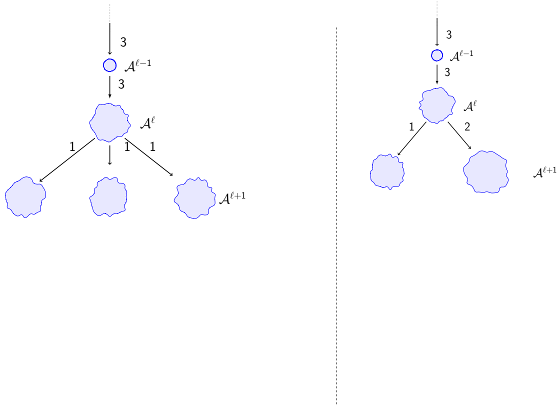
If active liftable path ✓: not Burnside.

Otherwise :

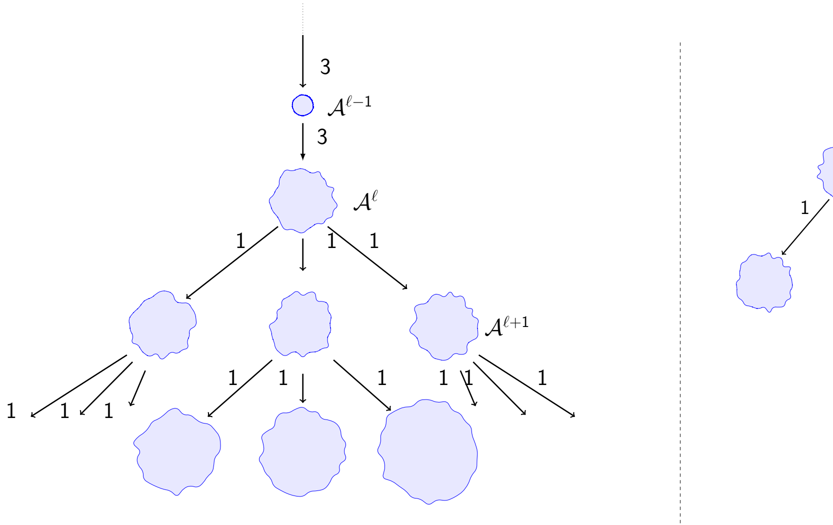
words in the jungle tree have bounded order.



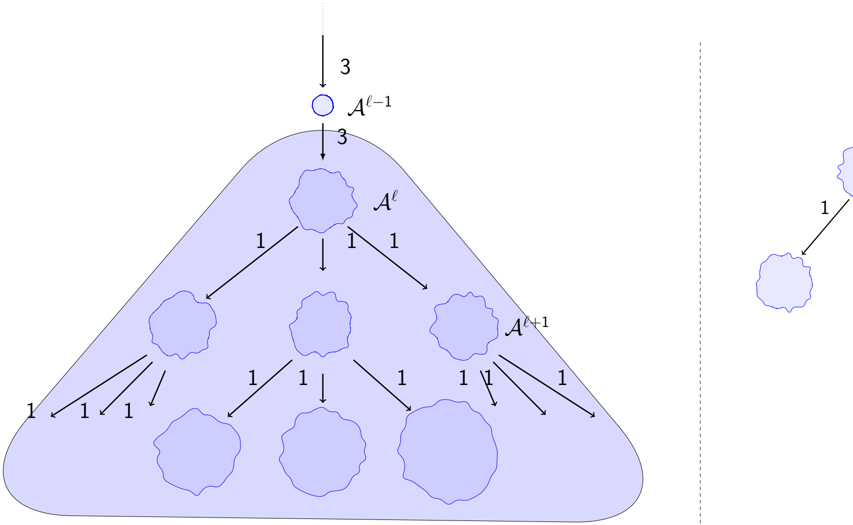
3-state case


3
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
1
1
$\mathcal{A}^{\ell+1}$
3
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
2
$\mathcal{A}^{\ell+1}$

