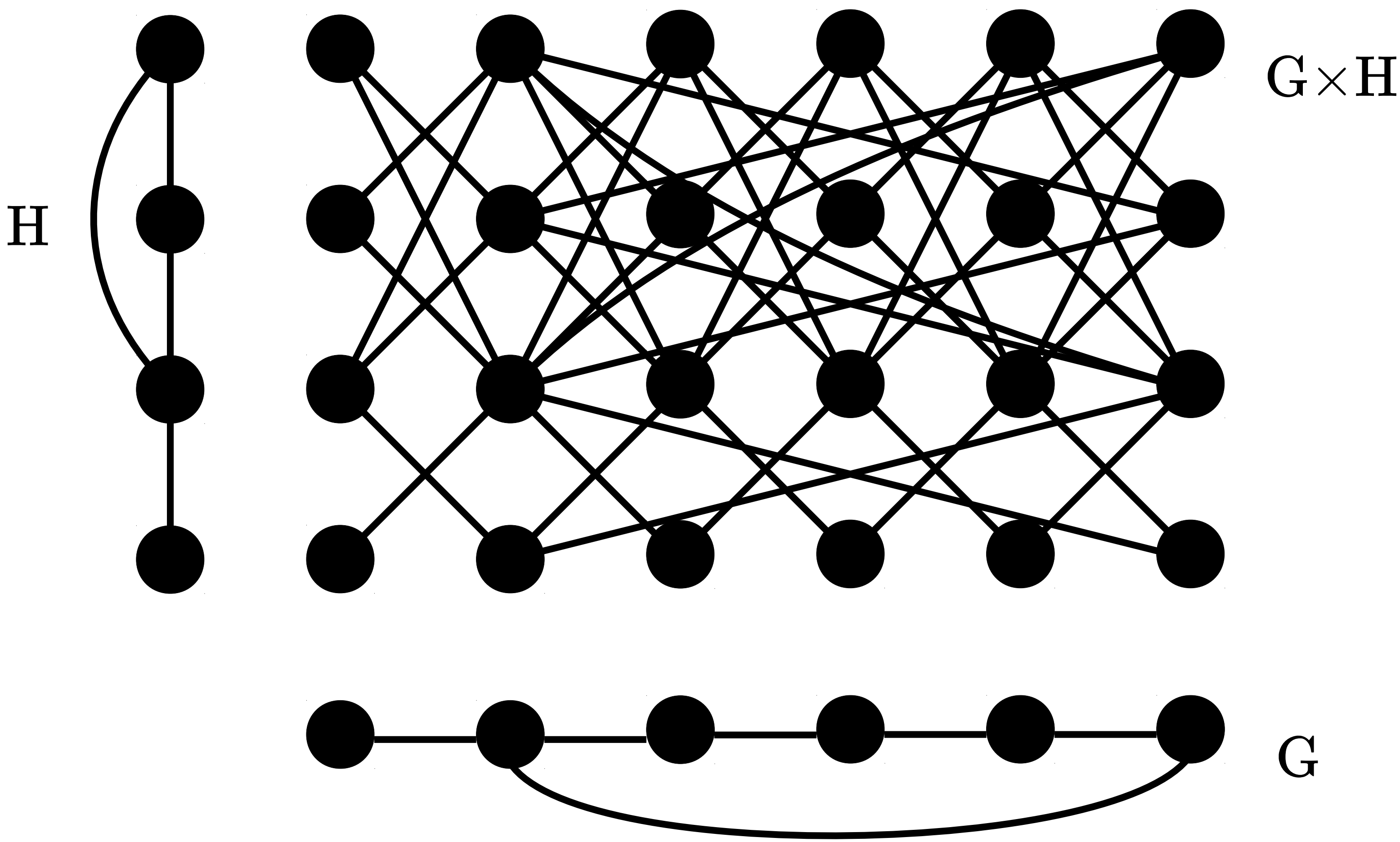
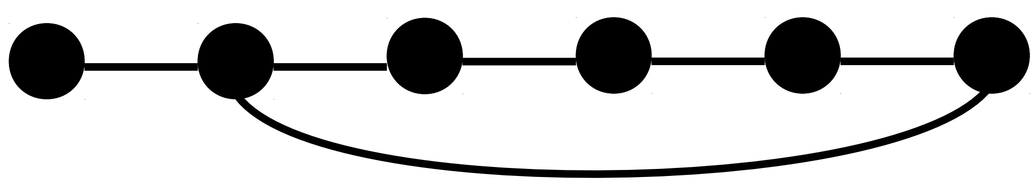
New counterexamples to Hedetniemi's conjecture

Claude Tardif, RMC



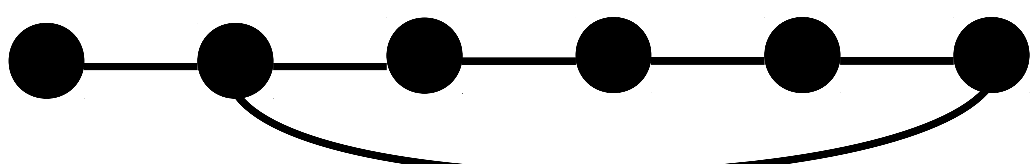
The categorical product

The categorical product G



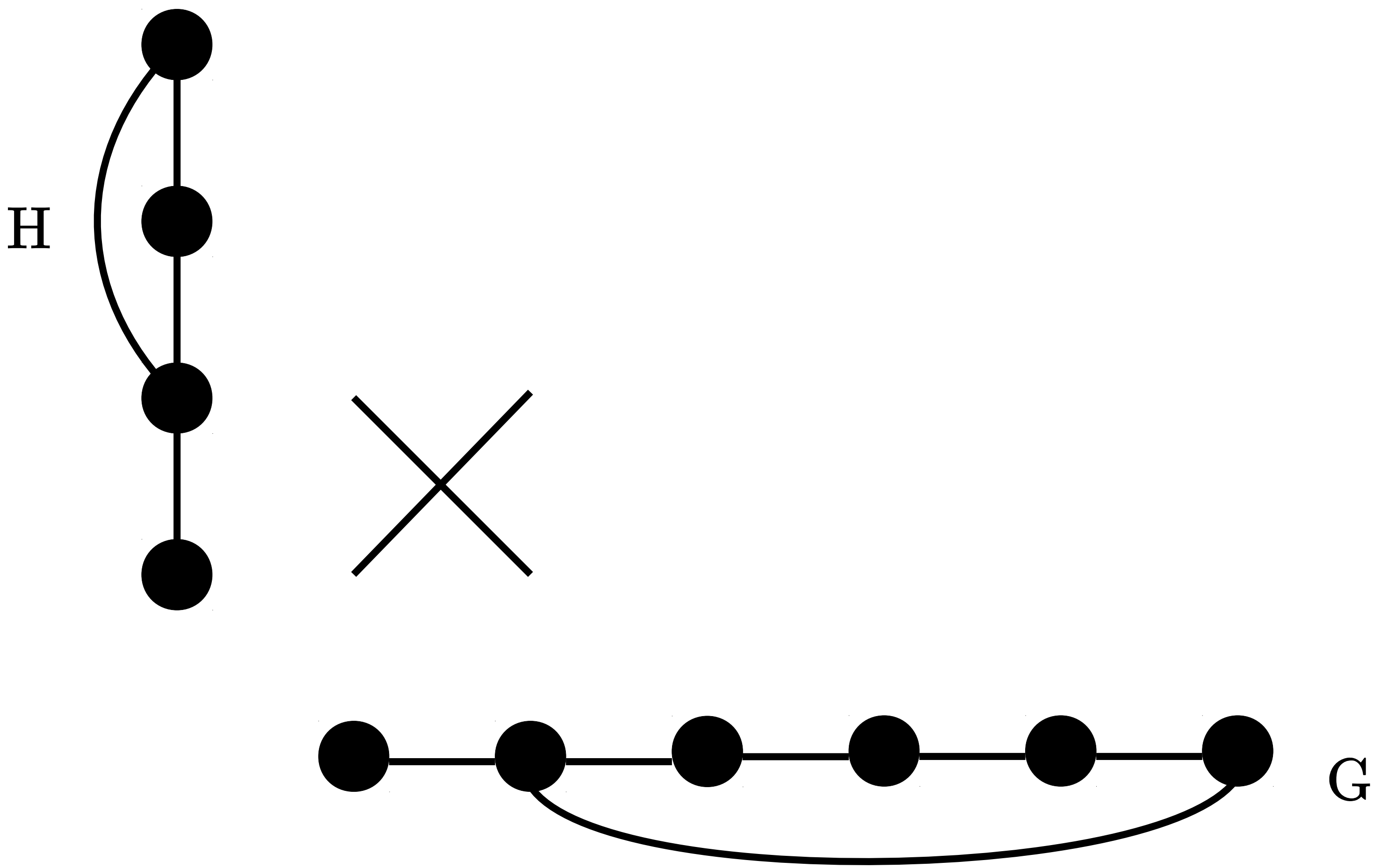
G

The categorical product G×



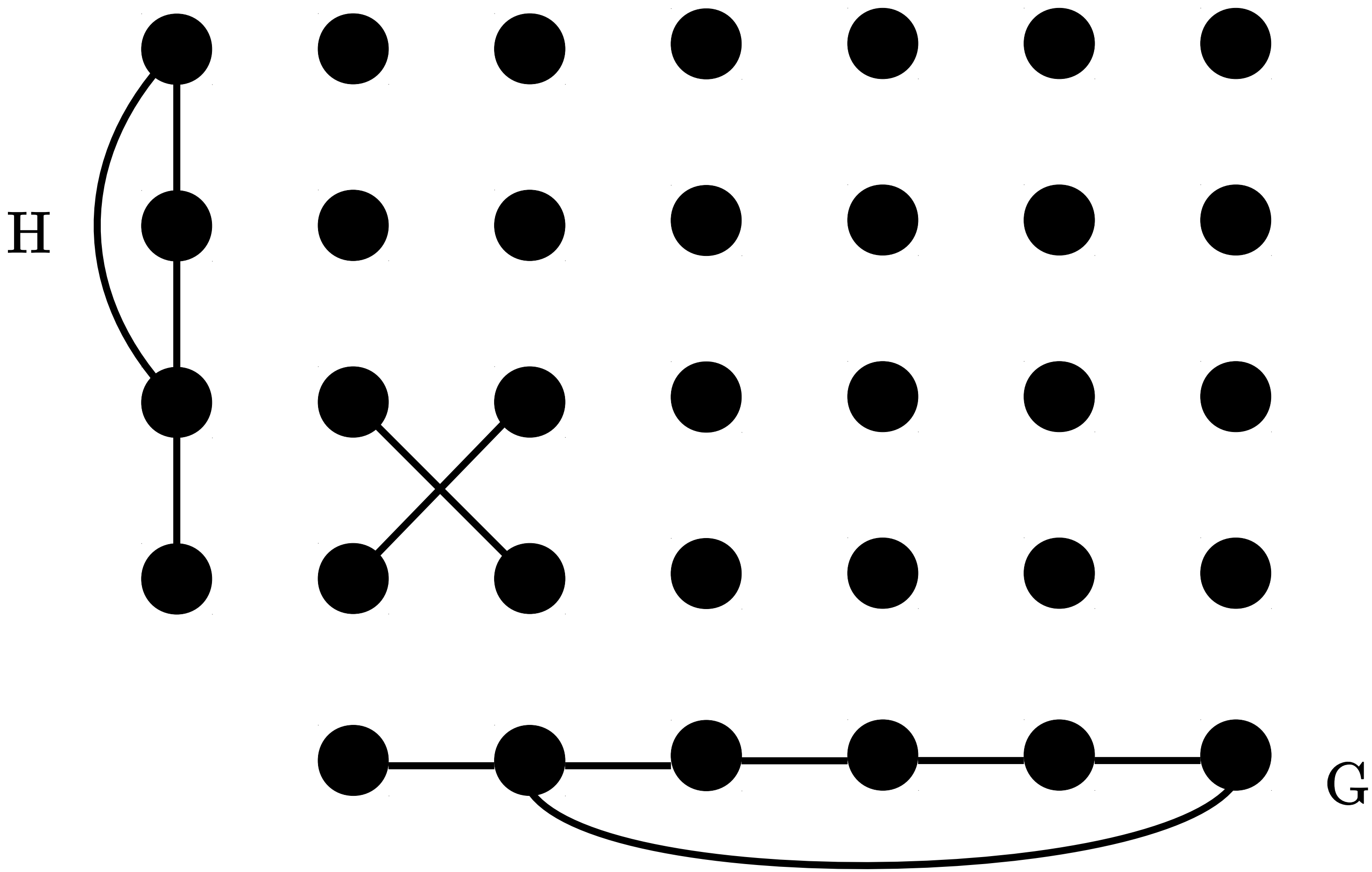
G

The categorical product G×H



The categorical product $\mathrm{G}\times\mathrm{H}$

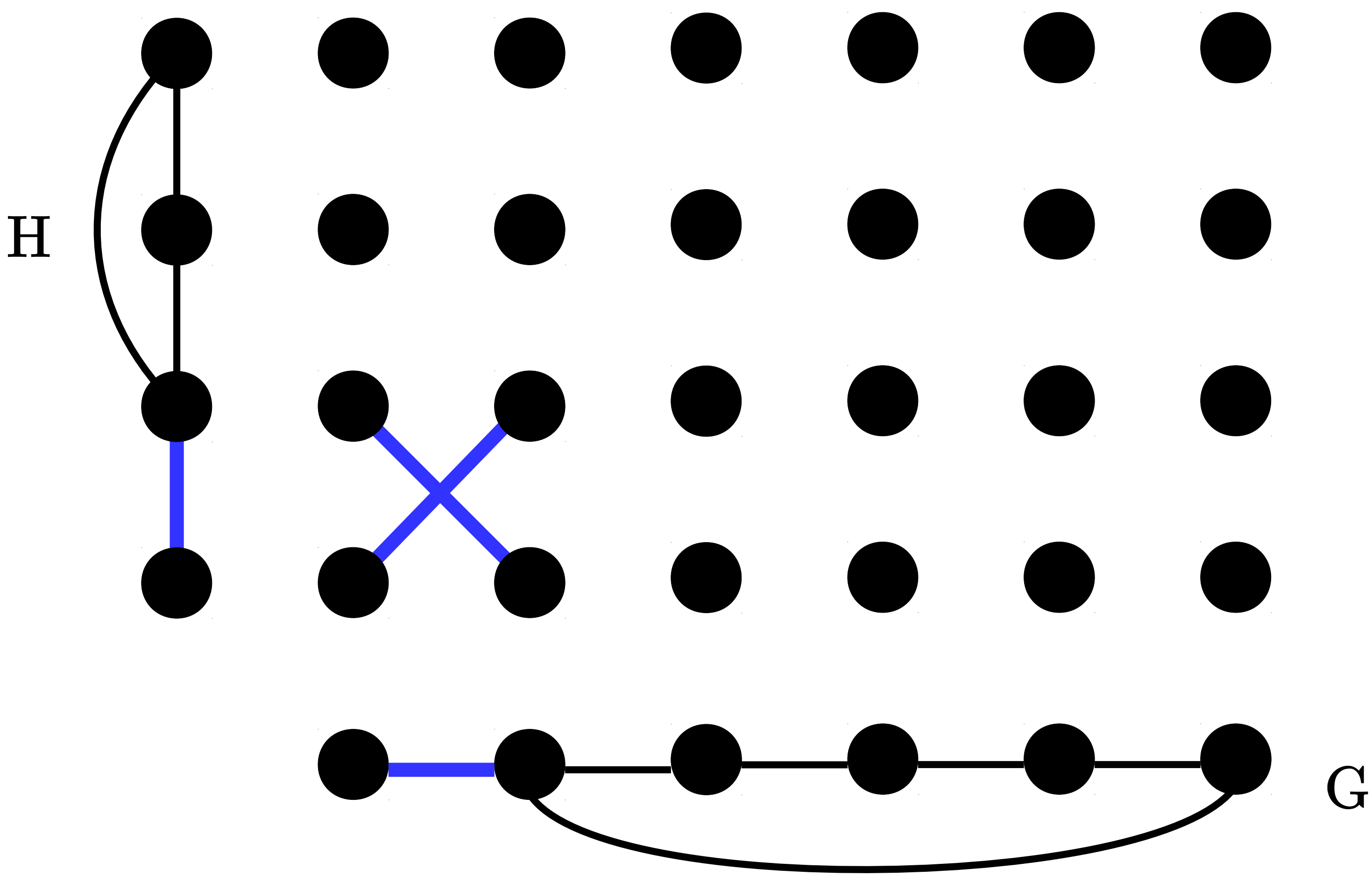
$\mathrm{V}(\mathrm{G}\times\mathrm{H}) = \mathrm{V}(\mathrm{G})\times\mathrm{V}(\mathrm{H})$



The categorical product $G \times H$

$V(G \times H) = V(G) \times V(H)$

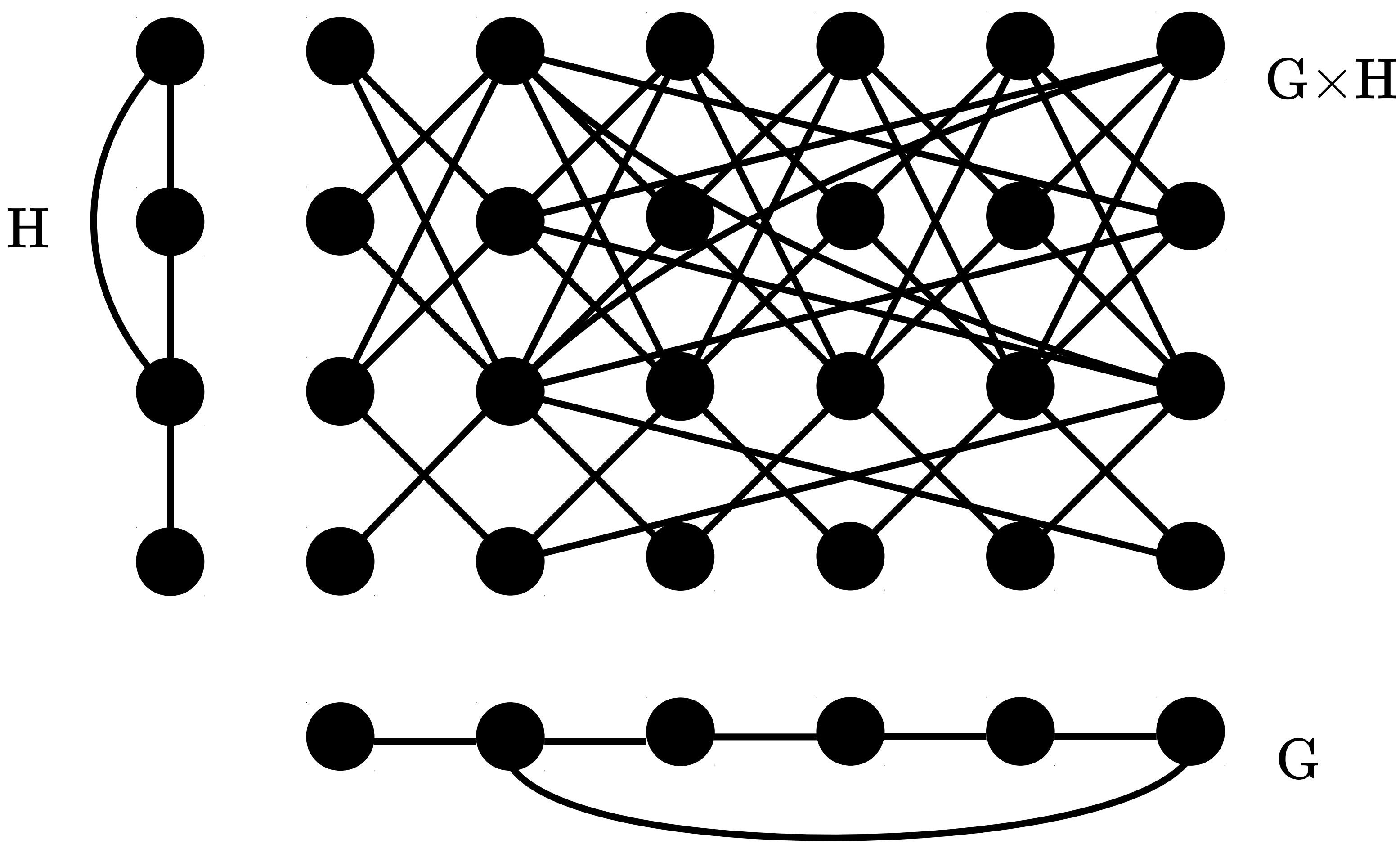
$E(G \times H) =$



The categorical product $G \times H$

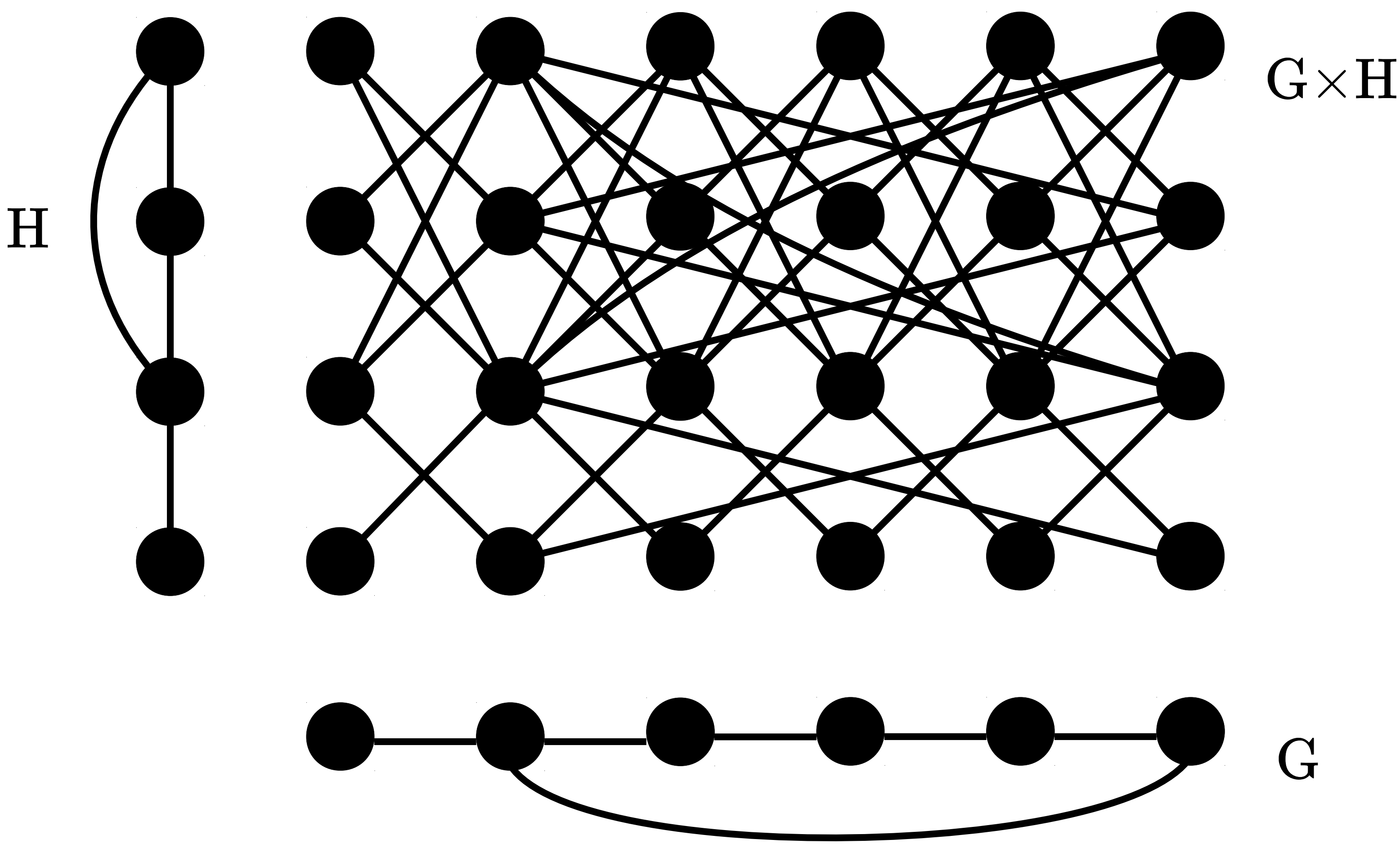
$$V(G \times H) = V(G) \times V(H)$$

$$E(G \times H) = \left\{ \{(u,v),(u'v')\} \mid \{u,u'\} \in E(G),\ \{v,v'\} \in E(H) \right\}$$



The categorical product G×H

(direct, tensor, cartesian)