3-state case

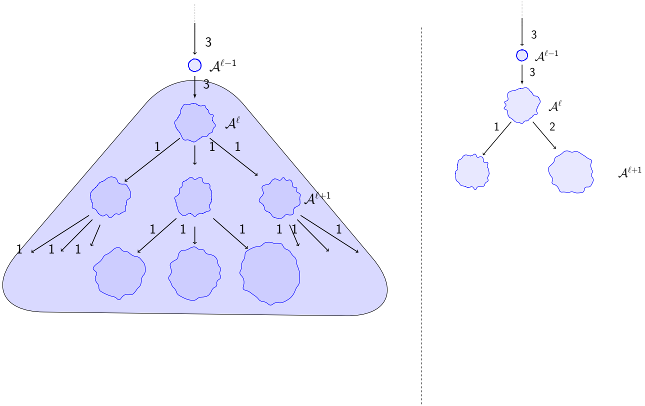

3
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
1
1
$\mathcal{A}^{\ell+1}$
1
1
1
1
1
1
1
1
1
1
1

3-state case

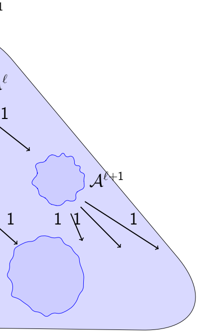


3
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
1
1
$\mathcal{A}^{\ell+1}$
1
1
1
1
1
1
1
1
1
1
1

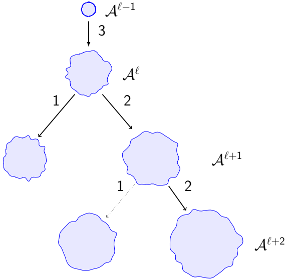
3-state case


3
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
1
1
$\mathcal{A}^{\ell+1}$
1
1
1
1
1
1
1
1
1
1
3
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
2
$\mathcal{A}^{\ell+1}$

3-state case

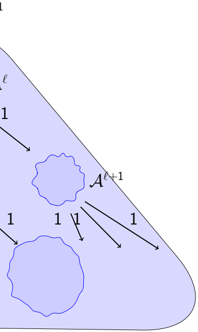


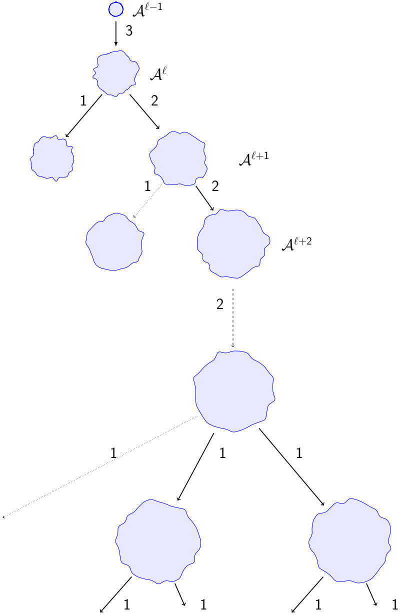
1
$\mathcal{A}^{\ell+1}$
1 1 1 1



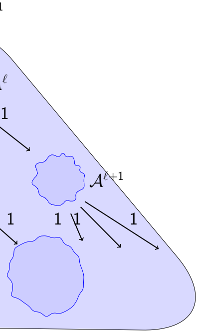
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1 2
$\mathcal{A}^{\ell+1}$
1 2
$\mathcal{A}^{\ell+2}$