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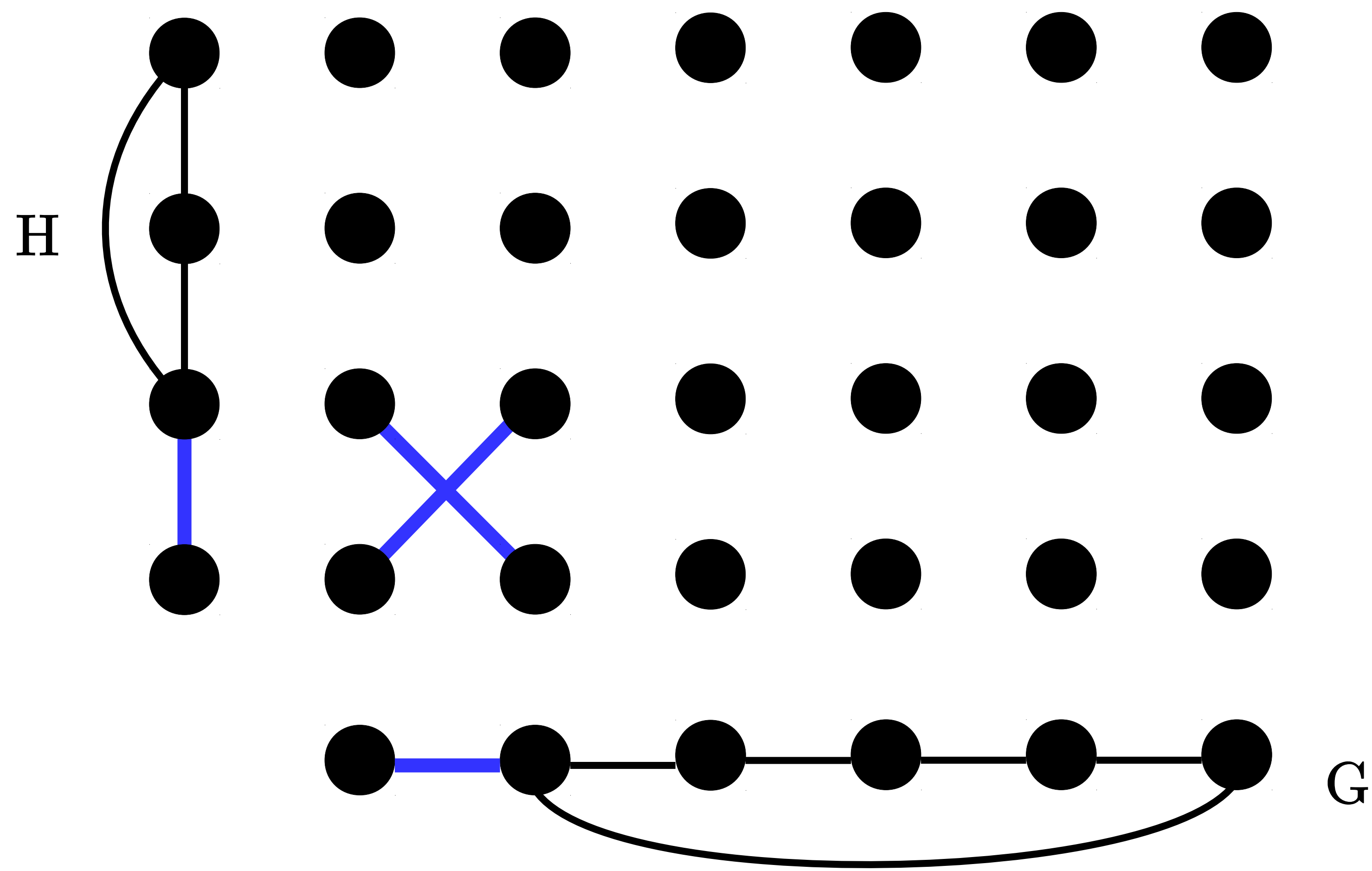
(Nešetřil, Rödl 1983: Products of graphs and their applications)



The categorical product G × H

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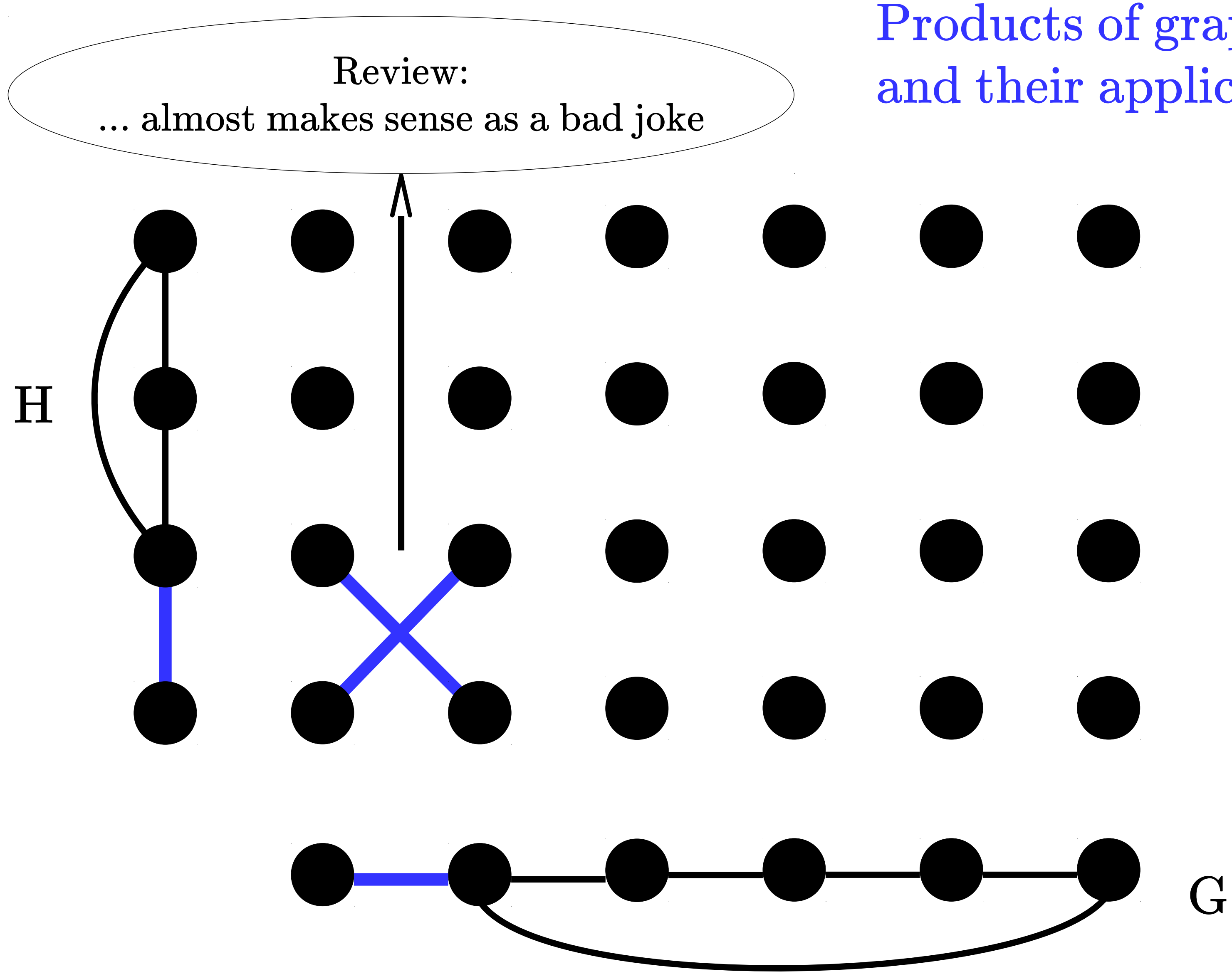
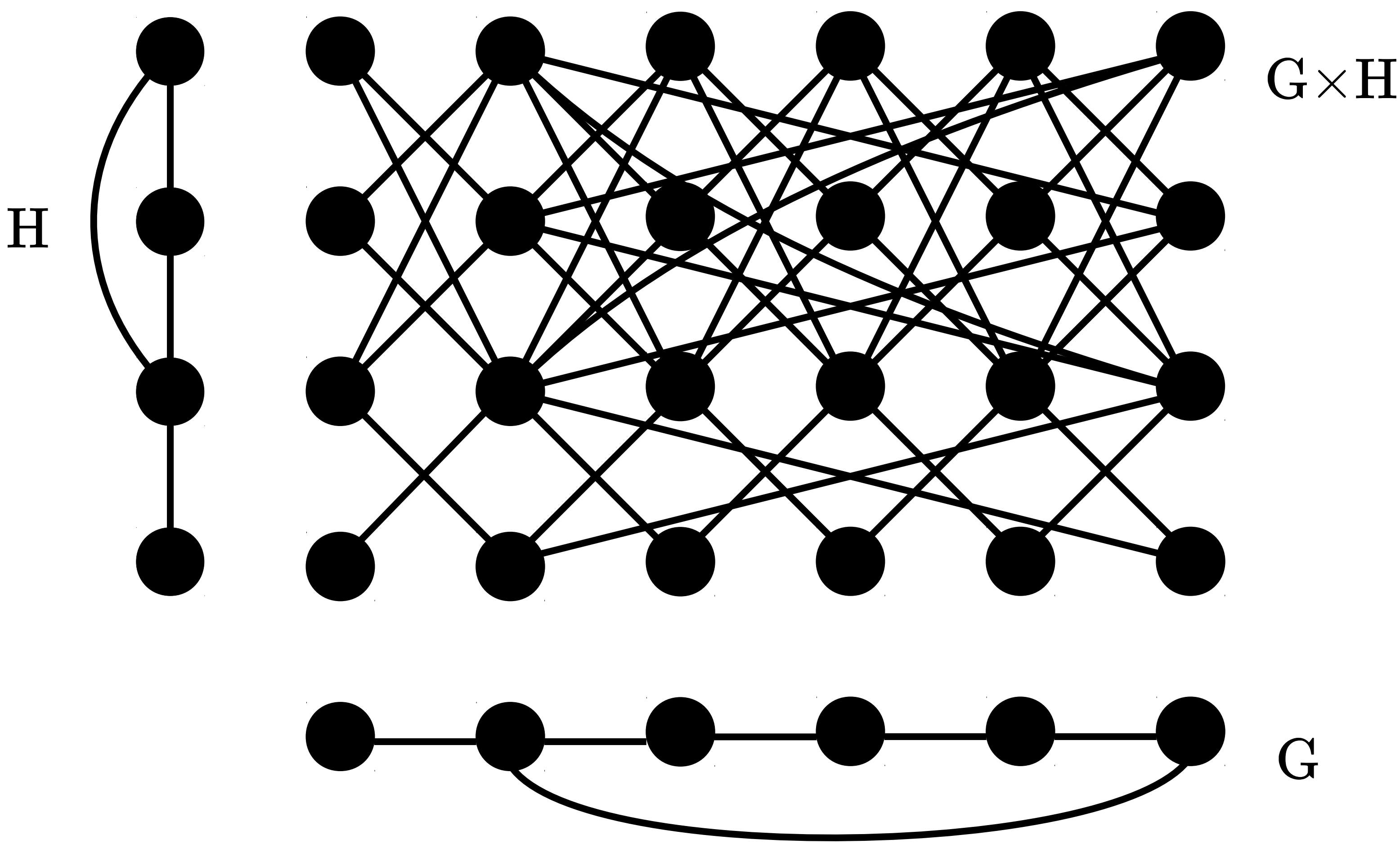
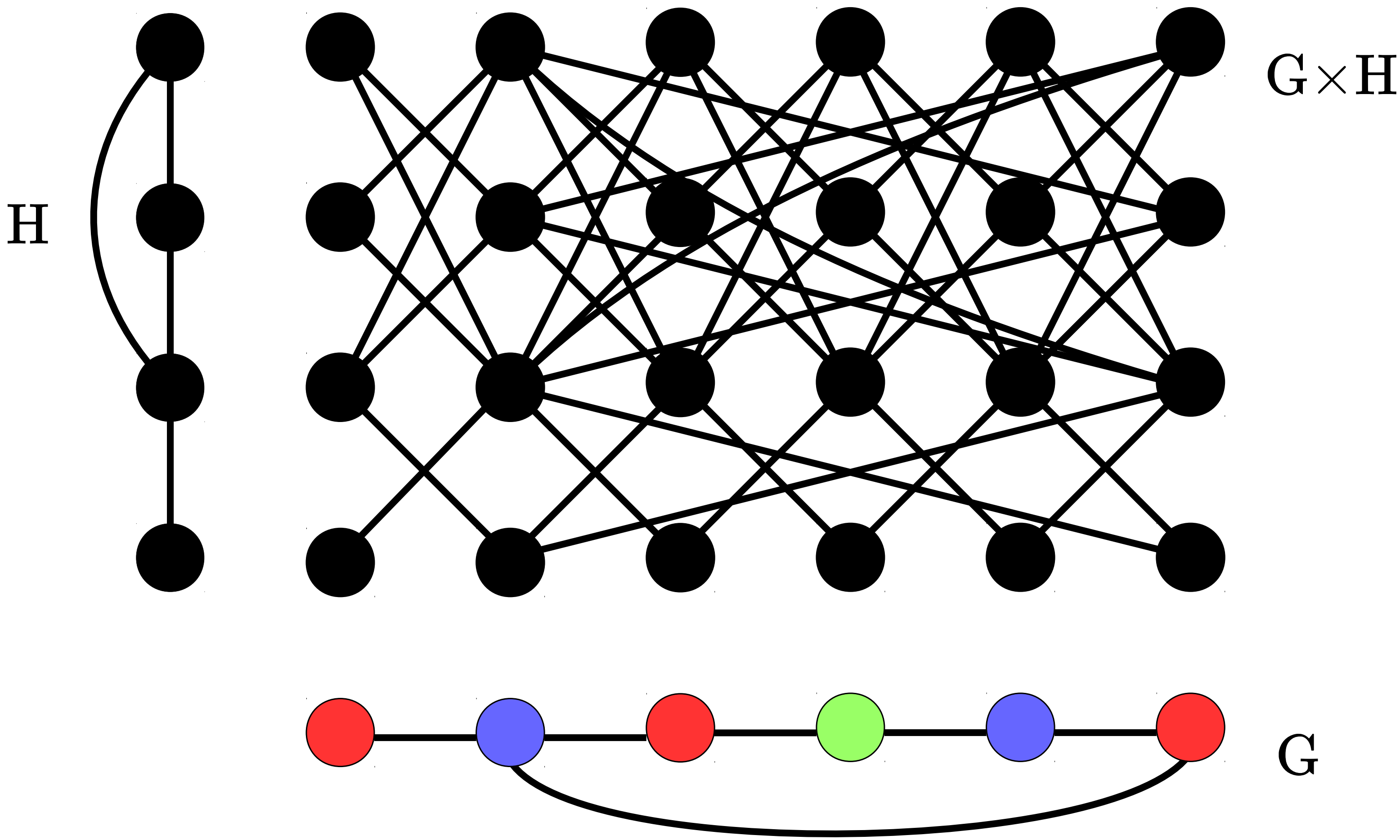