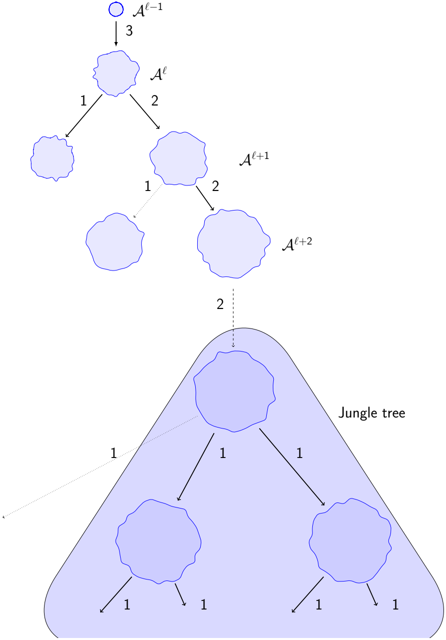
3-state case


$\mathcal{A}^{\ell+1}$
1
1
1
1
1
1

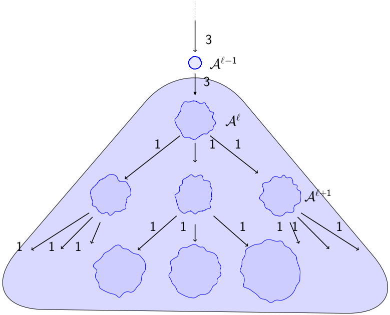

$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
2
$\mathcal{A}^{\ell+1}$
1
2
$\mathcal{A}^{\ell+2}$
2
1
1
1
1
1
1
1

3-state case

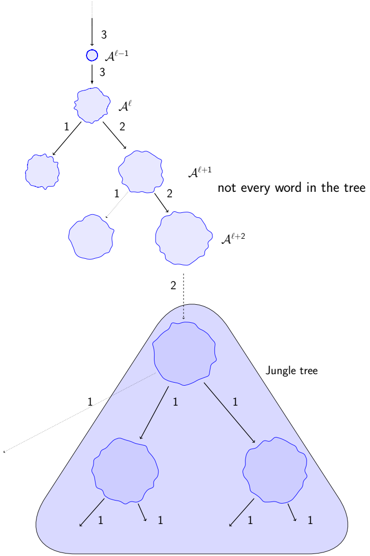

$\mathcal{A}^{\ell+1}$
1
1
1
1
1
1


$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
2
$\mathcal{A}^{\ell+1}$
1
2
$\mathcal{A}^{\ell+2}$
2
Jungle tree
1
1
1
1
1
1
1