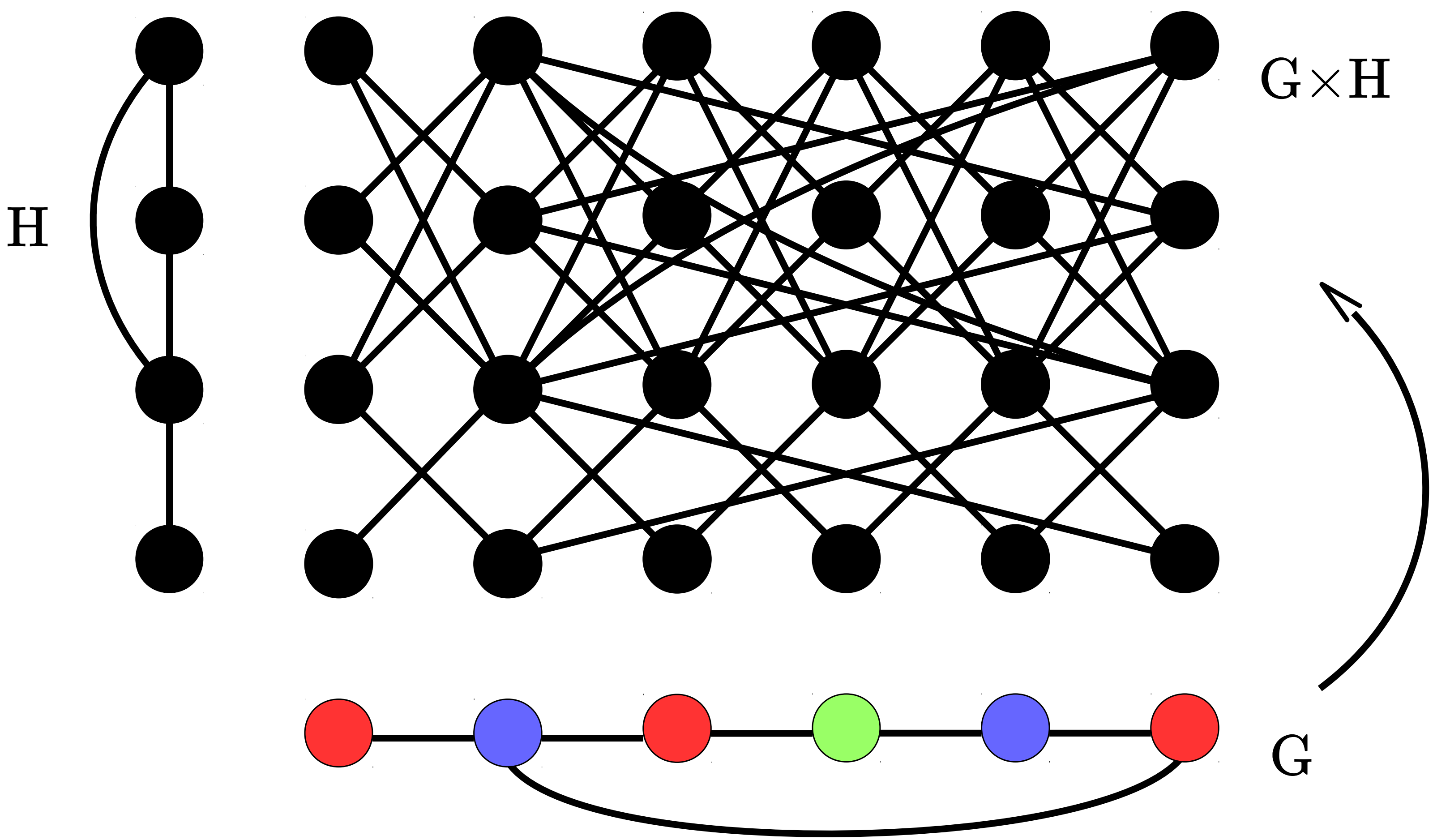
Colouring G×H



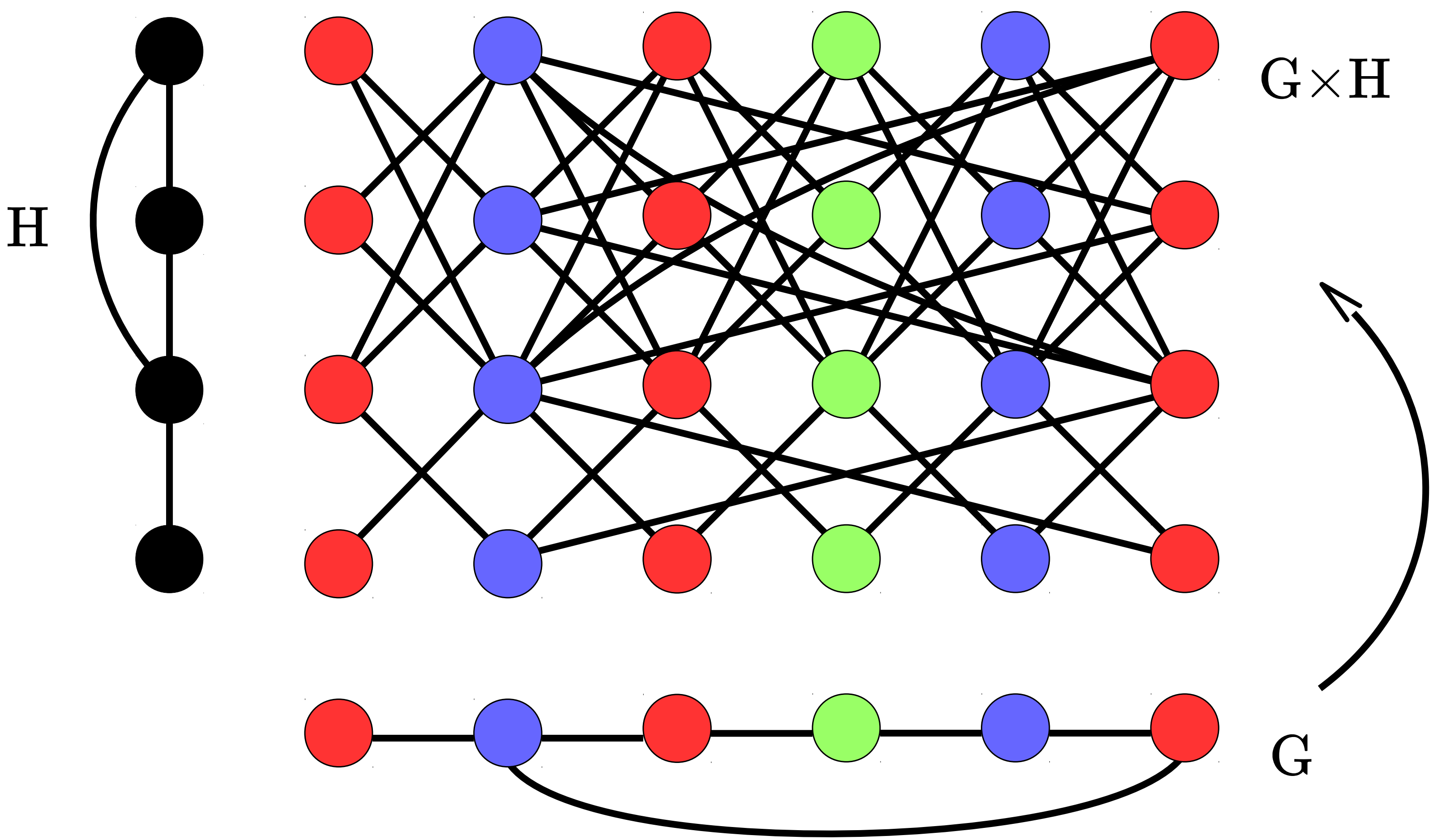
Colouring G×H *lazily*



Colouring G×H *lazily from*



Colouring G×H *lazily from* G



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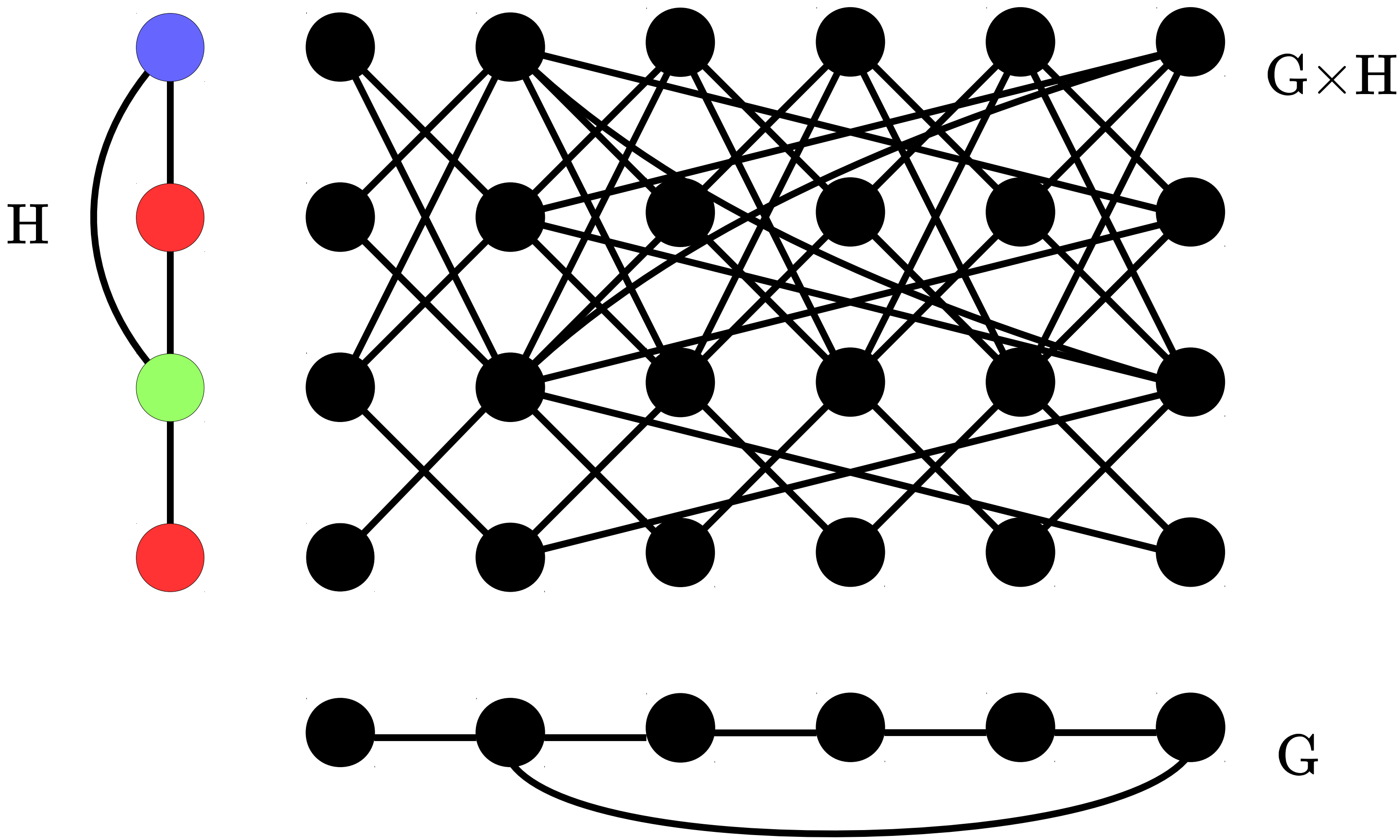
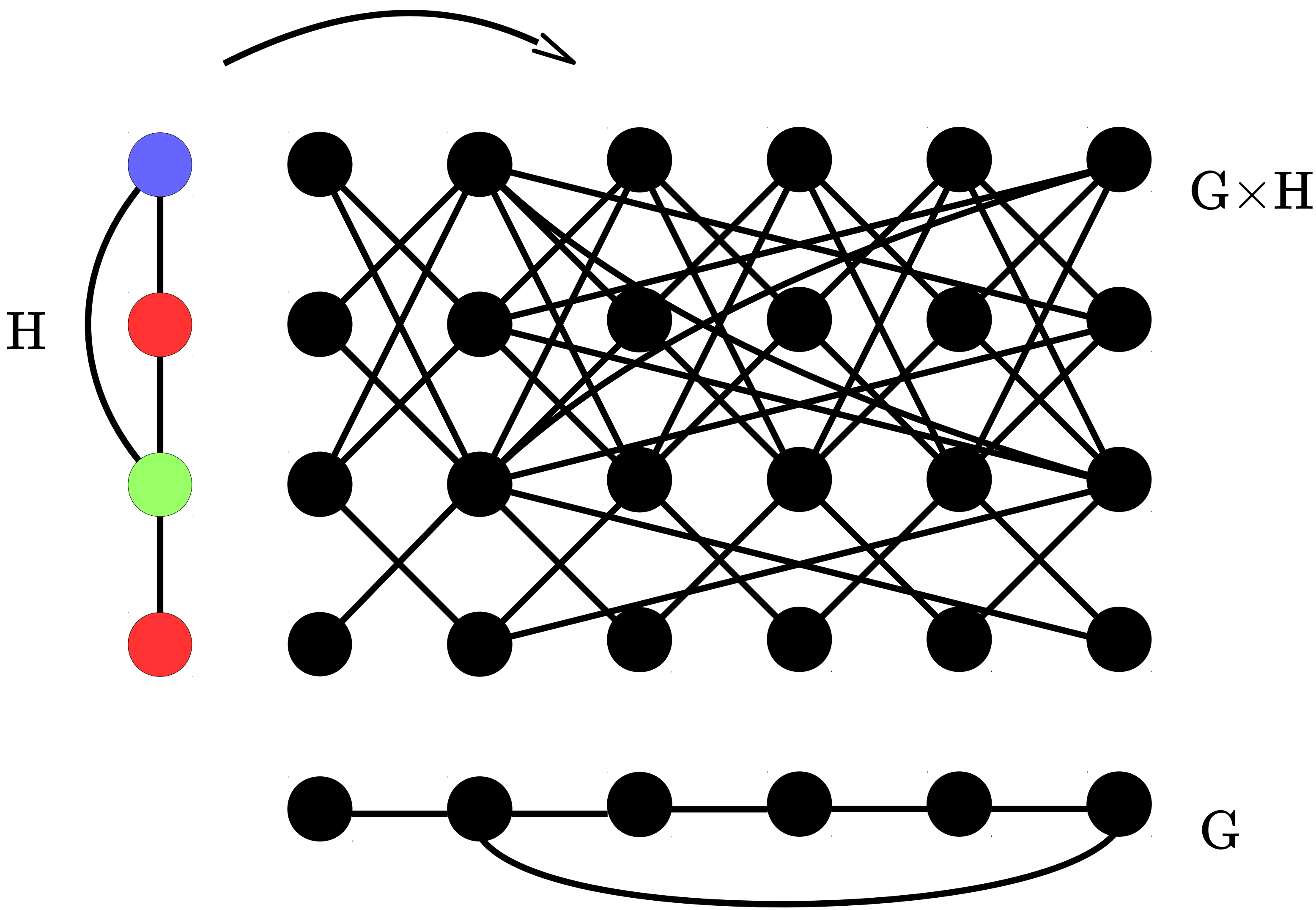
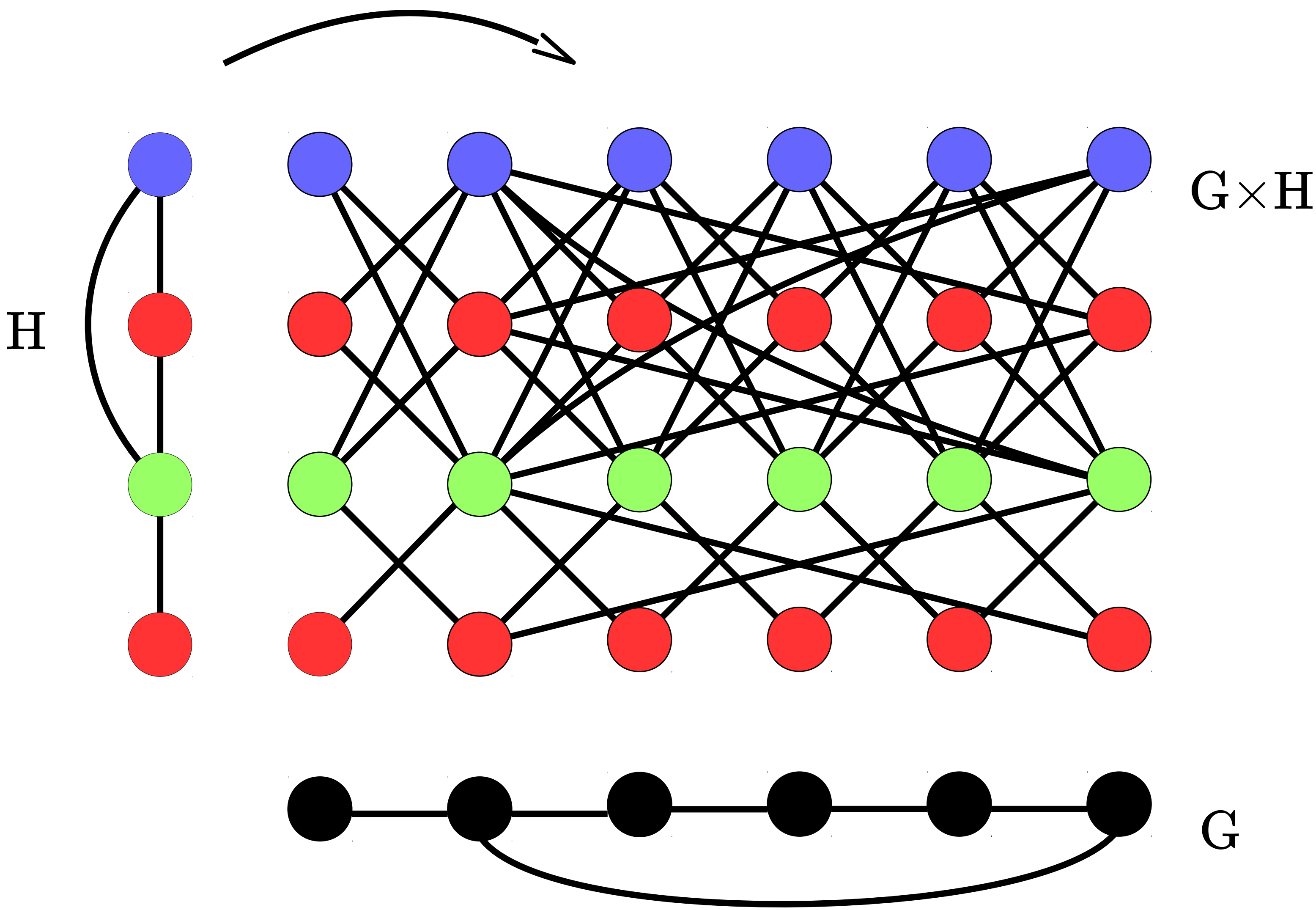


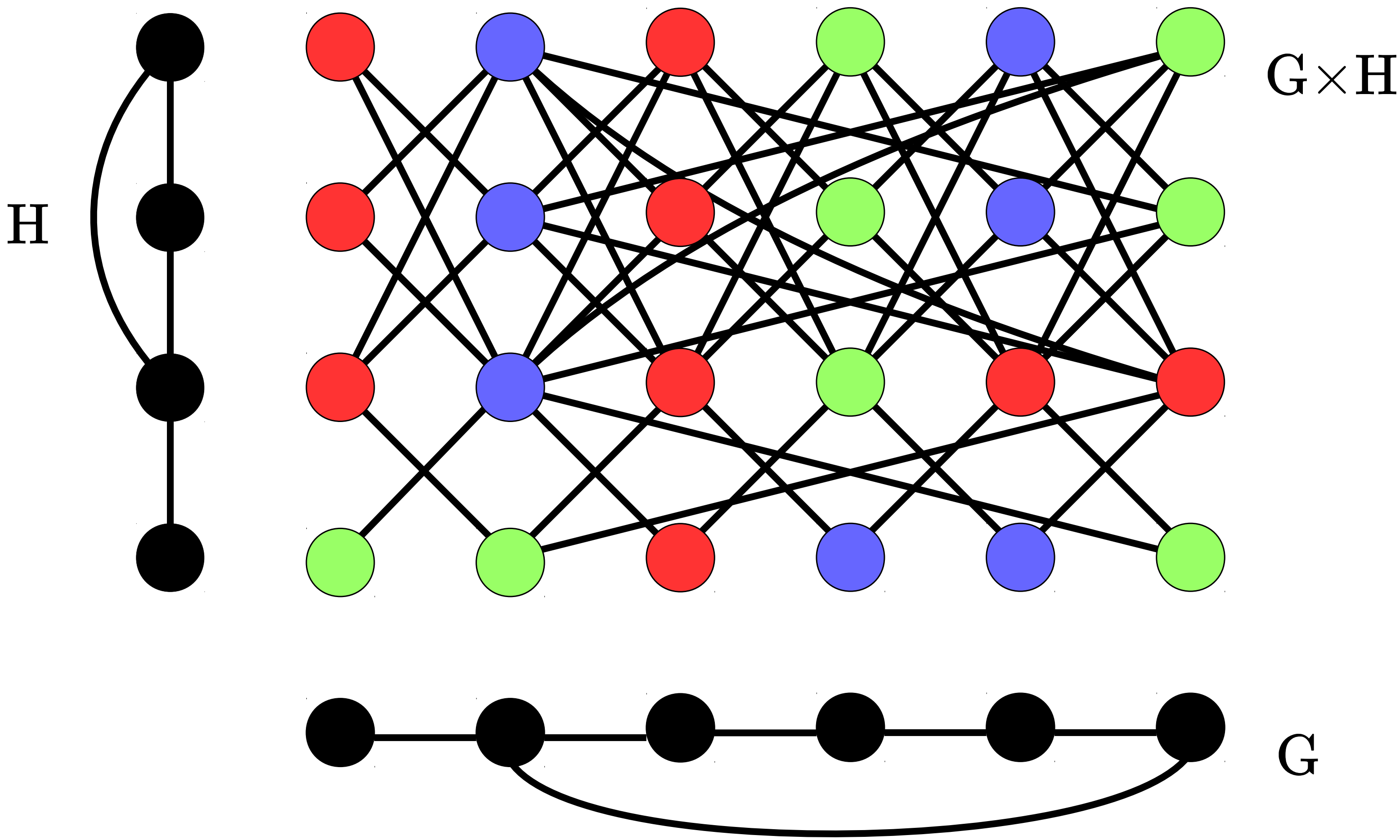
Colouring G × H *lazily from*



Colouring G × H *lazily from* H

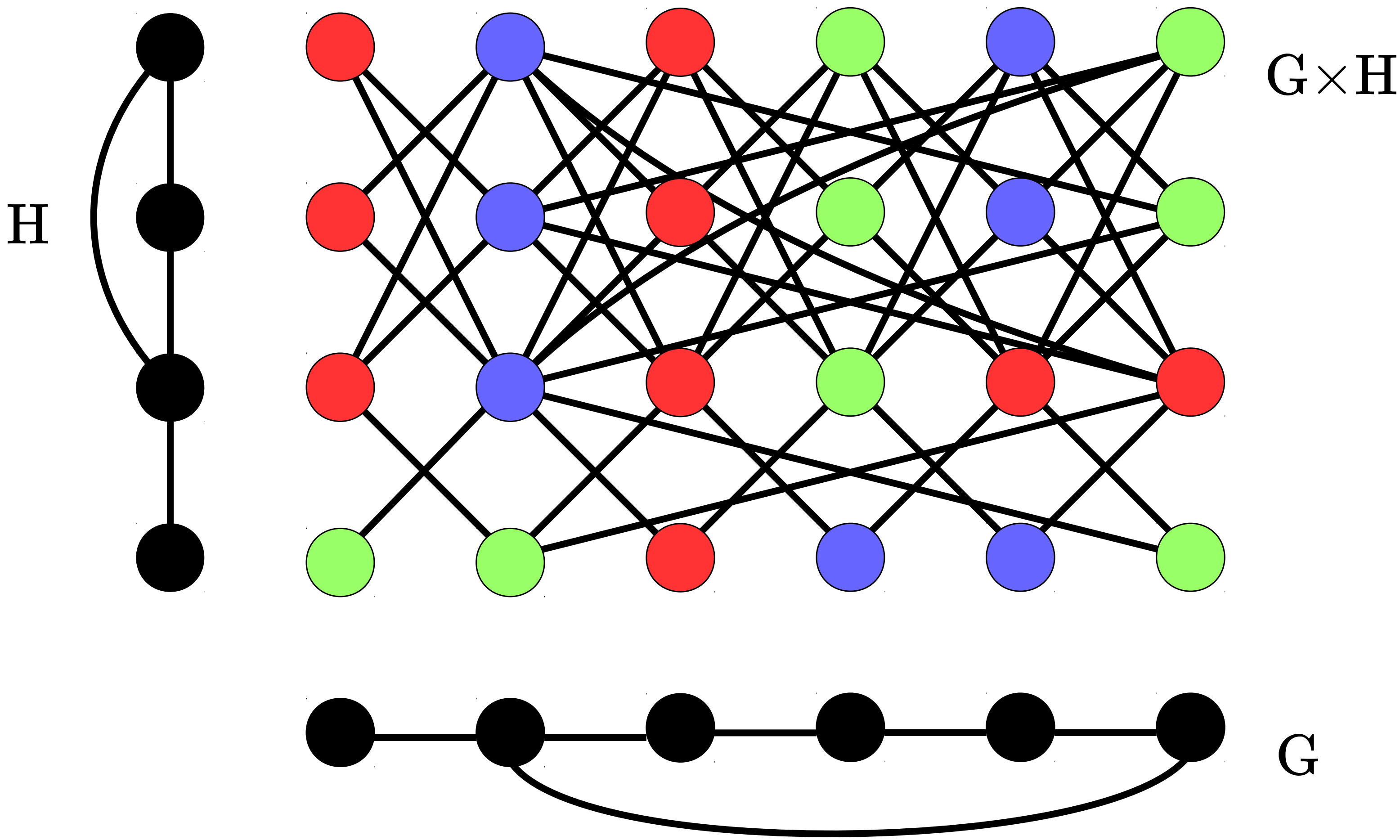


Colouring G×H *creatively*



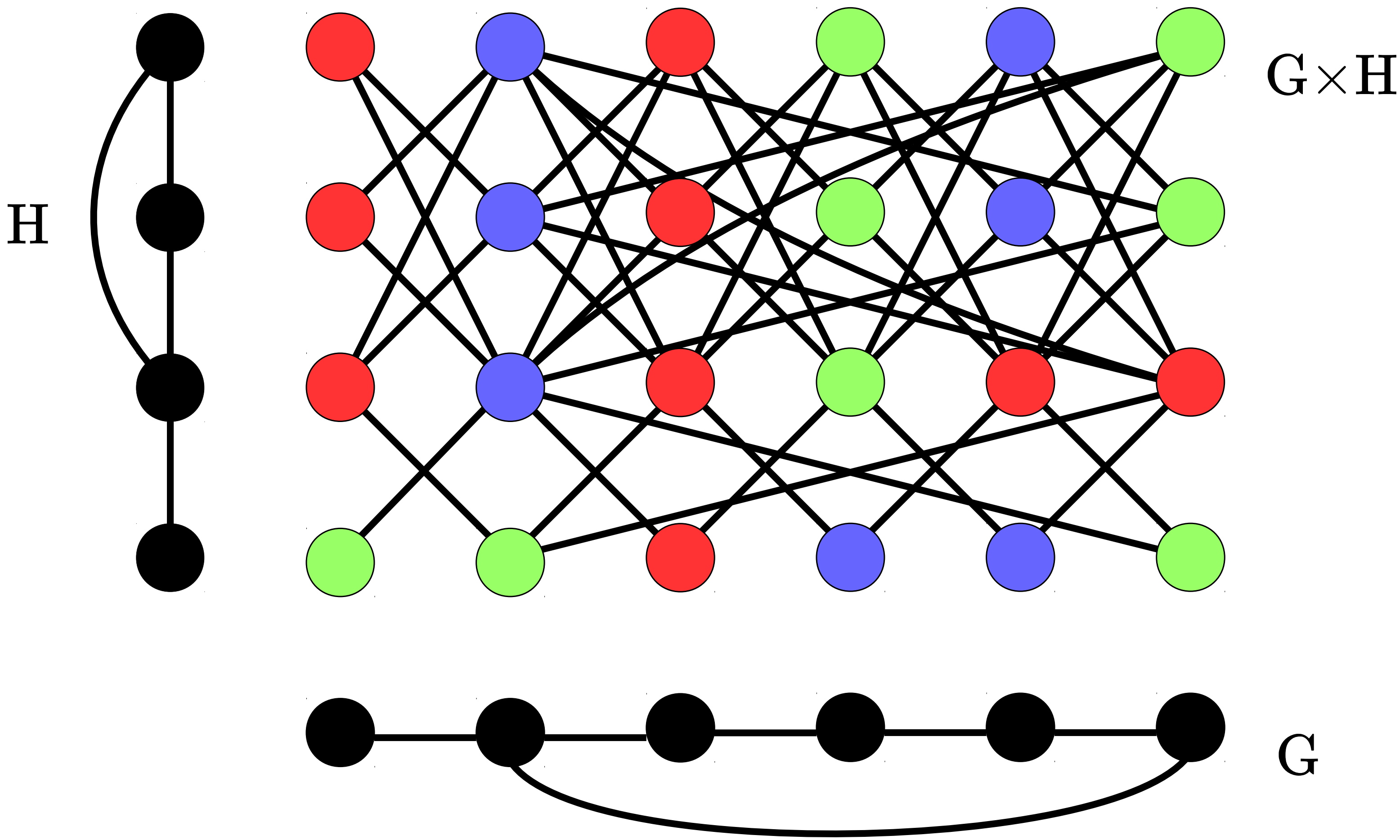
Colouring G×H *creatively*

Can we use fewer colours?



Hedetniemi's Conjecture (1966)

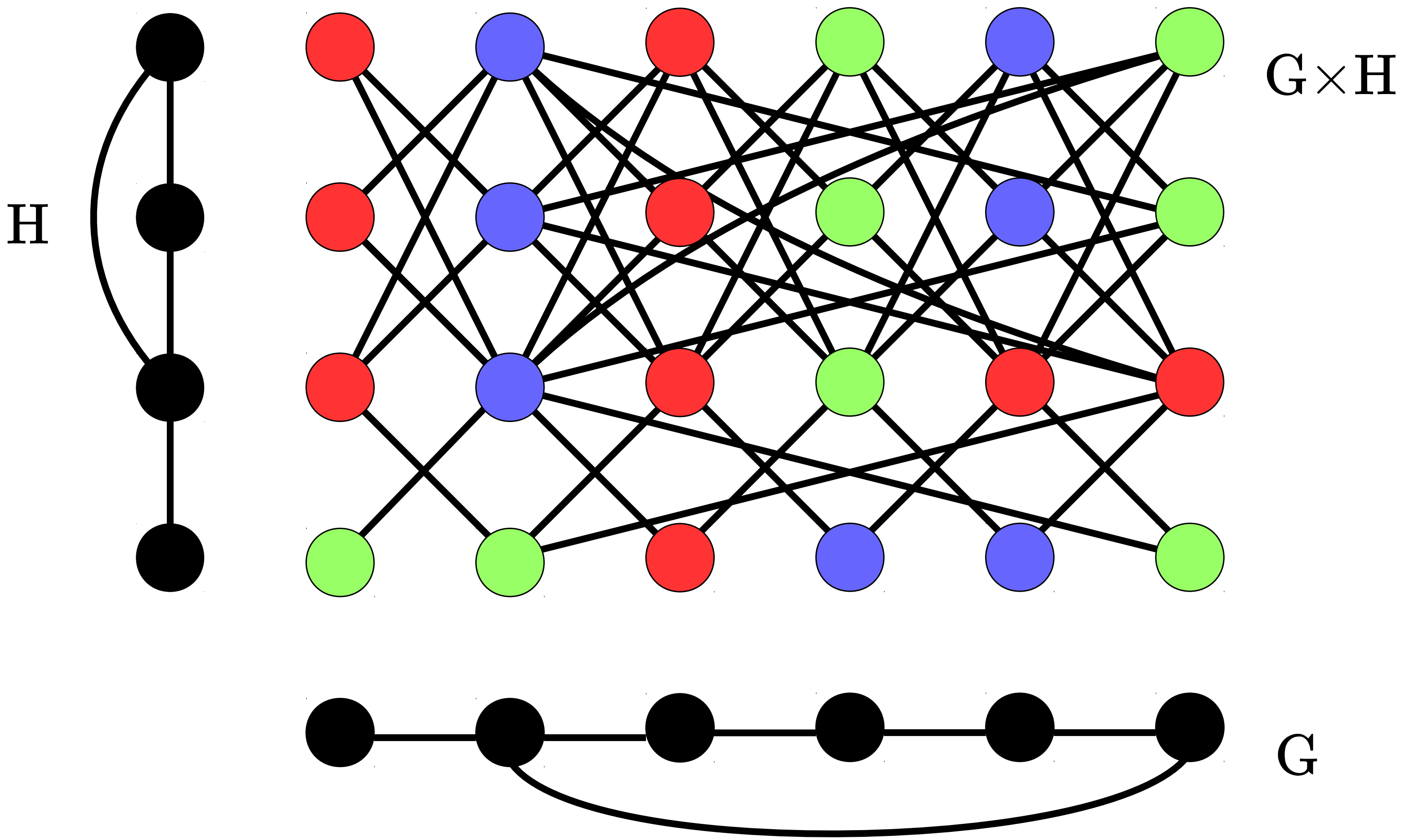
$$\chi(G \times H) = \min\{\ \chi(G), \chi(H)\ \}$$



Hedetniemi's Conjecture (1966)

$\chi(G \times H) = \min\{\ \chi(G), \chi(H)\ \}$

false! (Shitov 2019)



Hedetniemi's Conjecture (1966)

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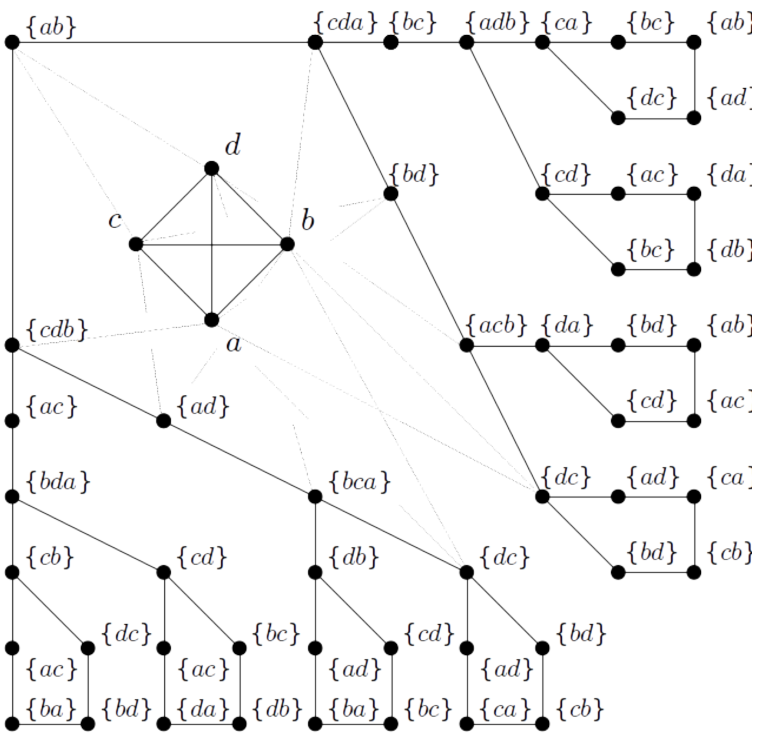
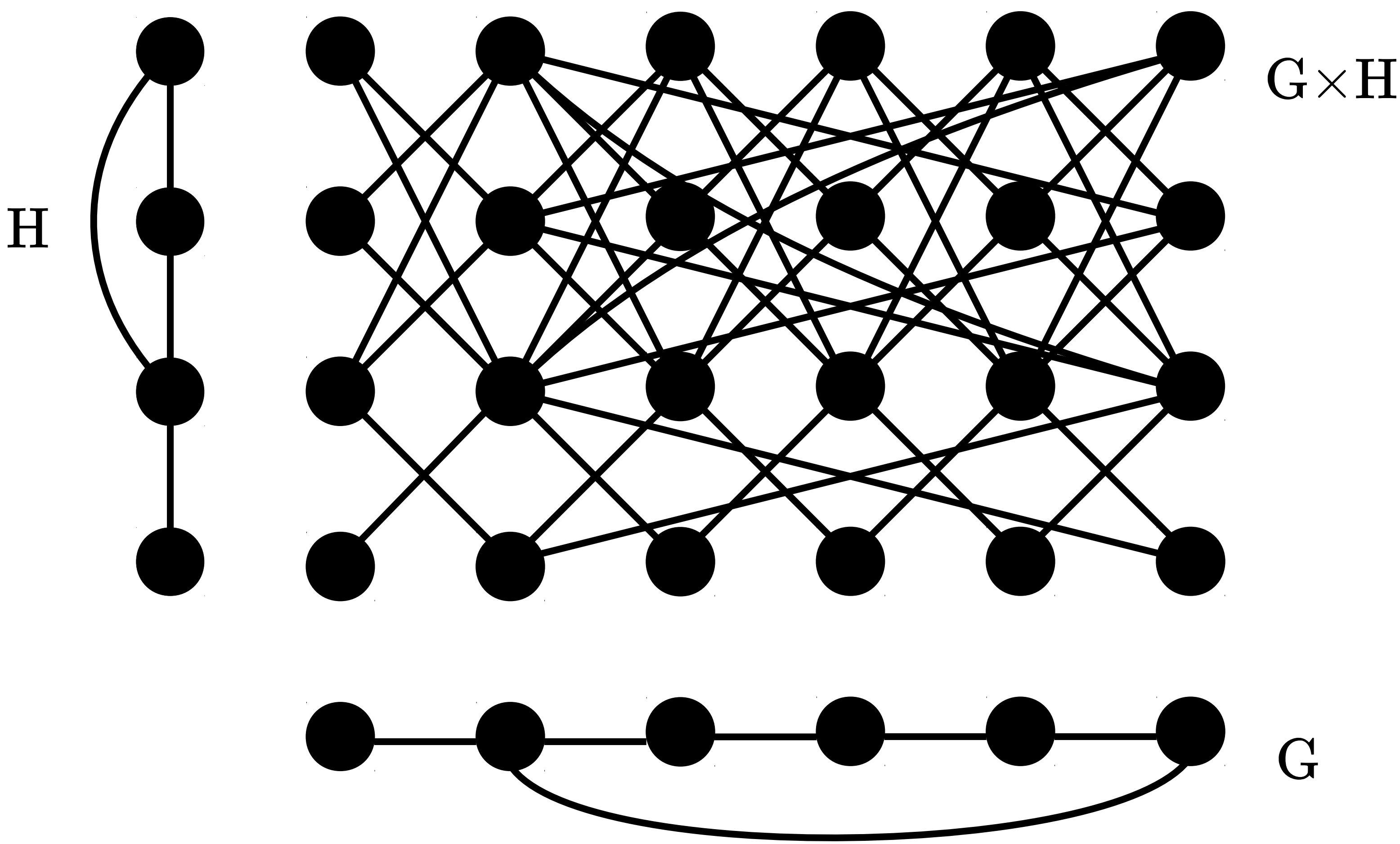
This talk:

- Why Hedetniemi's conjecture is still interesting
- The new counterexamples



$$\chi(G \times H) \geq \min\{\ \chi(G), \chi(H)\ \}\ ?$$

$$\chi \geq 2$$



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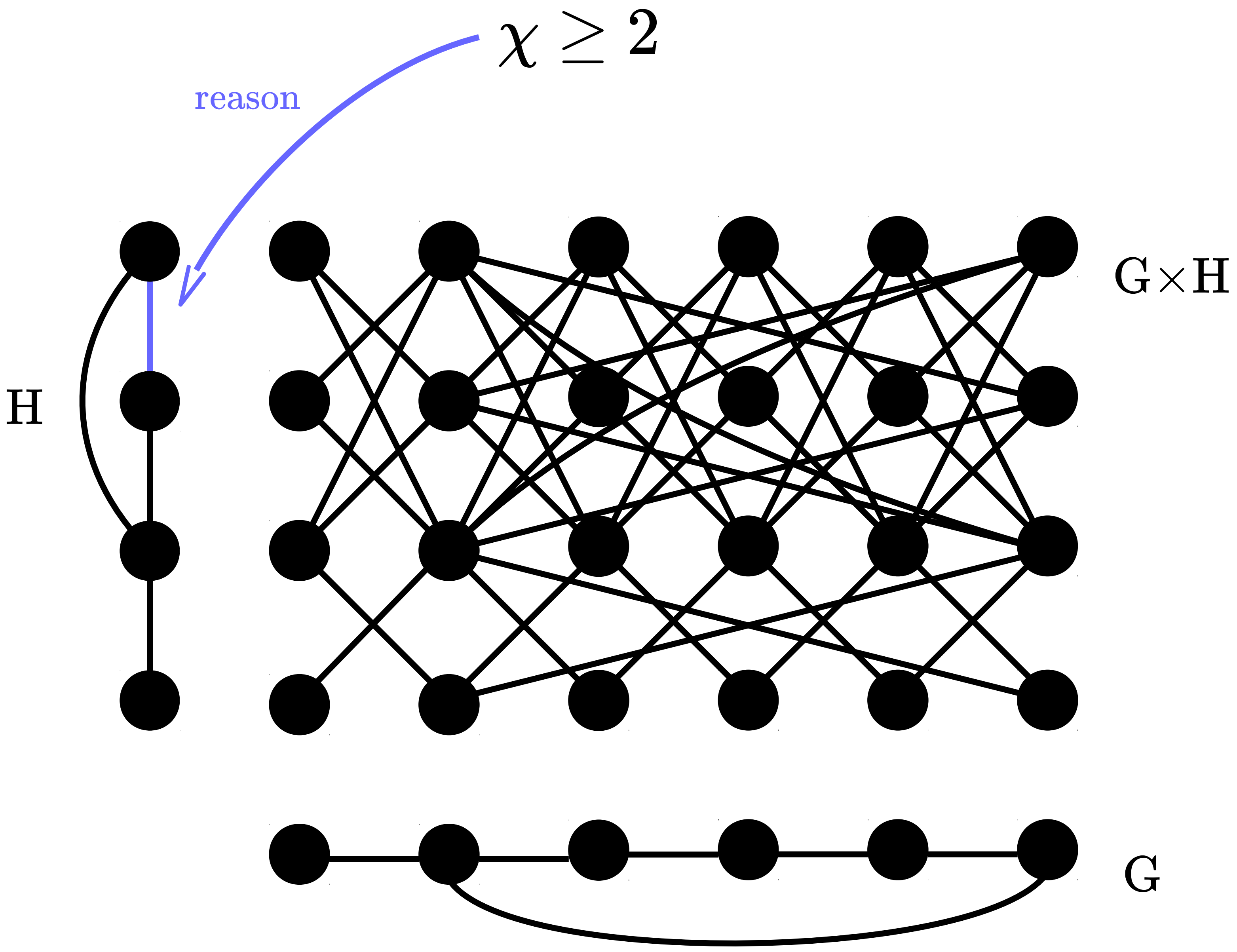

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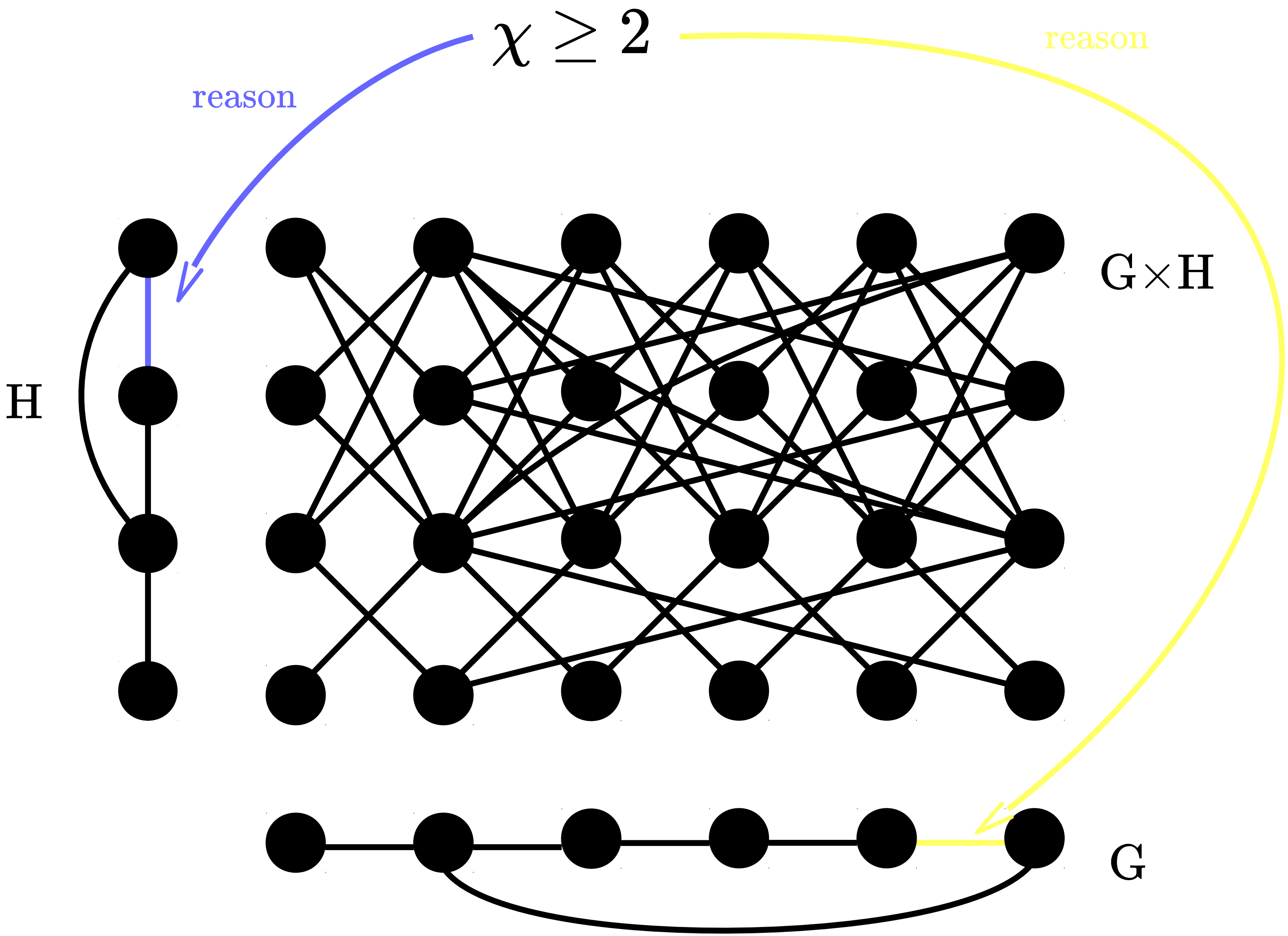

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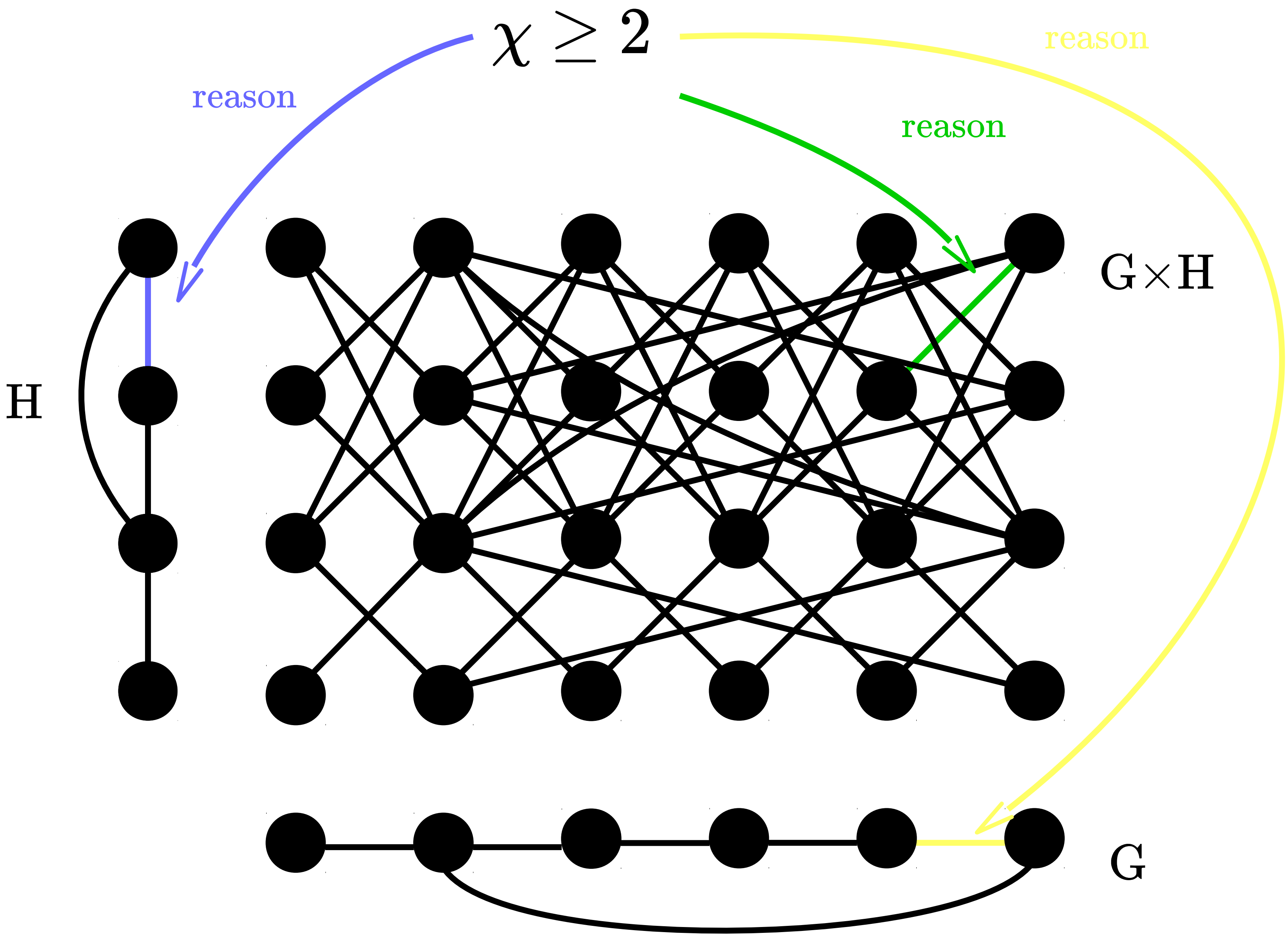
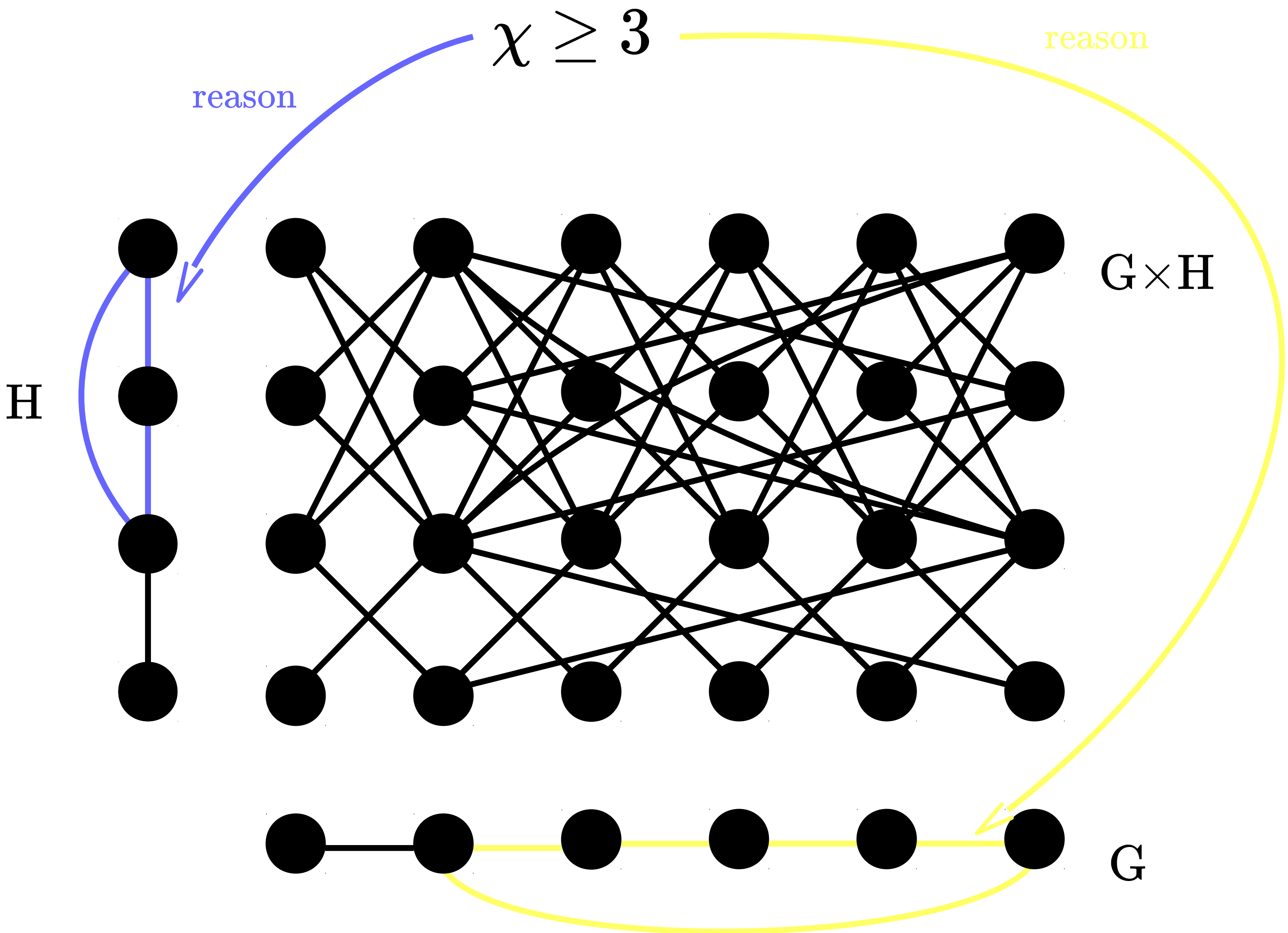


$\chi(G \times H) \geq \min\{\ \chi(G), \chi(H)\ \}$?



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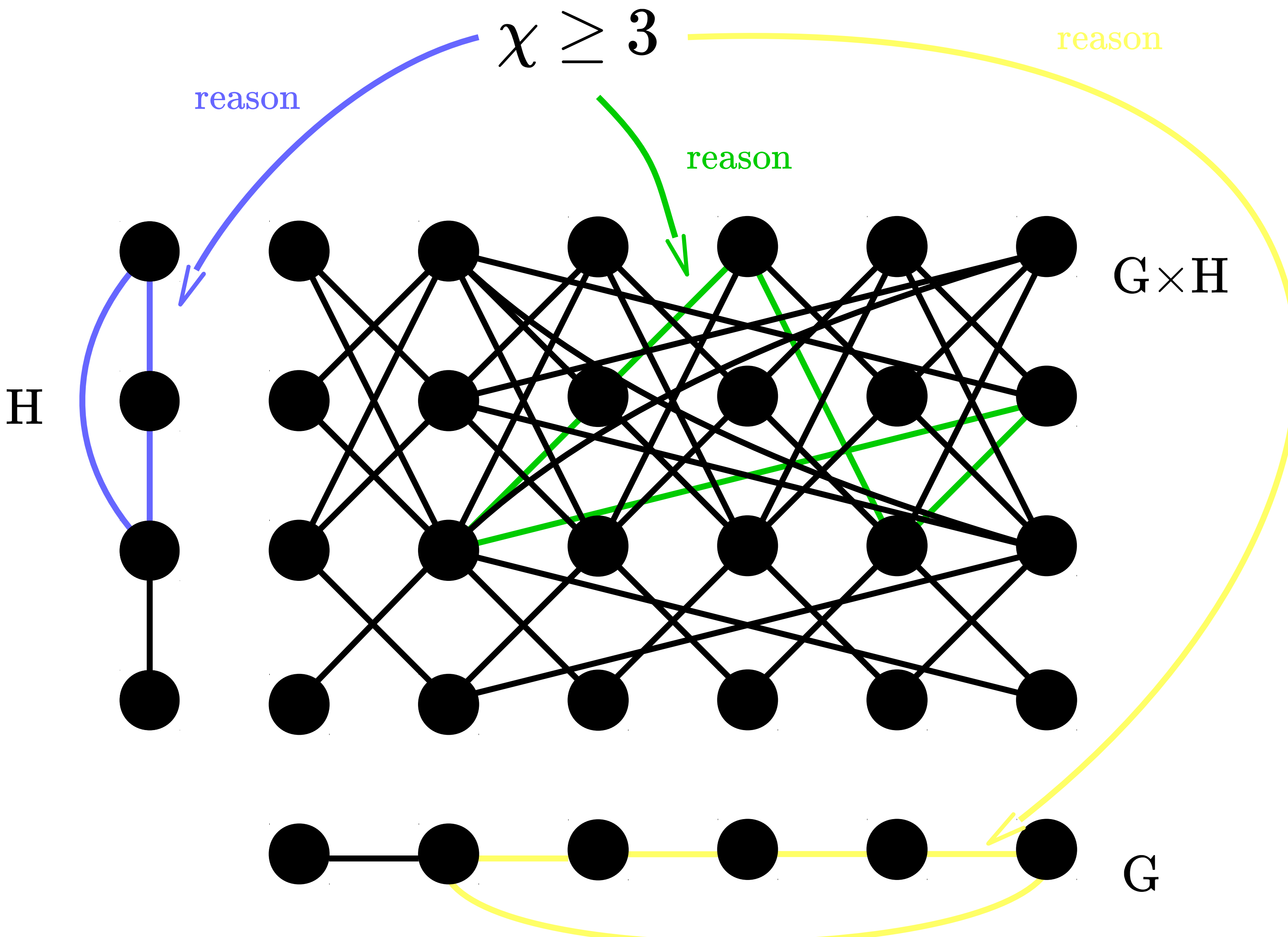

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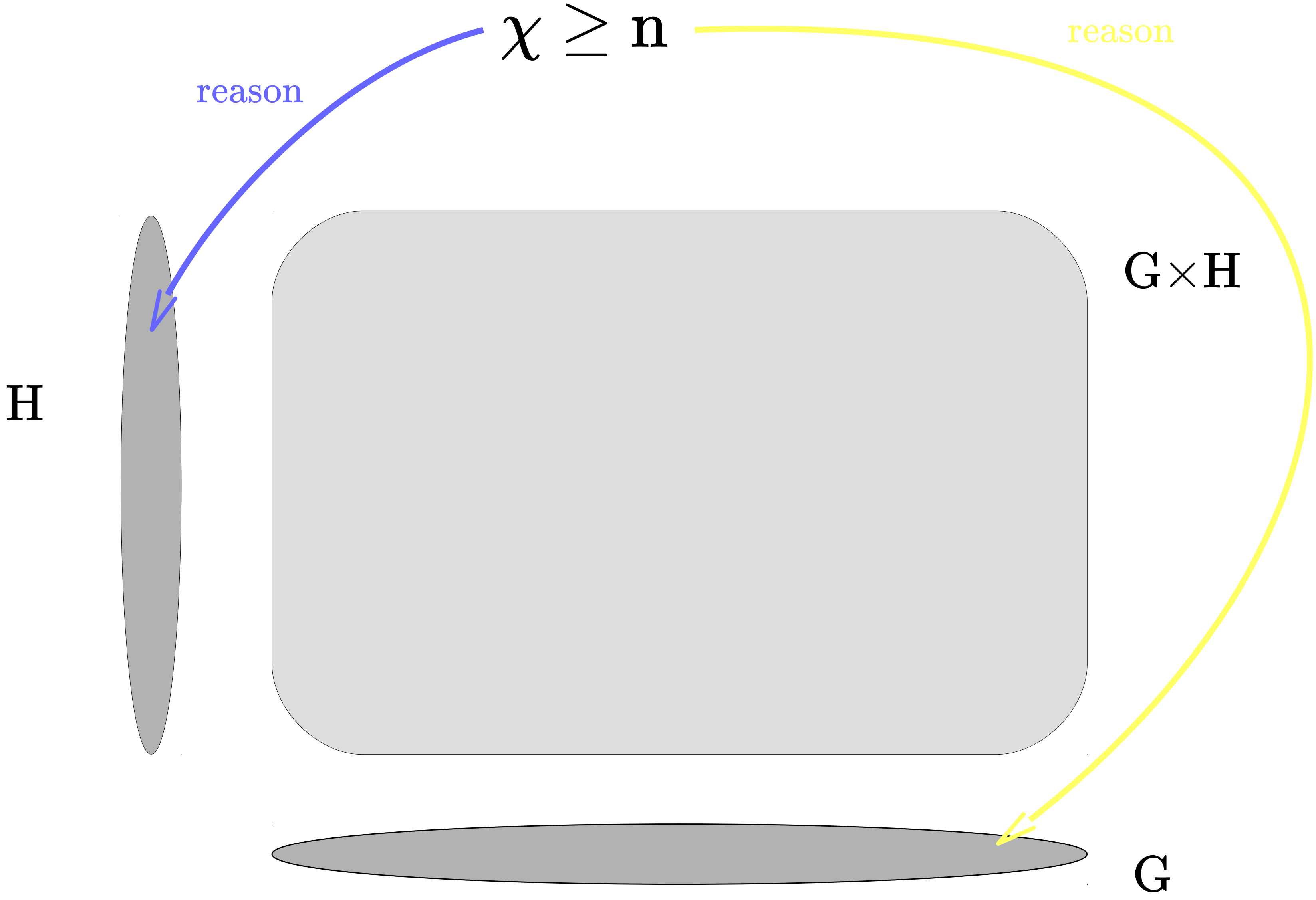