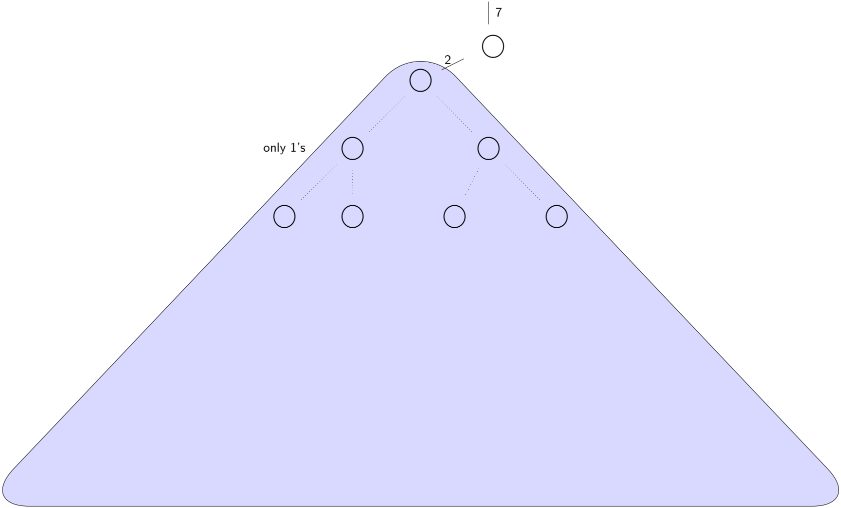
3-state case



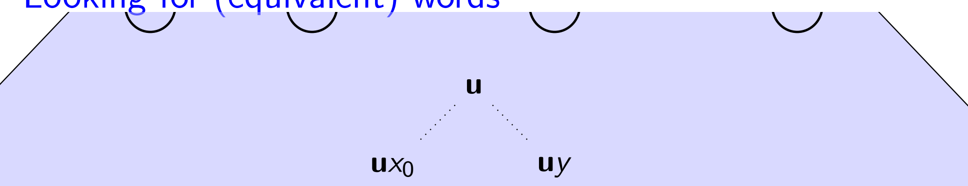
Every word in the tree ✓



Looking for (equivalent) words

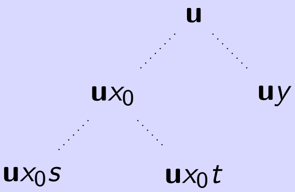


Looking for (equivalent) words



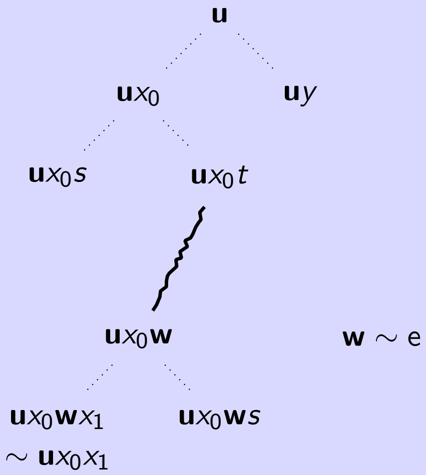
Idea: $\forall x_0 x_1 x_2 \cdots$ find a word with same action in the jungle tree

Looking for (equivalent) words



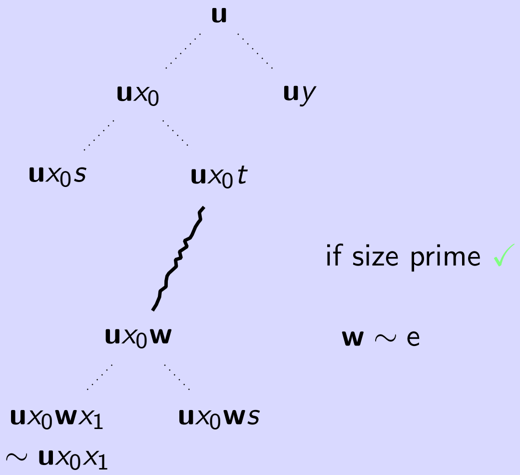
Idea: $\forall x_0 x_1 x_2 \cdots$ find a word with same action in the jungle tree

Looking for (equivalent) words



Idea: $\forall x_0 x_1 x_2 \cdots$ find a word with same action in the jungle tree

Looking for (equivalent) words



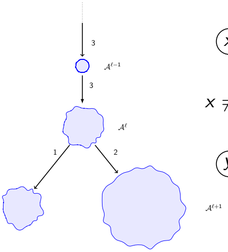
Idea: $\forall x_0x_1x_2\cdots$ find a word with same action in the jungle tree finiteness

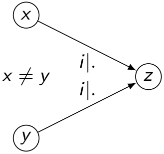
Invertible reversible non-coreversible automata generate infinite non Burnside groups
[LATA'15 w. Klimann and Picantin]

Bireversible automata of prime size cannot generate infinite Burnside groups
[MFCS'16 w. Klimann]

infinite Burnside

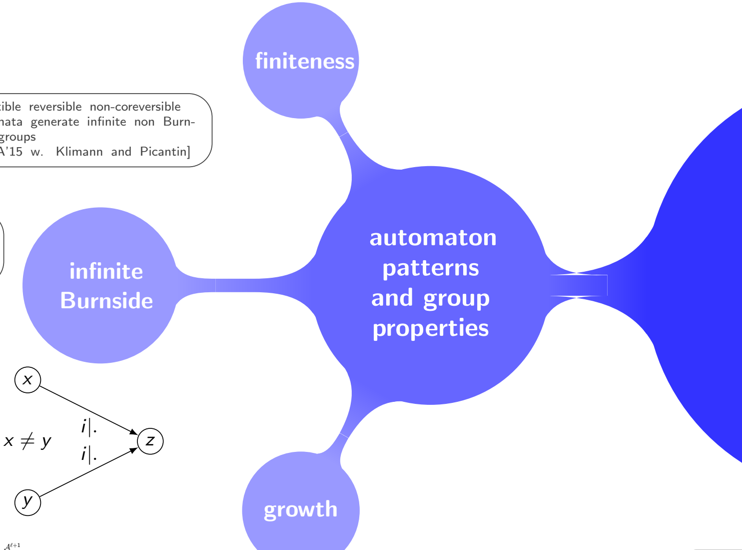
automaton patterns and group properties





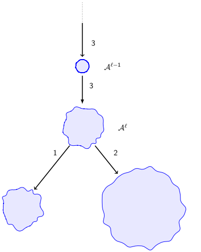
growth

The set o
of a contr
is describe
automaton



Invertible reversible non-coreversible automata generate infinite non Burnside groups
[LATA'15 w. Klimann and Picantin]

Bireversible automata of prime size cannot generate infinite Burnside groups
[MFCS'16 w. Klimann]



Level transitive reversible automata have exponential growth [Klimann'16]

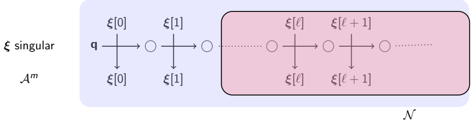
The set o
of a contr
is describe
automaton

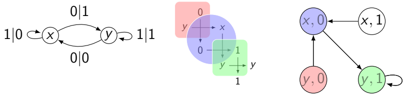
Mealy automata

dynamics of the action

singular points

The set of singular points of a contracting automaton is described by a Büchi automaton [DGKPR'16]

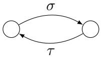




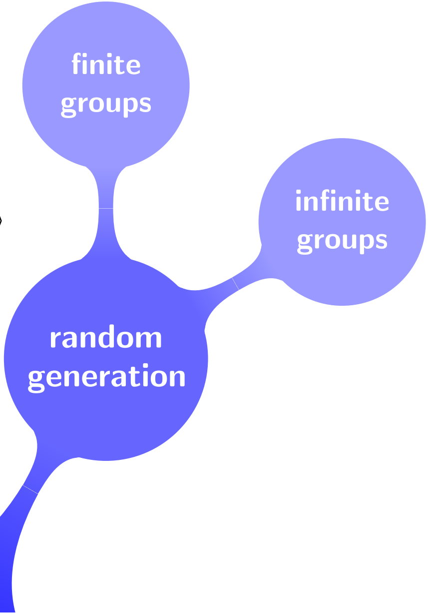
Wang tillings

Schreier graphs

Analogue to Dixon theorem [ANALCO'16]

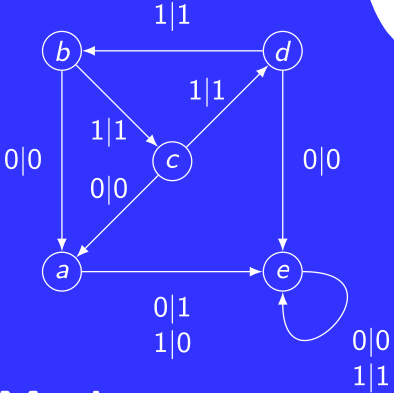


$$\left\langle \quad \right\rangle = \begin{cases} \mathfrak{S}_k \times \mathfrak{S}_k \\ (\mathfrak{A}_k \times \mathfrak{A}_k) \rtimes \langle (\pi, \pi) \rangle \\ \mathfrak{A}_k \times \mathfrak{A}_k \end{cases}$$



[LATA'15]
w. Klimann and Picantin

[ANALCO'16]



Mealy automata

[arXiv'16]
w. D'Angeli, Klimann, Picantin, and Rodaro

[MFCS'16 & ToCS'17]
w. Klimann

dy