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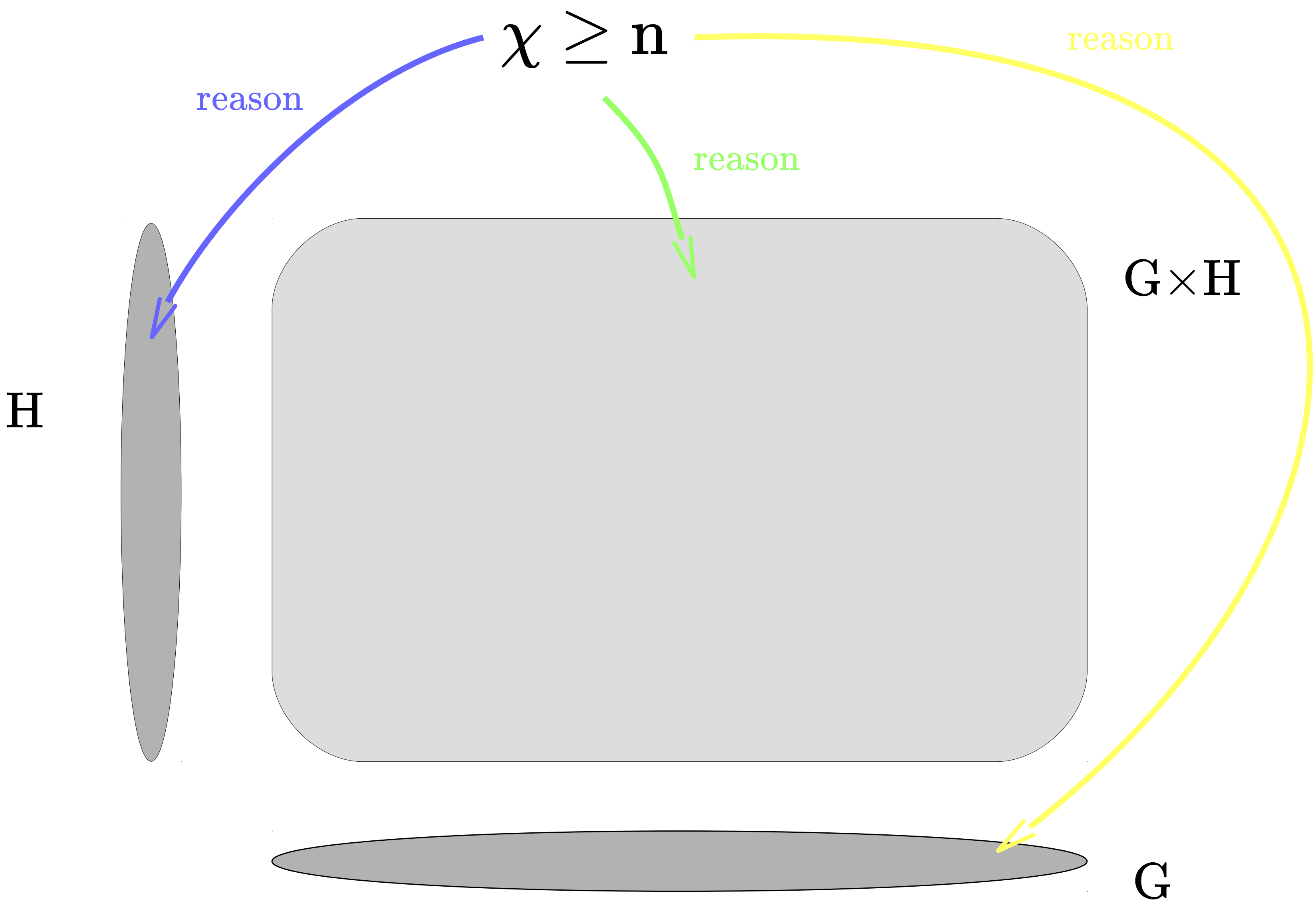

Reasons for $\chi(G) \geq n$

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Fractional chromatic number: $\chi_f(G) \geq n$

Zhu (2011): $\chi_f(G \times H) = \min\{ \chi_f(G), \chi_f(H) \}$

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Fractional chromatic number: $\chi_f(G) \geq n$

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Topological bound: $\mathrm{coind}(B(G)) + 2 \geq n$

Simonyi-Zsbán (2010):

$\mathrm{coind}(B(G \times H)) = \min\{ \mathrm{coind}(B(G)), \mathrm{coind}(B(H)) \}$

...

Hedetniemi's Conjecture

$\chi(\mathrm{G}\times\mathrm{H}) = \min\{\ \chi(\mathrm{G}), \chi(\mathrm{H})\ \}$

false!

Shitov 2019

Zhu 2020

Tardif 2020

Wrochna 2020

Hedetniemi's Conjecture

$\chi(G \times H) = \min\{ \chi(G), \chi(H) \}$

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Shitov 2019: $\chi(G \times H) \geq 3^{95}$

Zhu 2020: $\chi(G \times H) \geq 125$

Tardif 2020: $\chi(G \times H) \geq 13$

Wrochna 2020: $\chi(G \times H) \geq 5$

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Today: $\chi(G \times H) \geq 4$

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El-Zahar, Sauer 1985:

$$\chi(G \times H) = \min\{ \chi(G), \chi(H) \}$$

when $\min\{ \chi(G), \chi(H) \} \leq 4$

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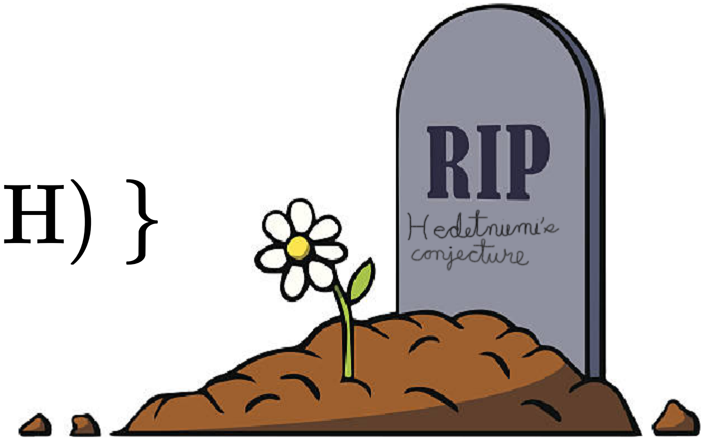

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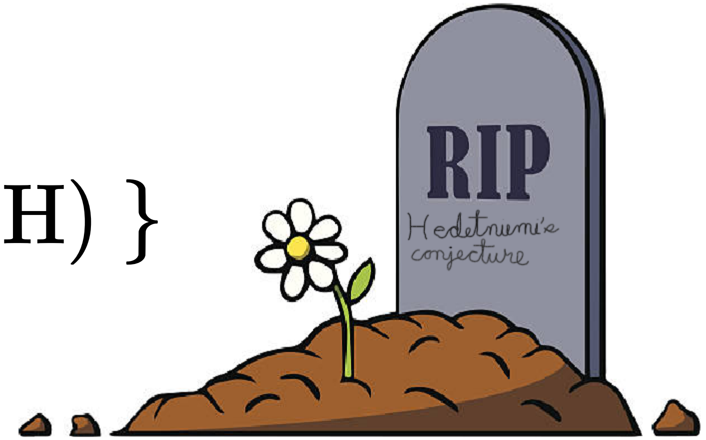

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Today: $\chi(G \times H) \geq 4$ Topological bound

El-Zahar, Sauer 1985:

$$\chi(G \times H) = \min\{ \chi(G), \chi(H) \} \quad \text{when } \min\{ \chi(G), \chi(H) \} \leq 4$$

Today: $\chi(\Omega_{13}(K_8) \times {K_4}^{\Omega_{13}(K_8)}) = 4$

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Monday: $\chi(\Omega_{17}(K_{16}) \times K_4^{\Omega_{17}(K_{16})}) = 4$

Tuesday: $\chi(\Omega_{21}(K_{32}) \times K_4^{\Omega_{21}(K_{32})}) = 4$

Wednesday: $\chi(\Omega_{25}(K_{64}) \times K_4^{\Omega_{25}(K_{64})}) = 4$

Thursday: $\chi(\Omega_{29}(K_{128}) \times K_4^{\Omega_{29}(K_{128})}) = 4$

Friday: $\chi(\Omega_{33}(K_{256}) \times K_4^{\Omega_{33}(K_{256})}) = 4$

Today: $\chi(\Omega_{13}(K_8)\times K_4^{\Omega_{13}(K_8)})= 4$

Monday: $\chi(\Omega_{17}(K_{16})\times K_4^{\Omega_{17}(K_{16})})= 4$

Tuesday: $\chi(\Omega_{21}(K_{32})\times K_4^{\Omega_{21}(K_{32})})= 4$

Wednesday: $\chi(\Omega_{25}(K_{64})\times K_4^{\Omega_{25}(K_{64})})= 4$

Thursday: $\chi(\Omega_{29}(K_{128})\times K_4^{\Omega_{29}(K_{128})})= 4$

Friday: $\chi(\Omega_{33}(K_{256})\times K_4^{\Omega_{33}(K_{256})})= 4$

For all m > n there exist graphs G, H with $\chi(G) = m$, $\chi(H) > 4$, $\chi(G\times H)= 4$

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$\Omega_w(K_m)$:

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$$\Omega_w(K_m):$$

Vertices: $(X_0, X_1, \ldots, X_v)$ where $w = 2v+1$

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$$\Omega_w(K_m):$$

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- $X_i \subseteq V(K_m)$, $i = 0, ..., v$

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Edges: $(X_0, X_1, \ldots, X_v)$--$(Y_0, Y_1, \ldots, Y_v)$ where

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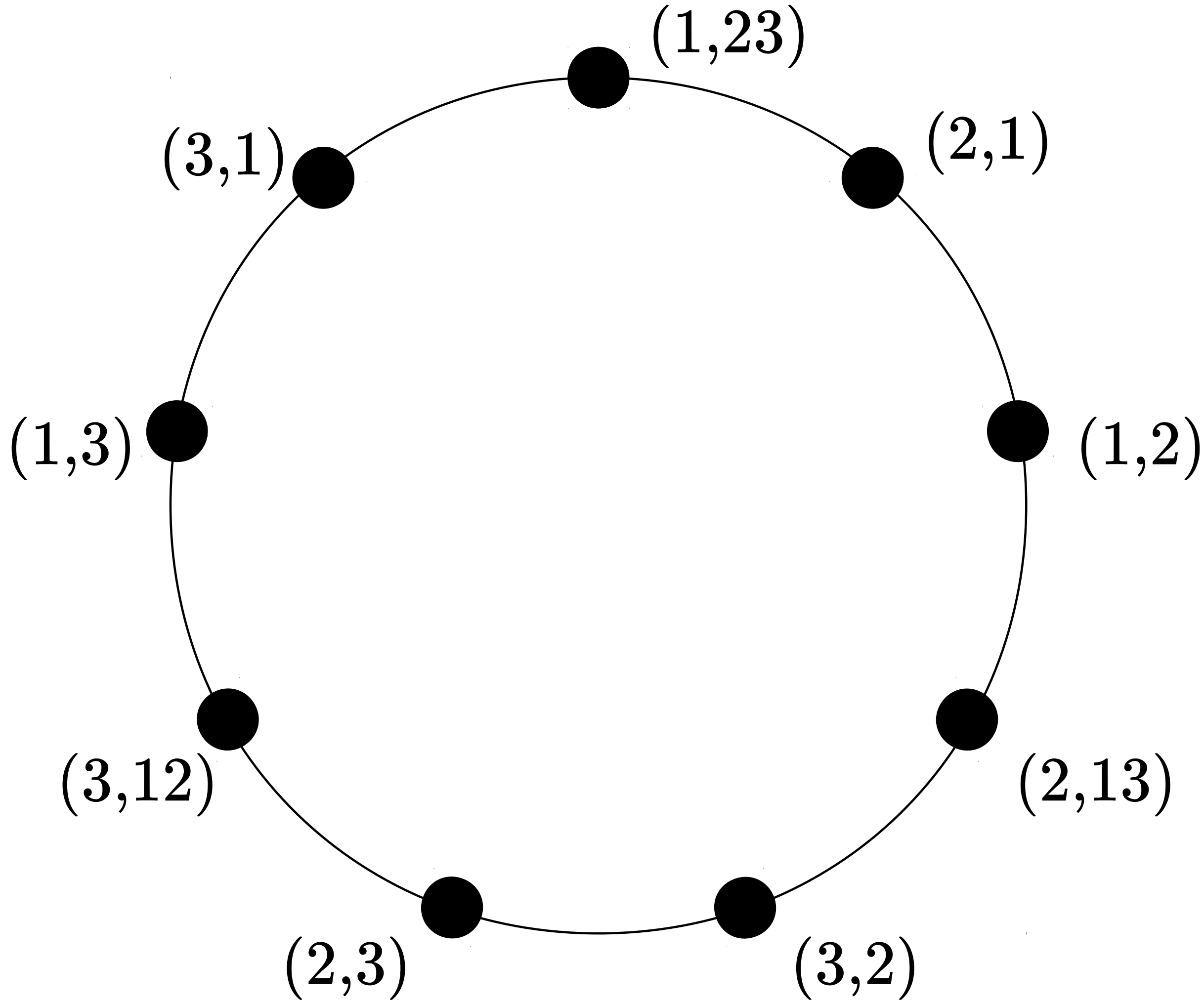
$\Omega_3(K_3)$:



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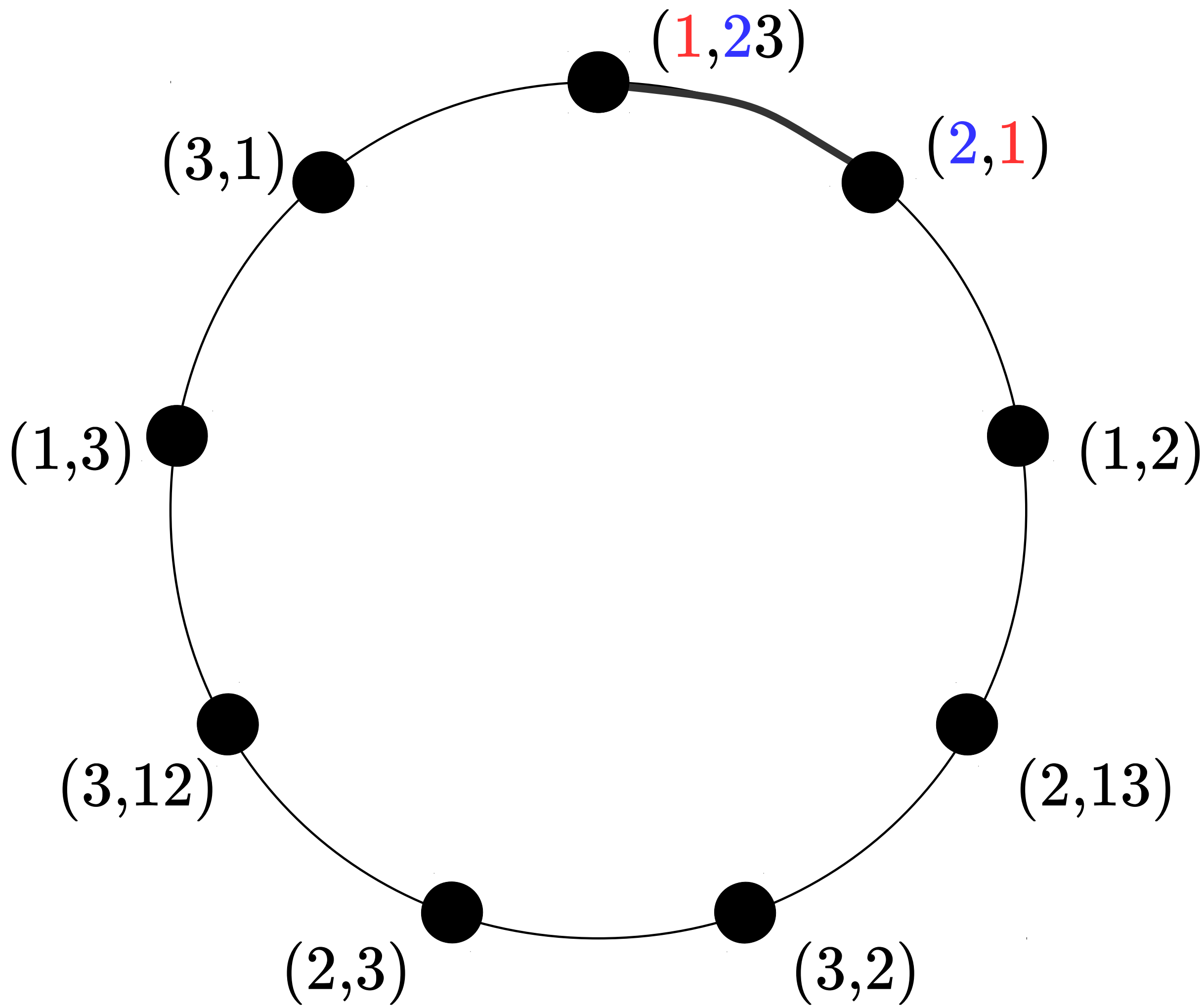

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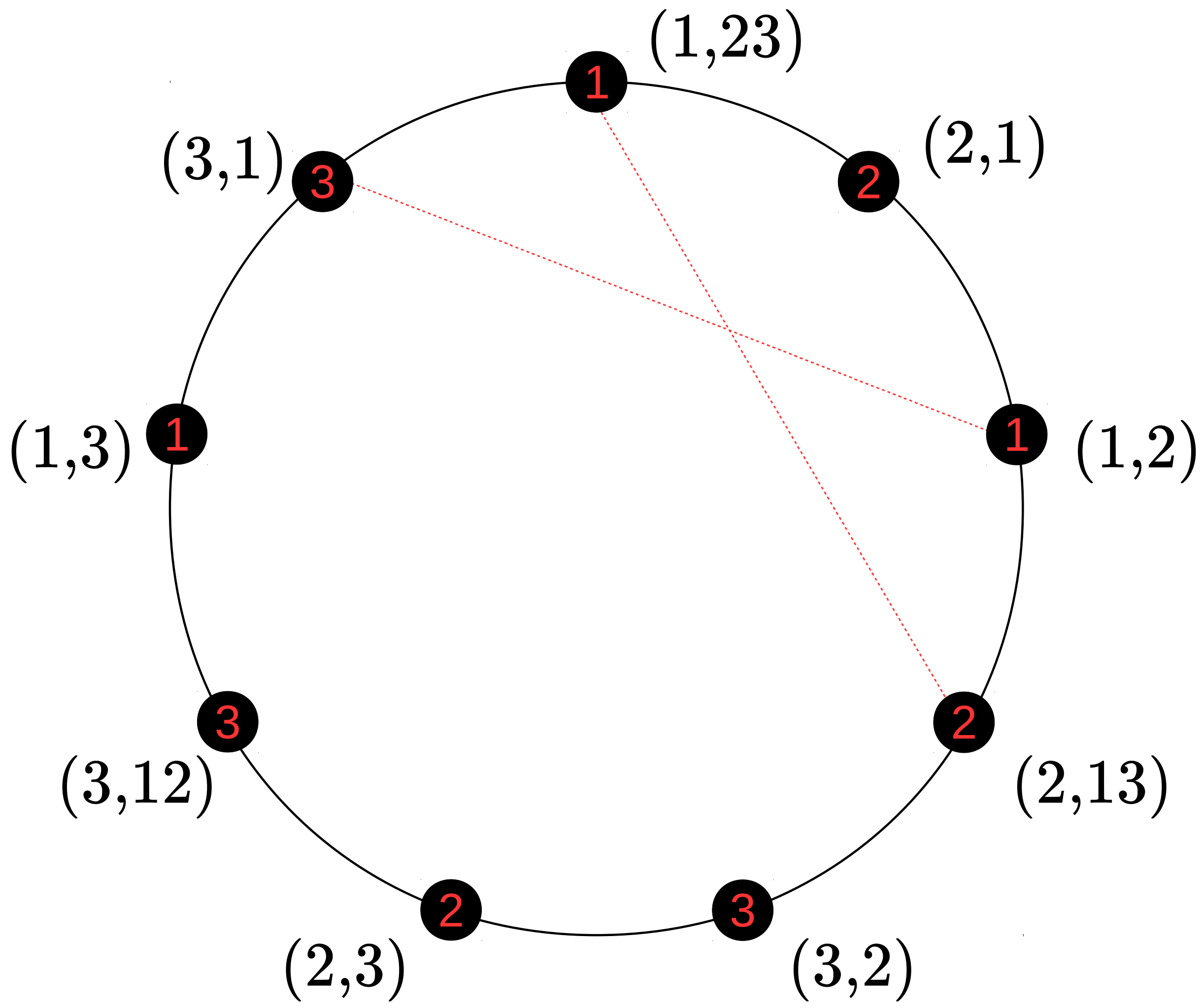
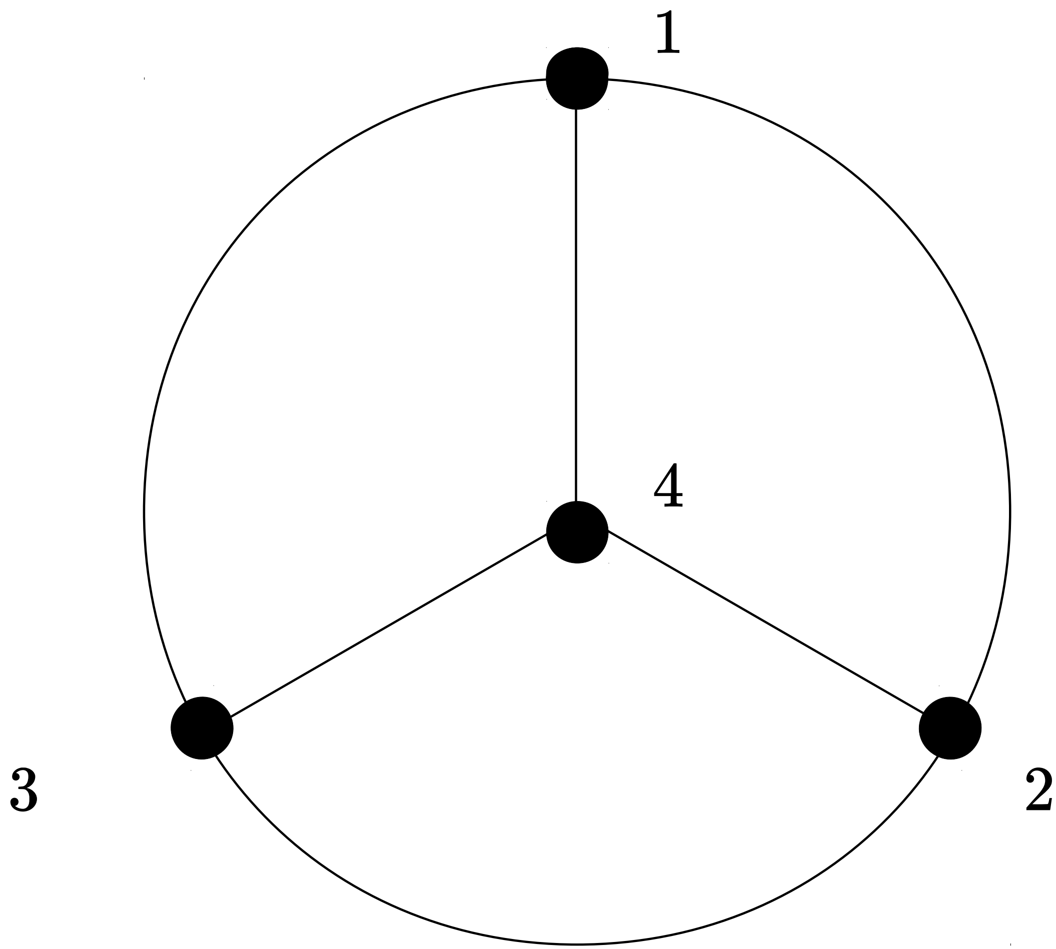
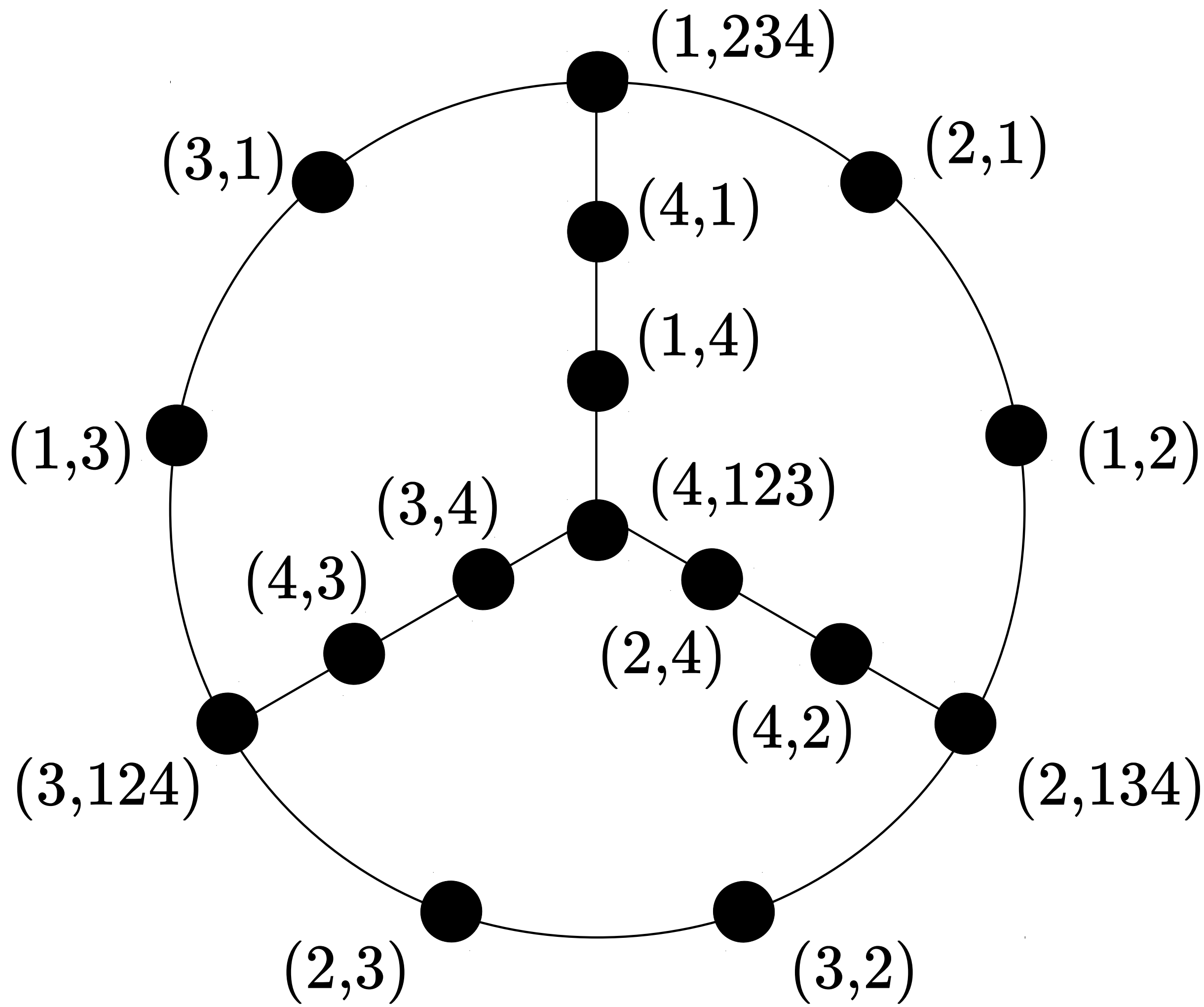


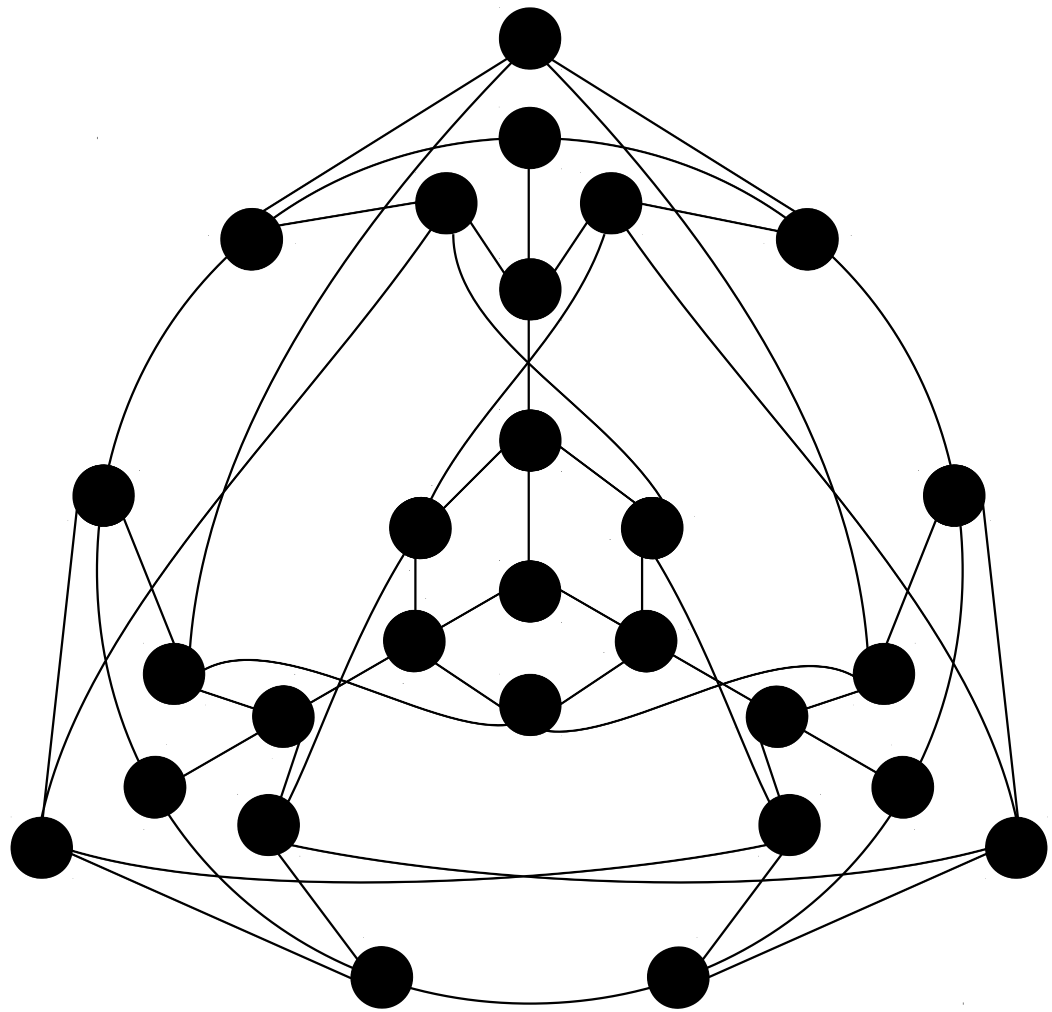
K_4 :



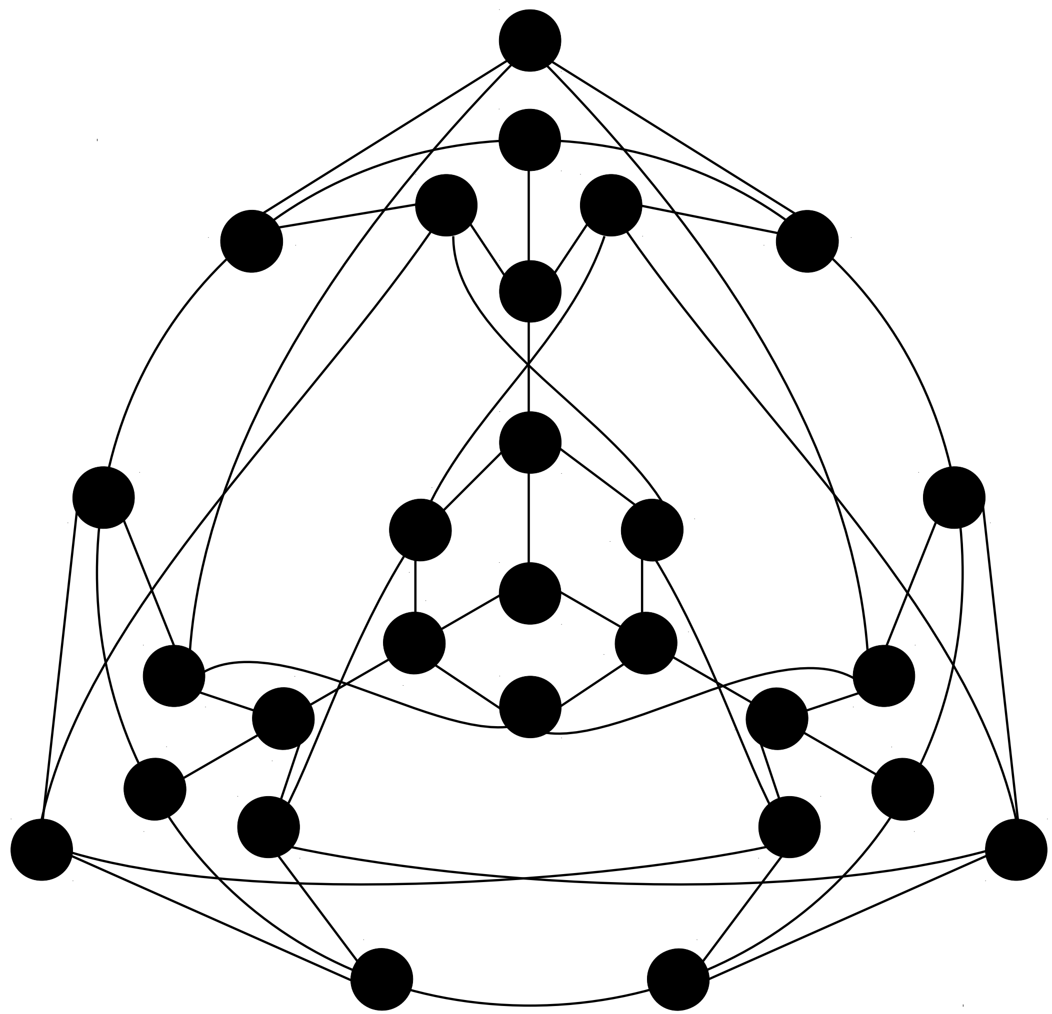
$\Lambda_3(\mathrm{K}_4)$:



$\Omega_3(K_4)$:



Theorem: $\chi(\Omega_w(K_m)) = m$ (Hajiabolhassan, Wrochna, ...)



Today: $\chi(\Omega_{13}(K_8) \times K_4^{\Omega_{13}(K_8)}) = 4$

$\Omega_{13}(K_8)$: 8-chromatic, 4348856 vertices

Today: $\chi(\Omega_{13}(K_8) \times {K_4}^{\Omega_{13}(K_8)}) = 4$

$\Omega_{13}(K_8)$: 8-chromatic, 4348856 vertices

${K_4}^{\Omega_{13}(K_8)}$: $4^{4348856}$ vertices

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$\Omega_{13}(K_8)$: 8-chromatic, 4348856 vertices

${K_4}^{\Omega_{13}(K_8)}$: $4^{4348856}$ vertices

vertices: all functions f: $V(\Omega_{13}(K_8)) \to V(K_4)$

Today: $\chi(\Omega_{13}(K_8) \times K_4^{\Omega_{13}(K_8)}) = 4$

$\Omega_{13}(K_8)$: 8-chromatic, 4348856 vertices

$K_4^{\Omega_{13}(K_8)}$: $4^{4348856}$ vertices

vertices: all functions f: $V(\Omega_{13}(K_8)) \rightarrow V(K_4)$

edges: f--g such that for all u--v in $\Omega_{13}(K_8)$, we have f(u)--g(v) in K_4

Special functions in $K_n^{\Omega_w(K_m)}$

Special functions in $K_n^{\Omega_w(K_m)}$

$\sigma^{i,}{}_{a,}{}^{S}{}_{b}$, $i \in \{0, ..., v\}$, $S \subseteq V(K_m)$, $a, b \in V(K_n)$

Special functions in $K_n^{\Omega_w(K_m)}$

$\sigma^{i,}{}_{a,}{}^{S}{}_{b}$, $i \in \{0, \ldots, v\}$, $S \subseteq V(K_m)$, $a, b \in V(K_n)$

$$\sigma^{i,}{}_{a,}{}^{S}{}_{b}(X_0, X_1, \ldots, X_v) =$$

Special functions in $K_n^{\Omega_w(K_m)}$

$\sigma^{i,}{}_{a,}{}^{S}{}_{b}$, $i \in \{0, ..., v\}$, $S \subseteq V(K_m)$, $a, b \in V(K_n)$

$$\sigma^{i,}{}_{a,}{}^{S}{}_{b}(X_0, X_1, ..., X_v) =$$
$$a \text{ if } X_i \cap S \neq \emptyset$$

Special functions in $K_n^{\Omega_w(K_m)}$

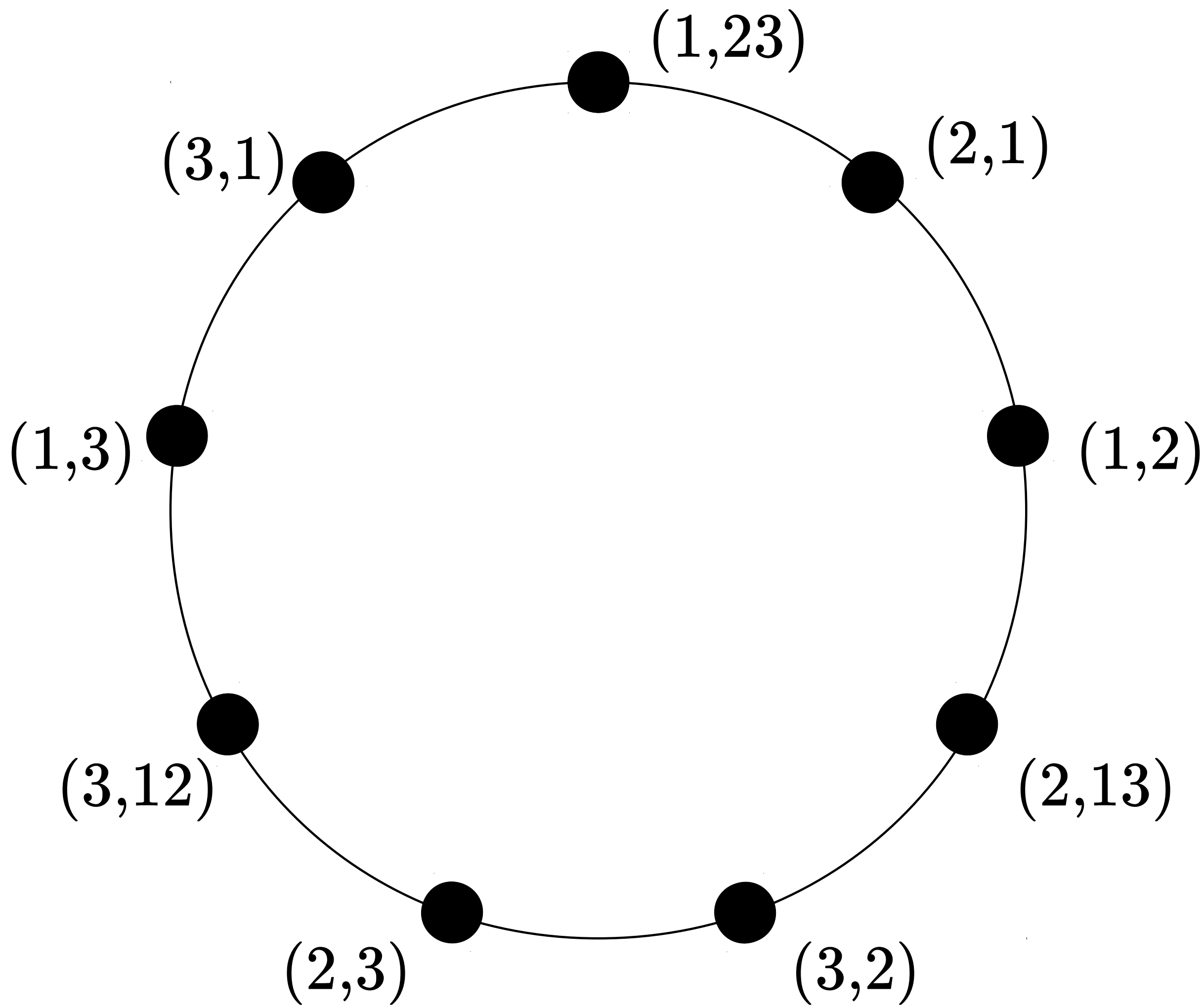
$\sigma^{i,}{}_{a,}{}^{S}{}_{b}$, $i \in \{0, \ldots, v\}$, $S \subseteq V(K_m)$, $a, b \in V(K_n)$

$\sigma^{i,}{}_{a,}{}^{S}{}_{b}(X_0, X_1, \ldots, X_v) =$

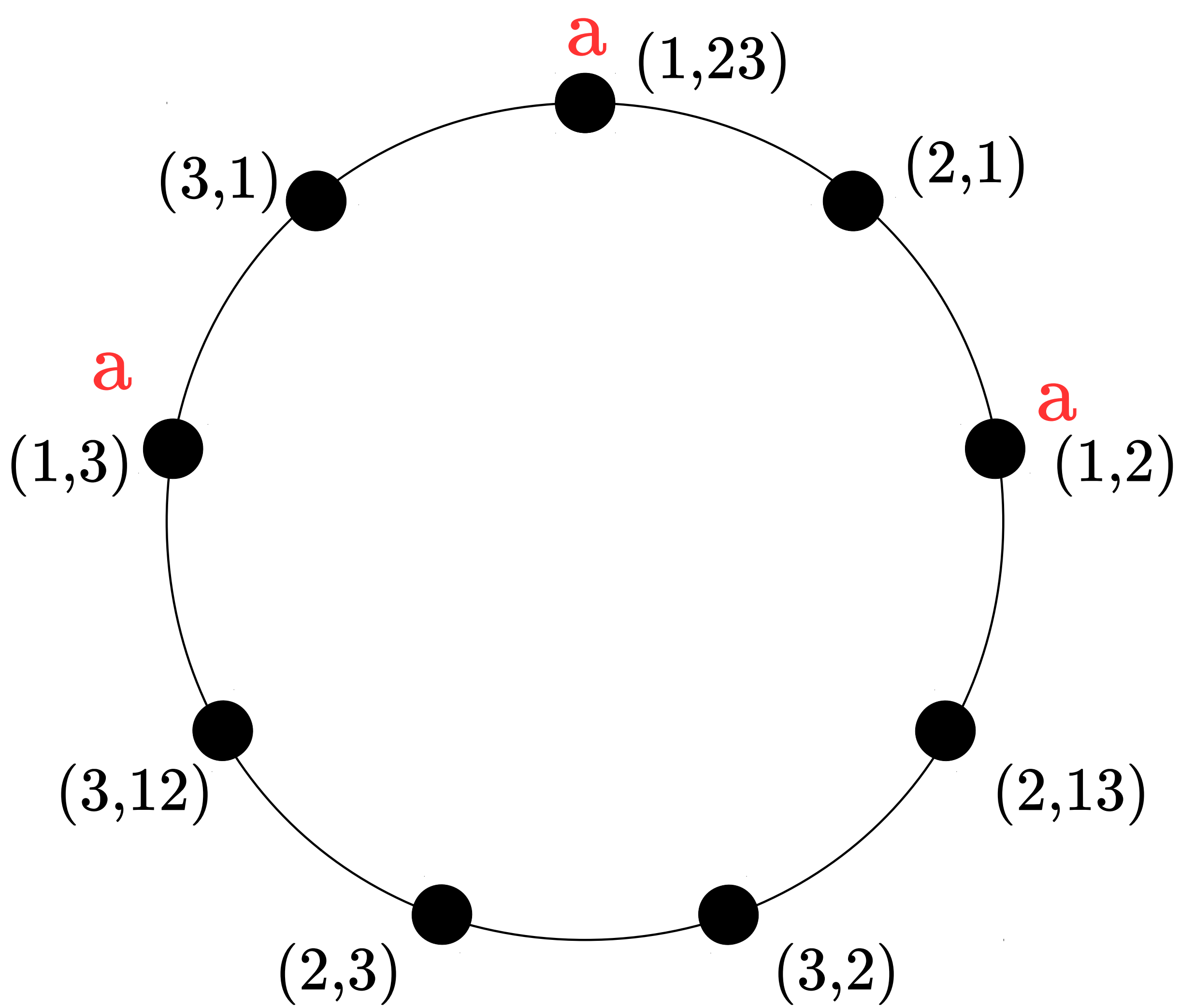
a if $X_i \cap S \neq \emptyset$

b otherwise

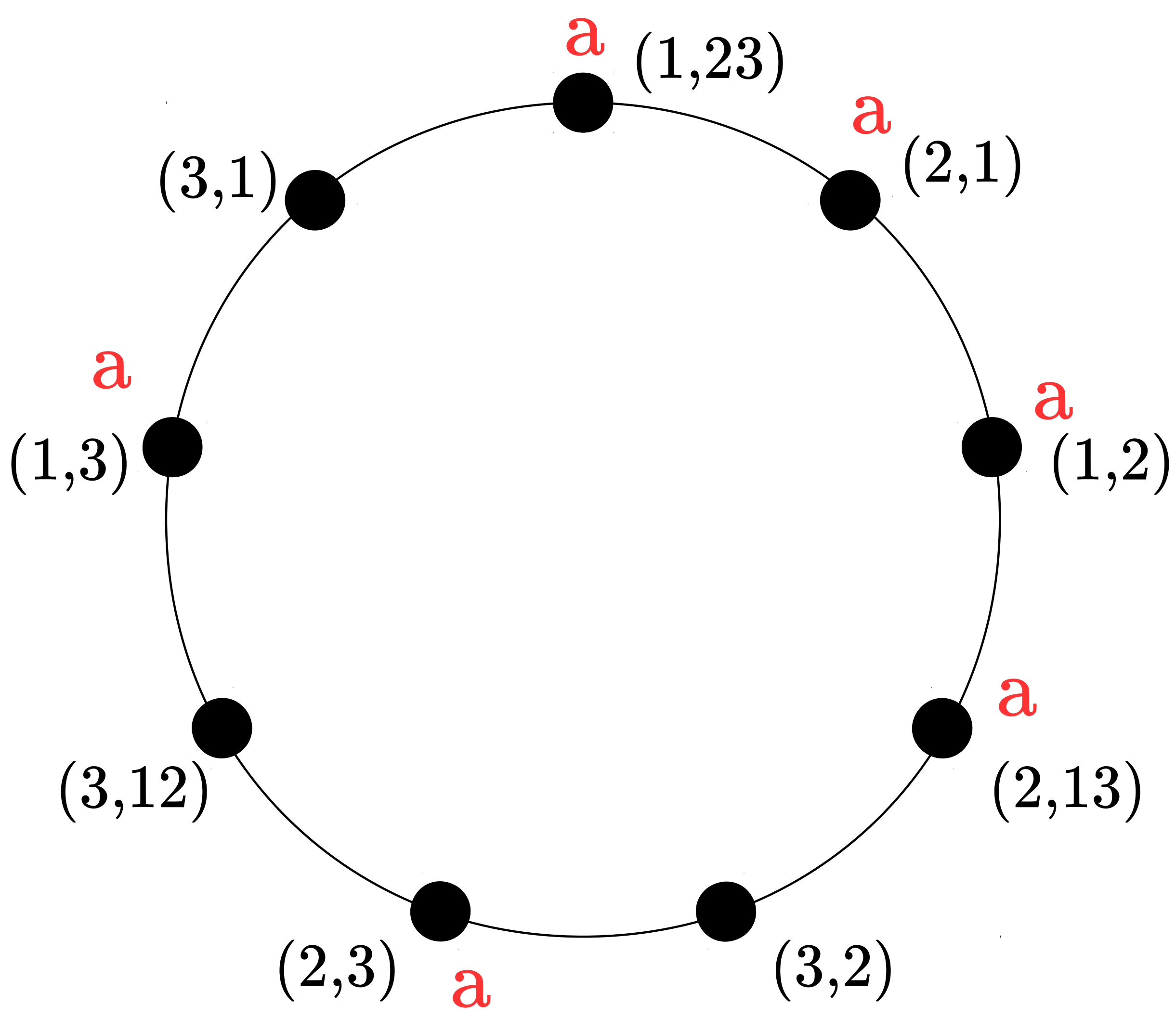
$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$



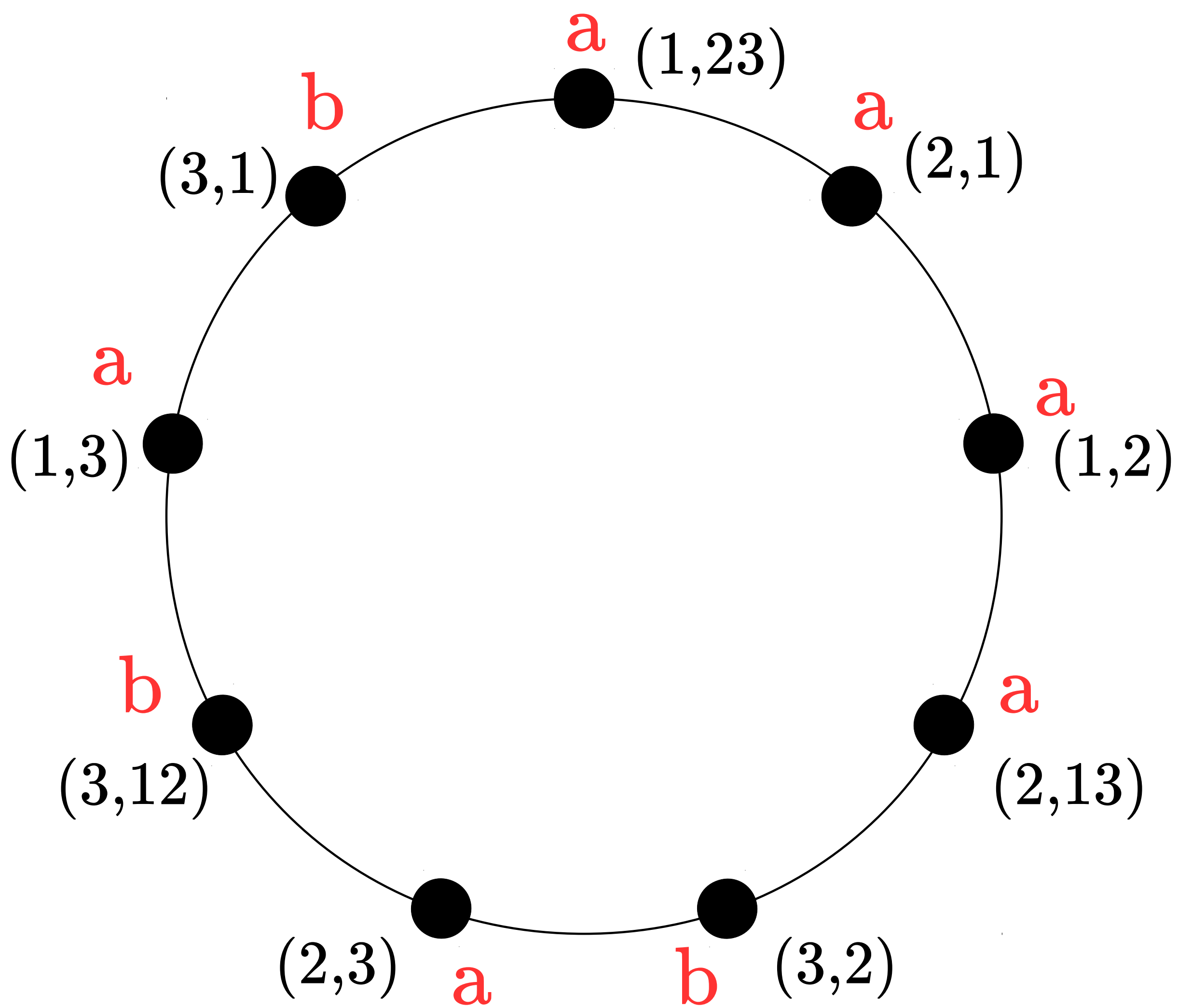
$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$



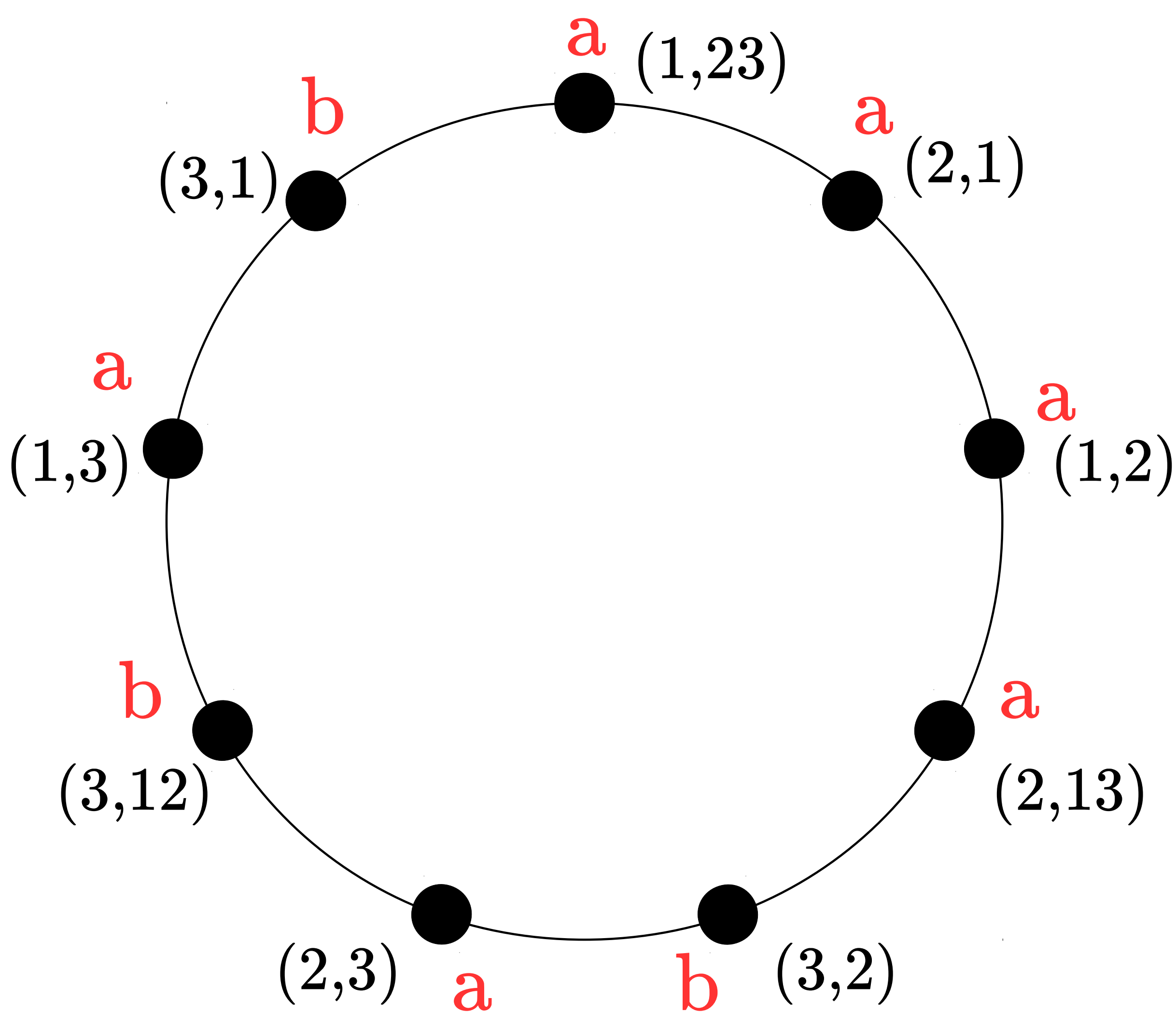
$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$



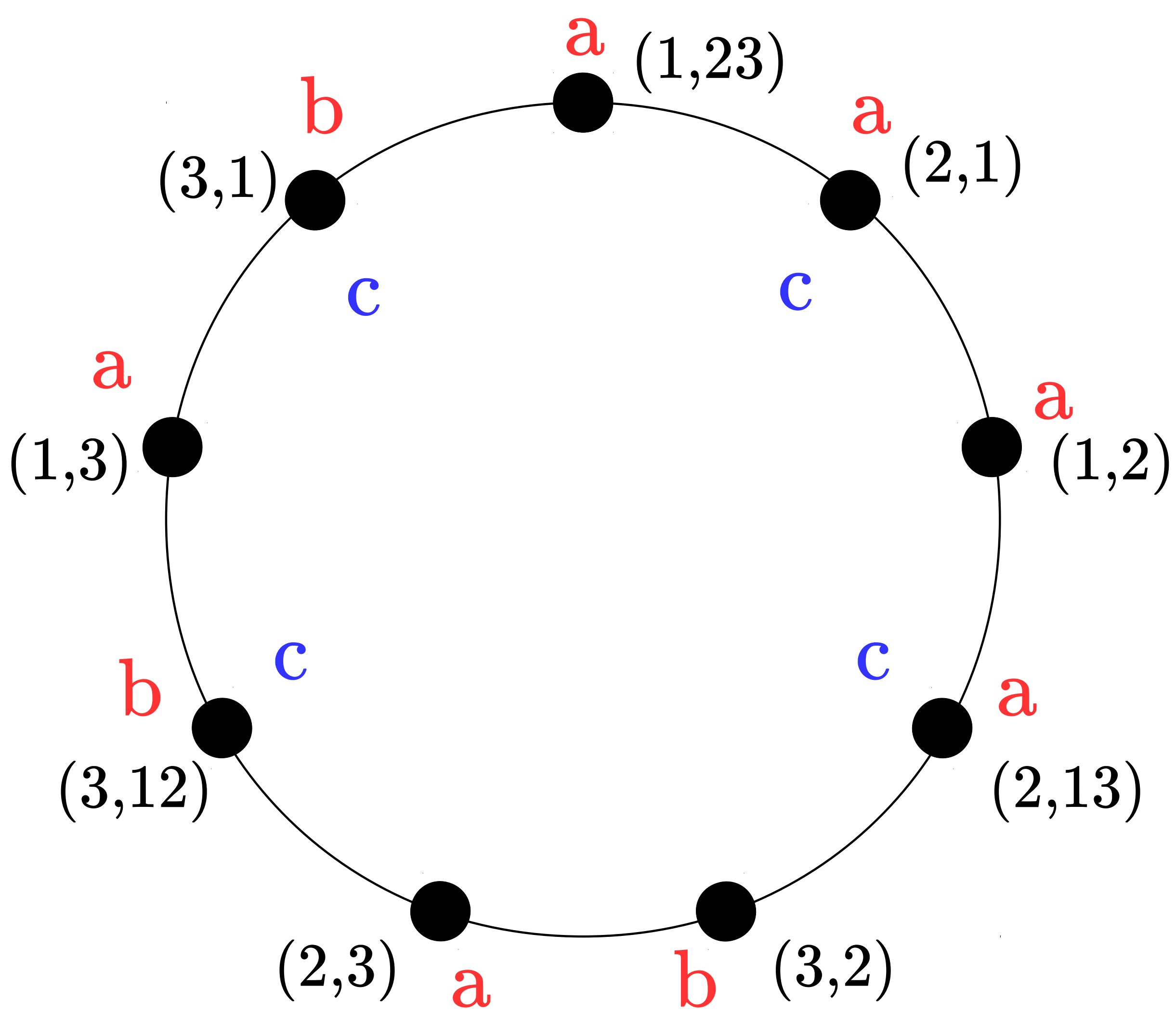
$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$



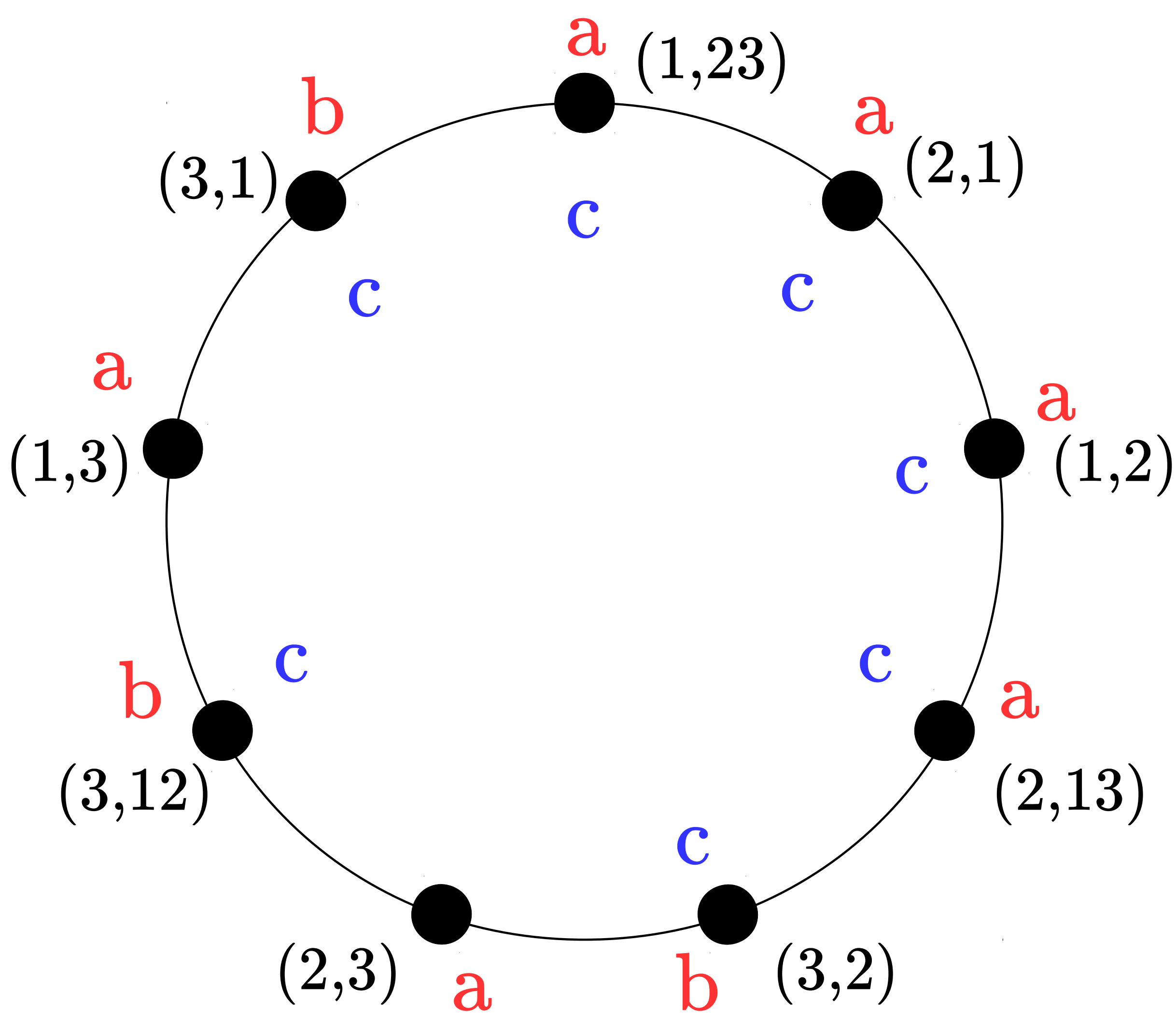
$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$ $\sigma^{1,}{}_{c,}{}^{\{1,2\}}{}_{a}$



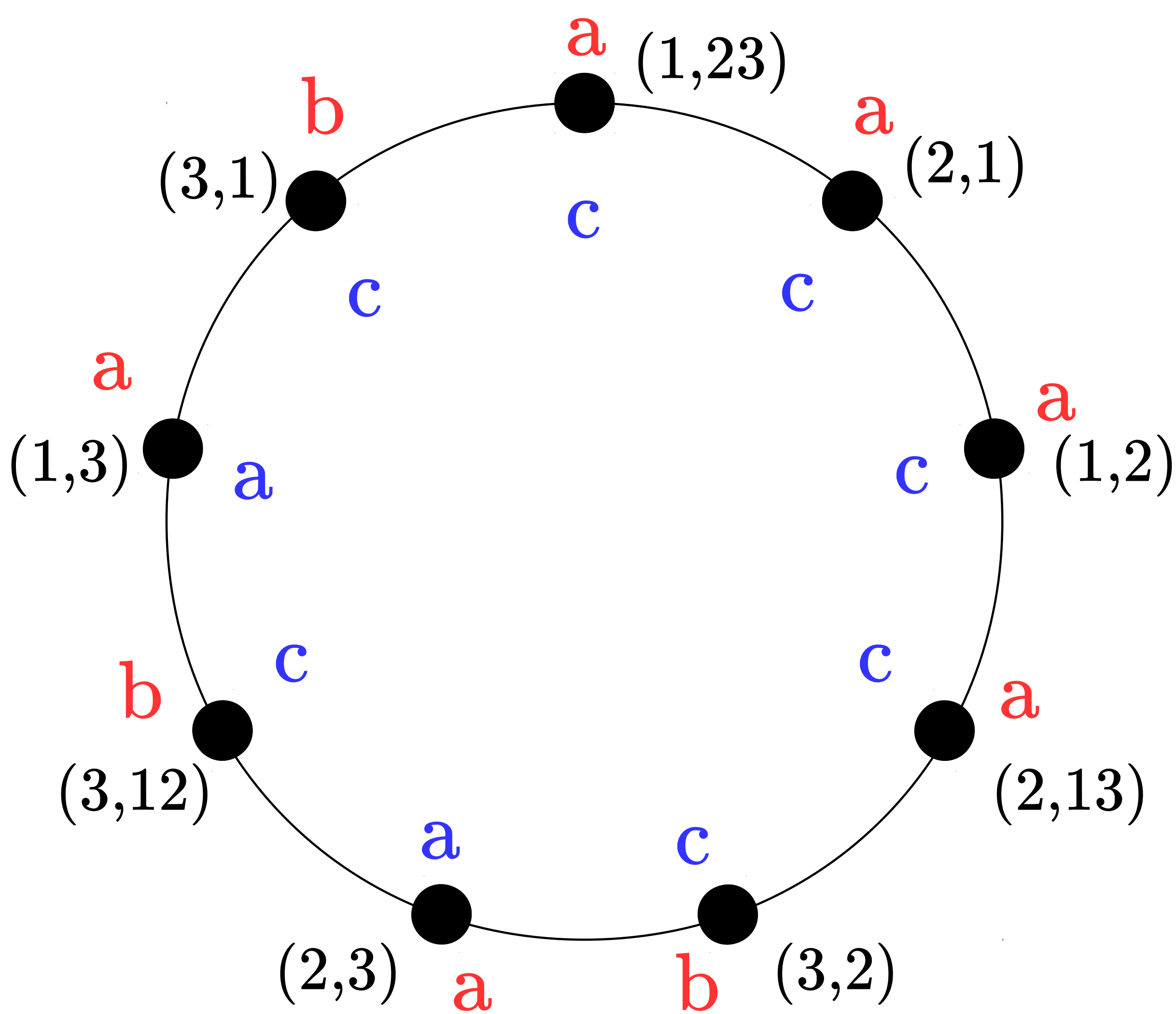
$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$ $\sigma^{1,}{}_{c,}{}^{\{1,2\}}{}_{a}$



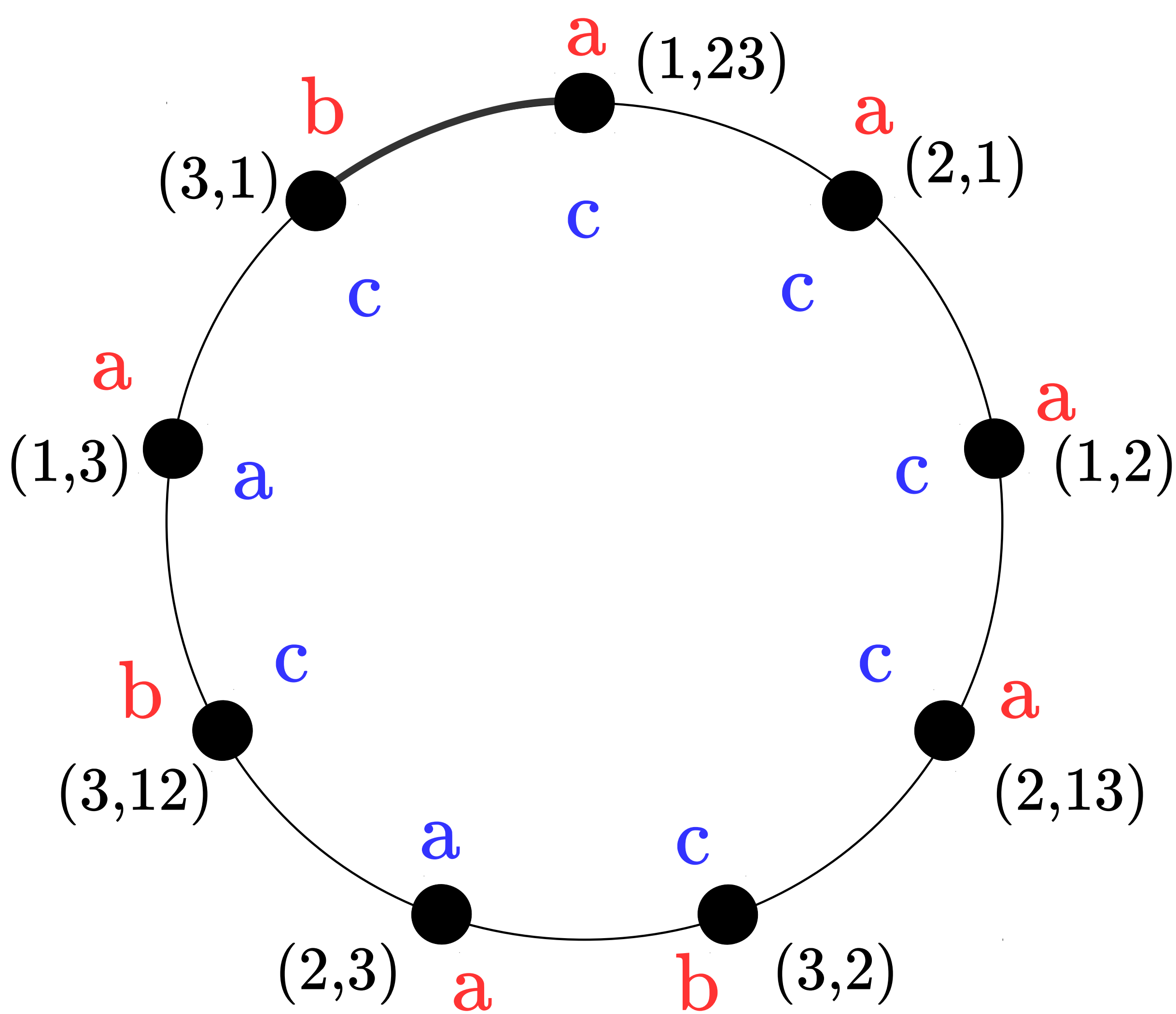
$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$ $\sigma^{1,}{}_{c,}{}^{\{1,2\}}{}_{a}$



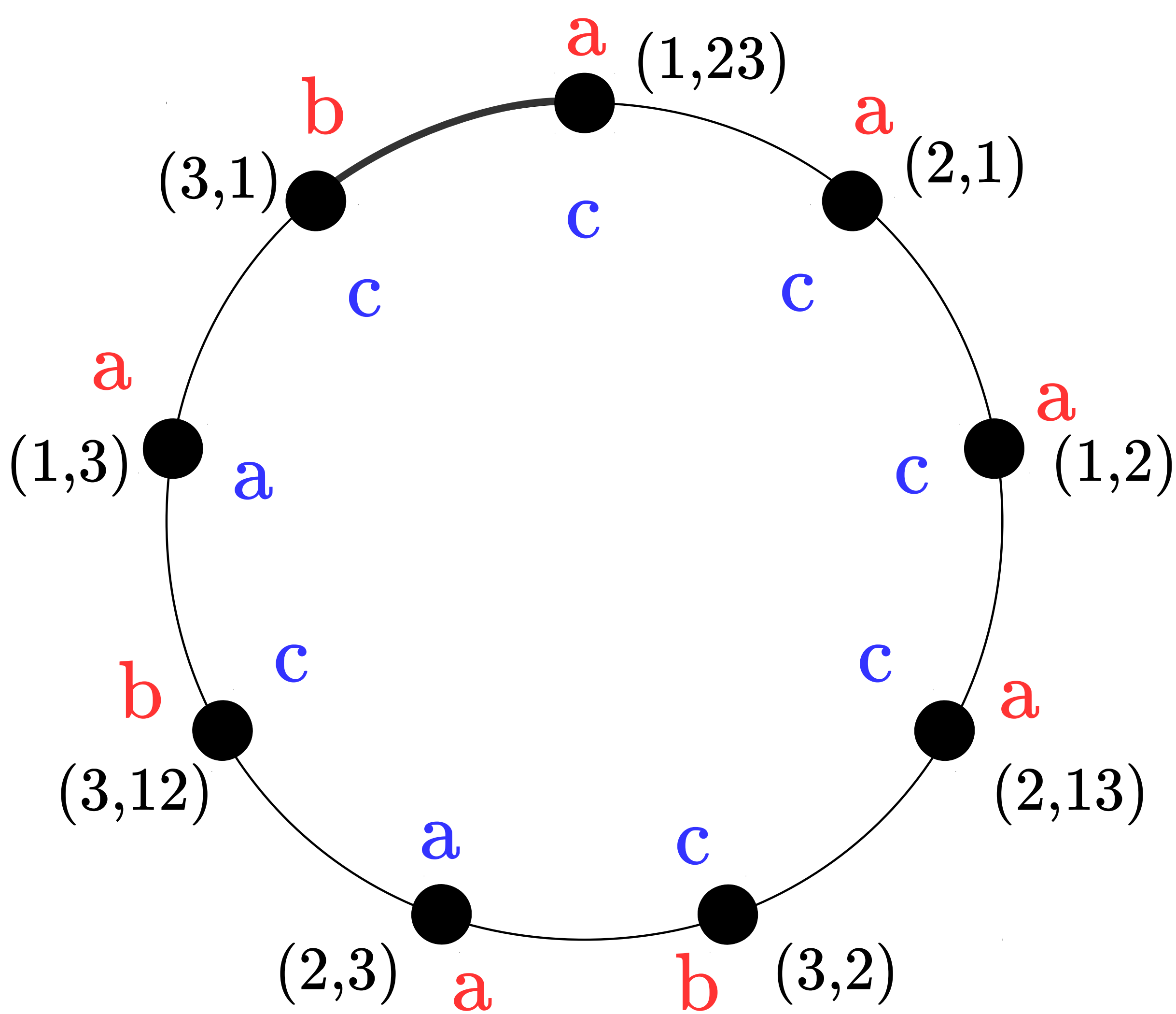
$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$ $\sigma^{1,}{}_{c,}{}^{\{1,2\}}{}_{a}$



$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$ $\sigma^{1,}{}_{c,}{}^{\{1,2\}}{}_{a}$



$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$ ——— $\sigma^{1,}{}_{c,}{}^{\{1,2\}}{}_{a}$ in $K_n^{\Omega_w(K_m)}$



Special functions in ${K_n}^{\Omega_w(K_m)}$

$\sigma^{i,}{}_{a,}{}^{S}{}_{b}$, $i \in \{0, ..., v\}$, $S \subseteq V(K_m)$, $a, b \in V(K_n)$

$\sigma^{i,}{}_{a,}{}^{S}{}_{b}(X_0, X_1, ..., X_v) =$

a if $X_i \cap S \neq \emptyset$

b otherwise

$\sigma^{i,}{}_{a,}{}^{S}{}_{b}$ is adjacent to $\sigma^{i+1,}{}_{c,}{}^{S}{}_{a}$

Special functions in $K_n^{\Omega_w(K_m)}$

$\tau^{i,}{}_{a,}{}^{S,}{}_{b}{}^{T}{}_{c}$, $i \in \{0, ..., v\}$, $S, T \subseteq V(K_m)$, $a, b, c \in V(K_n)$

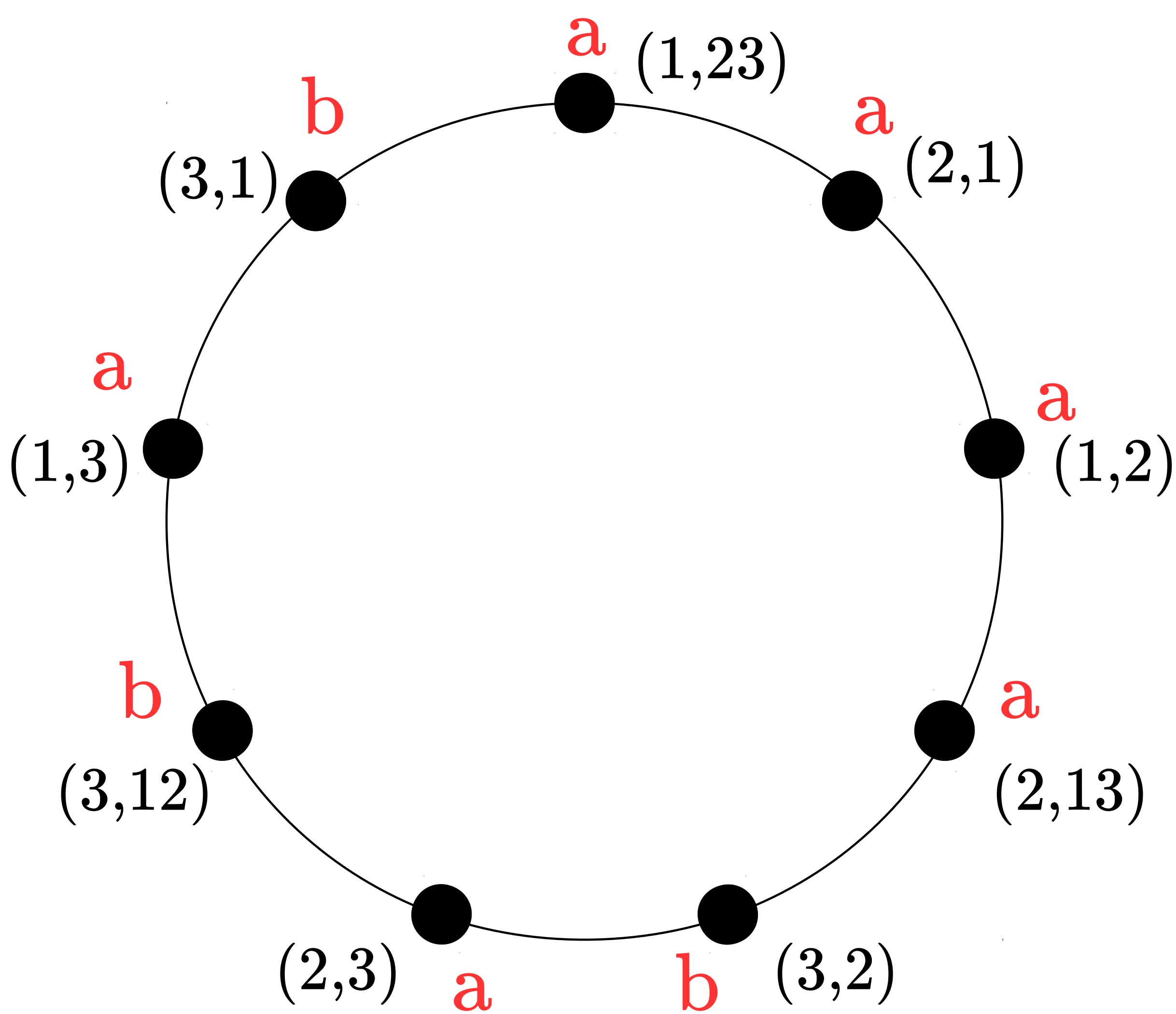
$\tau^{i,}{}_{a,}{}^{S,}{}_{b}{}^{T}{}_{c}(X_0, X_1, ..., X_v) =$

a if $X_i \cap S \neq \emptyset$

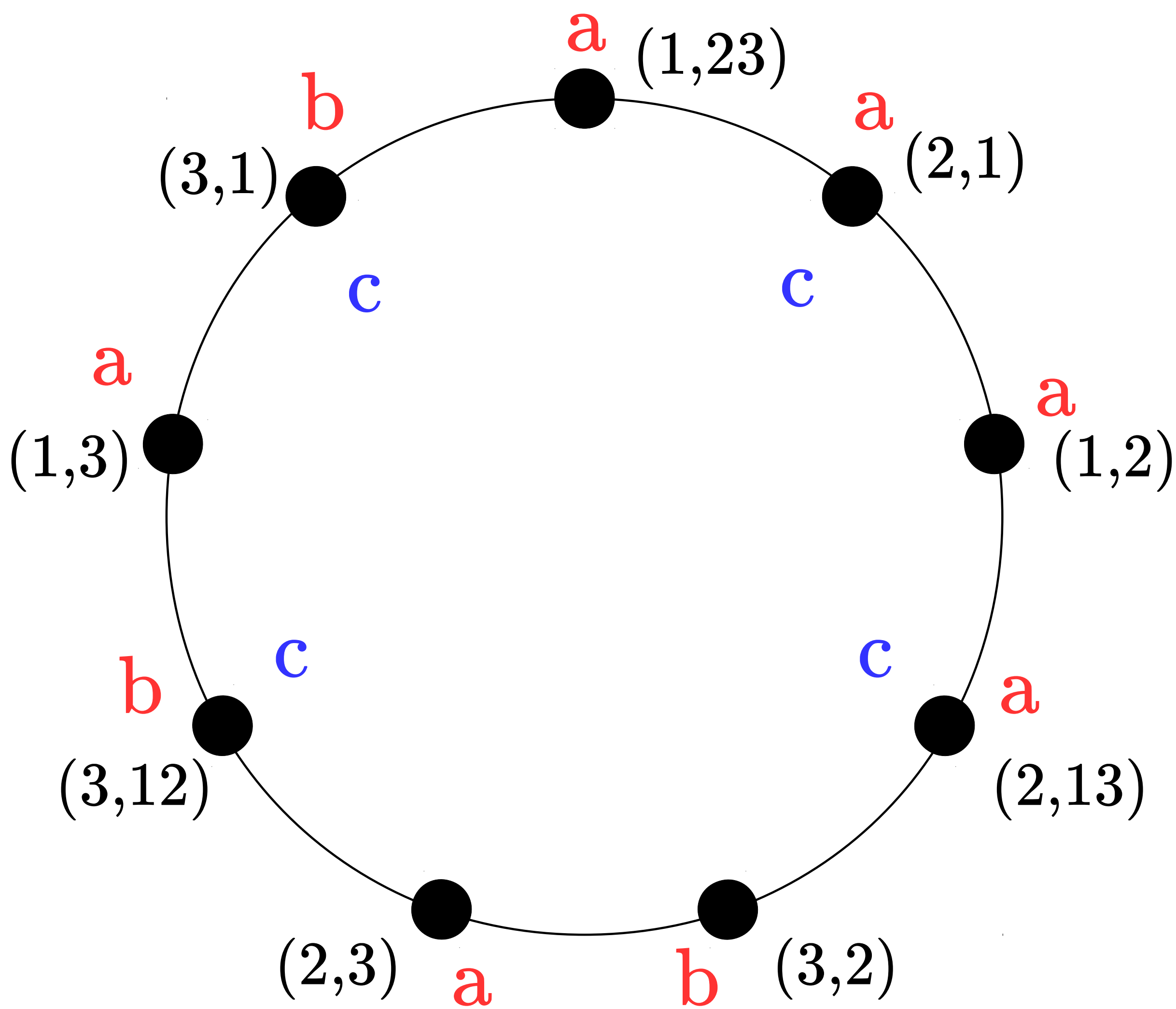
b if $X_i \cap S = \emptyset$, $X_i \cap T \neq \emptyset$

c otherwise

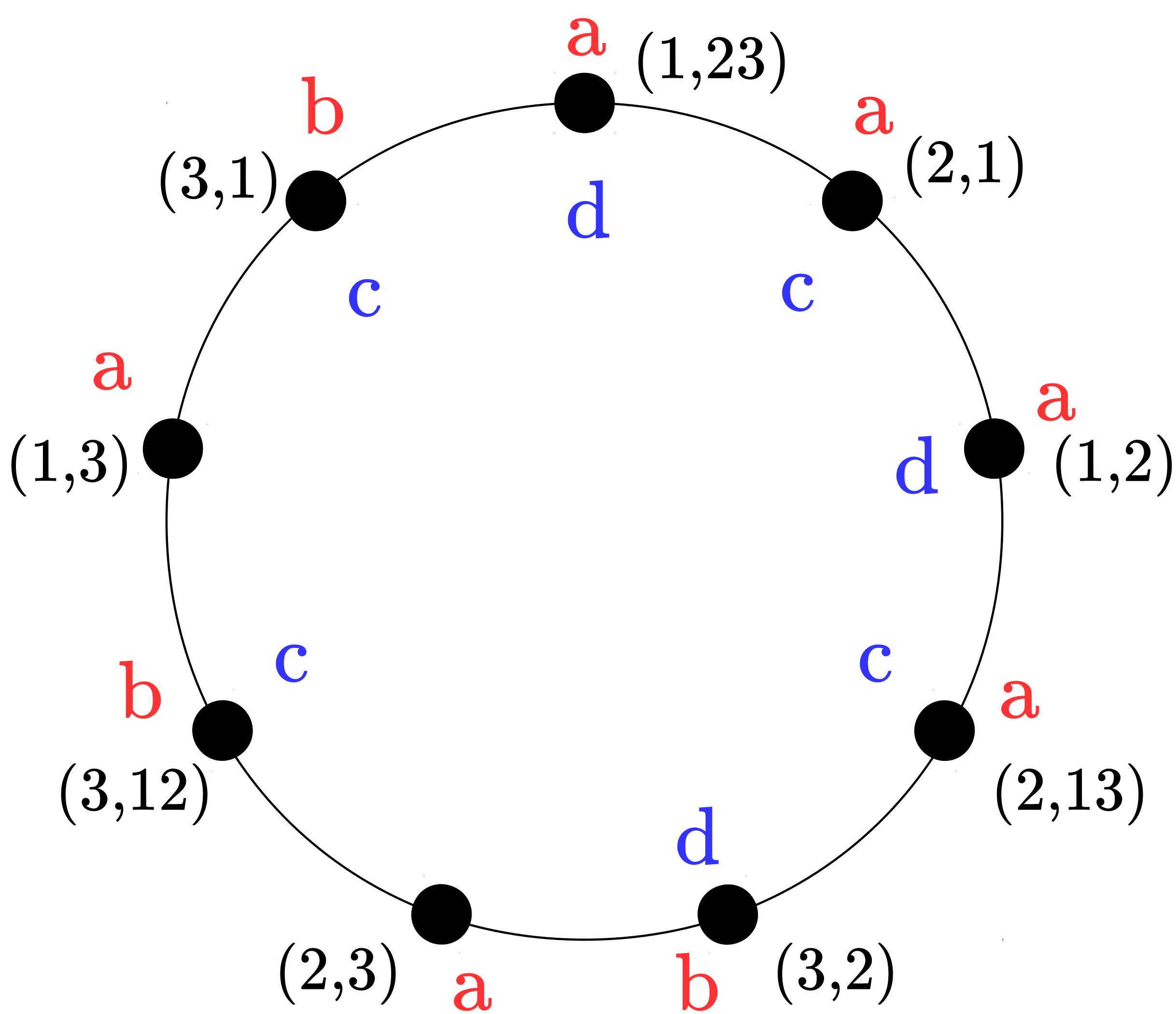
$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$ $\tau^{1,}{}_{c,}{}^{\{1\}}{}_{d,}{}^{\{2\}}{}_{a}$



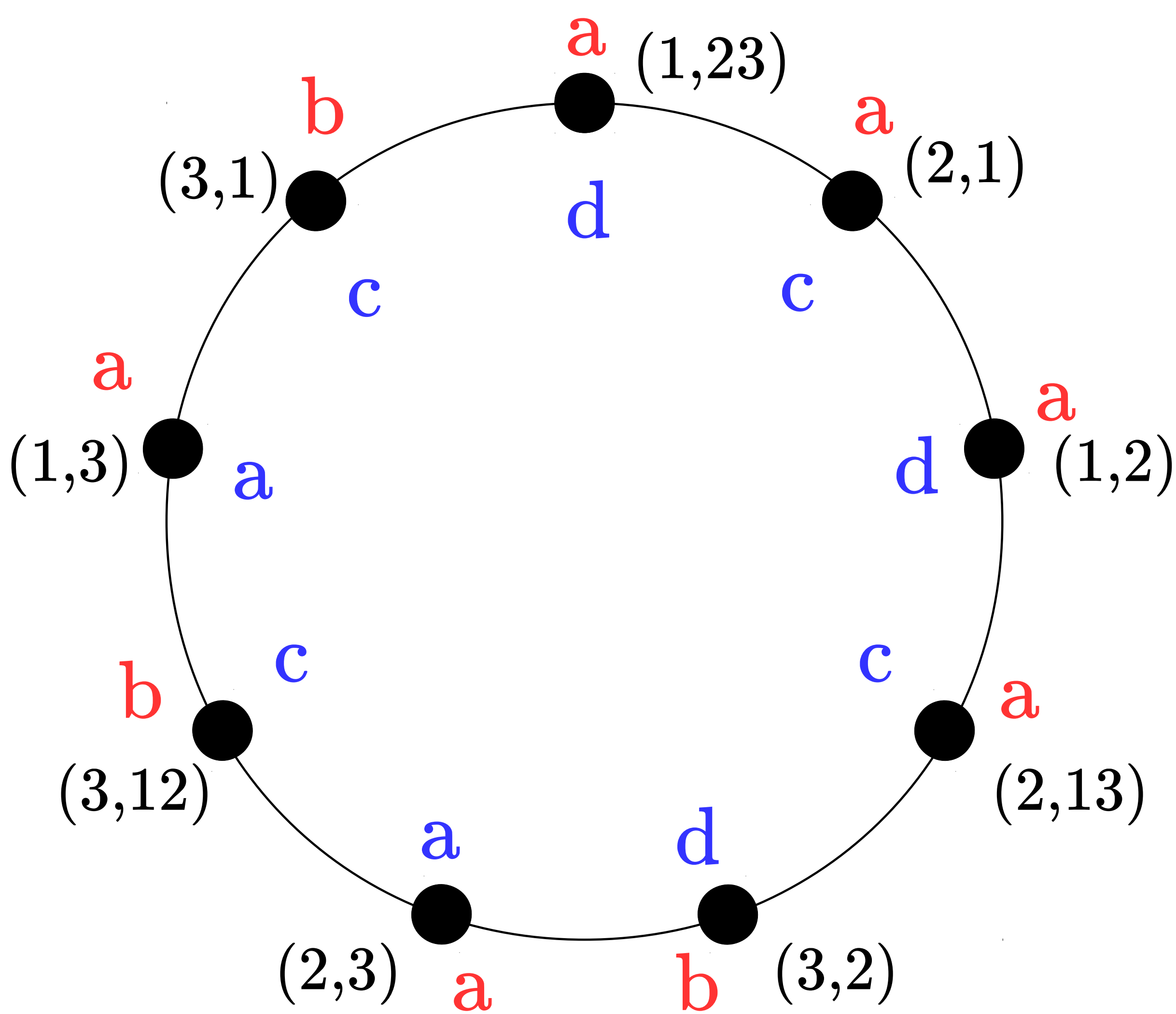
$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$ $\tau^{1,}{}_{c,}{}^{\{1\}}{}_{d,}{}^{\{2\}}{}_{a}$



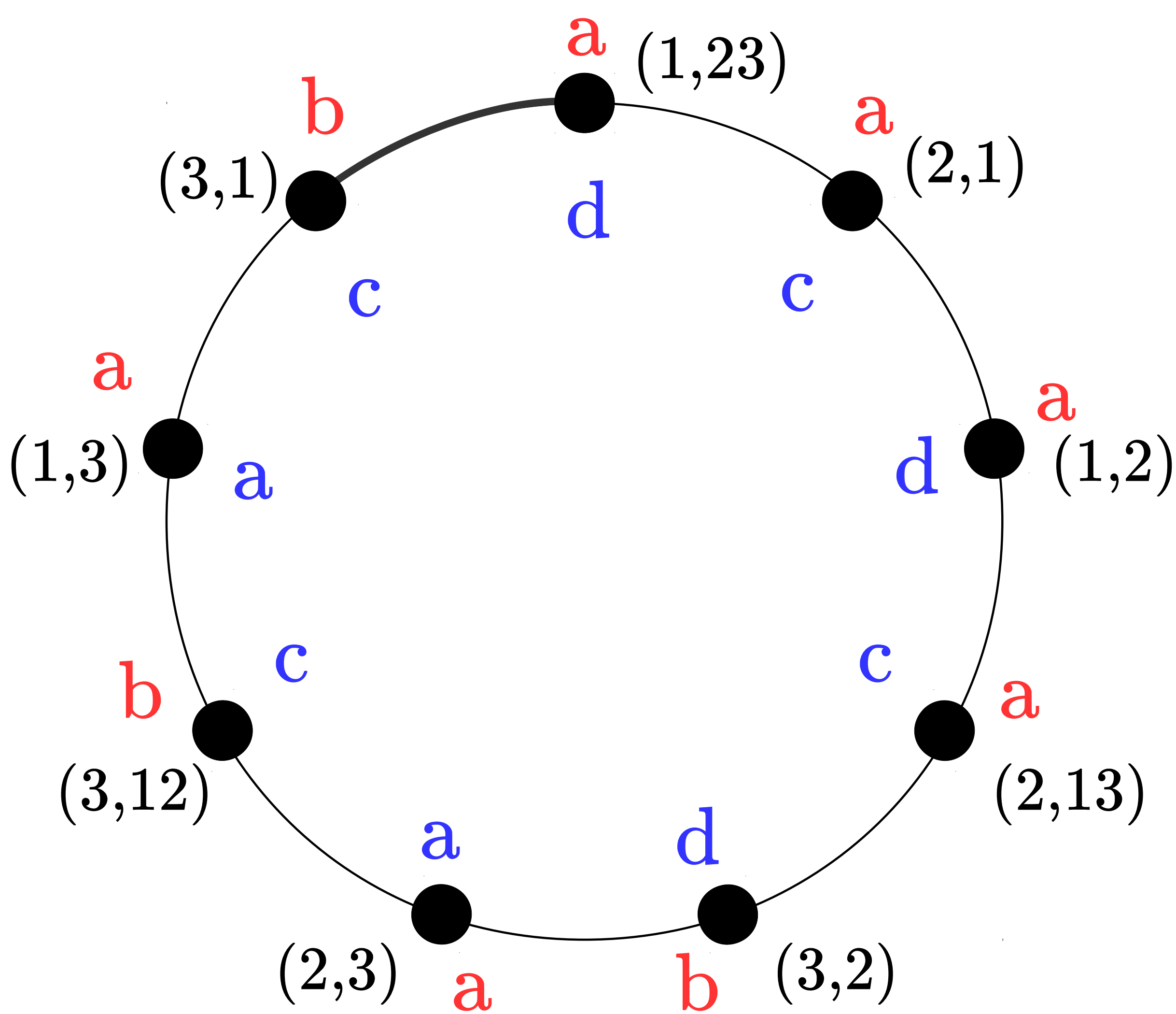
$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$ $\tau^{1,}{}_{c,}{}^{\{1\}}{}_{d,}{}^{\{2\}}{}_{a}$



$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$ $\tau^{1,}{}_{c,}{}^{\{1\}}{}_{d,}{}^{\{2\}}{}_{a}$



$\Omega_3(K_3)$: $\sigma^{0,}{}_{a,}{}^{\{1,2\}}{}_{b}$ ——— $\tau^{1,}{}_{c,}{}^{\{1\}}{}_{d,}{}^{\{2\}}{}_{a}$ in $K_n{}^{\Omega_w(K_m)}$



Special functions in $K_n^{\Omega_w(K_m)}$

$\tau^{i,}{}_{a,}{}^{S,}{}_{b}{}^{T}{}_{c}$, $i \in \{0, ..., v\}$, $S, T \subseteq V(K_m)$, $a, b, c \in V(K_n)$

$\tau^{i,}{}_{a,}{}^{S,}{}_{b}{}^{T}{}_{c}(X_0, X_1, ..., X_v) =$

a if $X_i \cap S \neq \emptyset$

b if $X_i \cap S = \emptyset$, $X_i \cap T \neq \emptyset$

c otherwise

$\sigma^{i,}{}_{a,}{}^{S \cup T}{}_{b}$ is adjacent to $\tau^{i+1,}{}_{c,}{}^{S}{}_{d,}{}^{T}{}_{a}$

Special functions in $K_n^{\Omega_w(K_m)}$

$\tau^{i,}{}_{a,}{}^{S,}{}_{b}{}^{T}{}_{c}$, $i \in \{0, \ldots, v\}$, $S, T \subseteq V(K_m)$, $a, b, c \in V(K_n)$

$\tau^{i,}{}_{a,}{}^{S,}{}_{b}{}^{T}{}_{c}(X_0, X_1, \ldots, X_v) =$

a if $X_i \cap S \neq \emptyset$

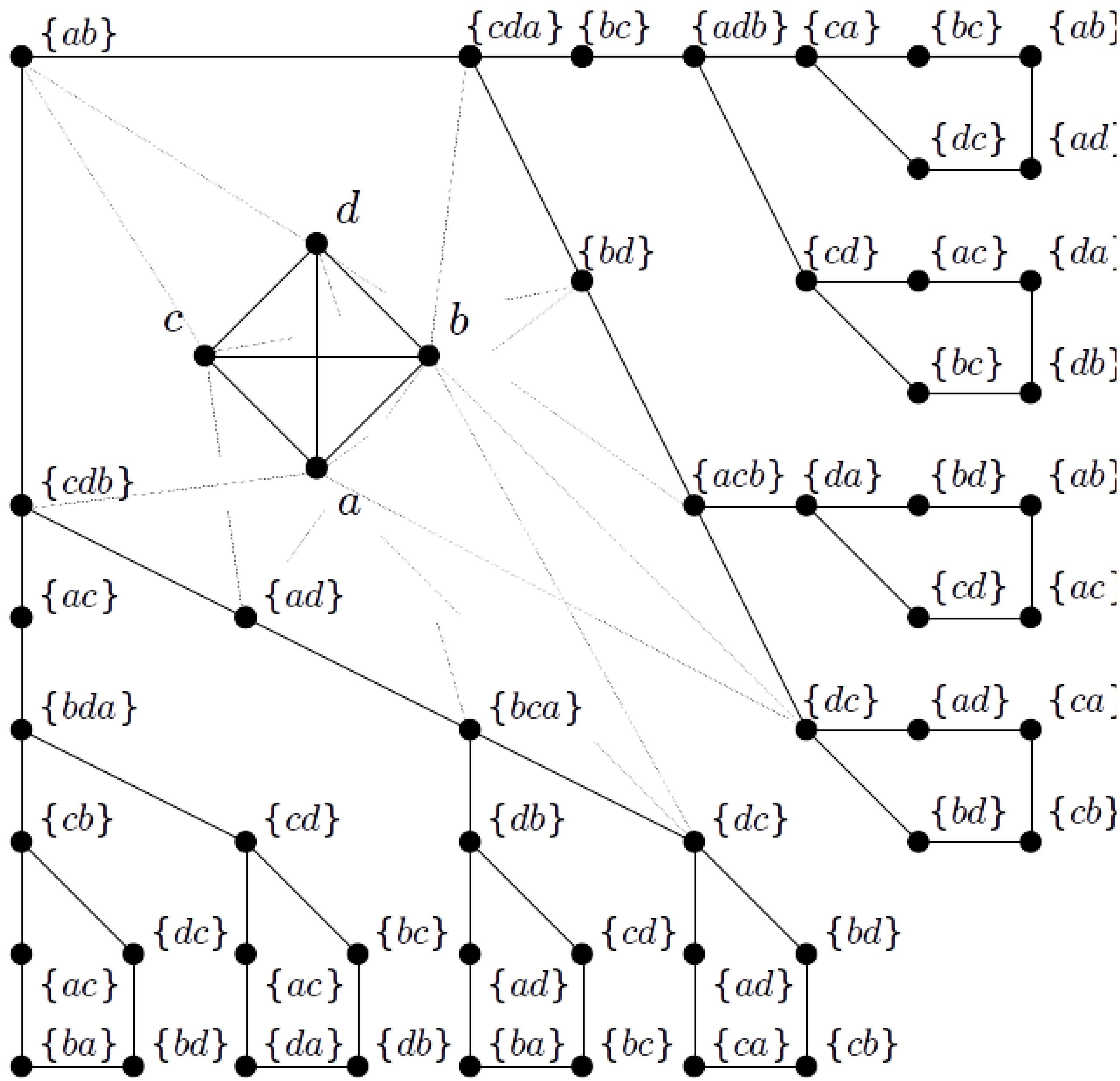
b if $X_i \cap S = \emptyset$, $X_i \cap T \neq \emptyset$

c otherwise

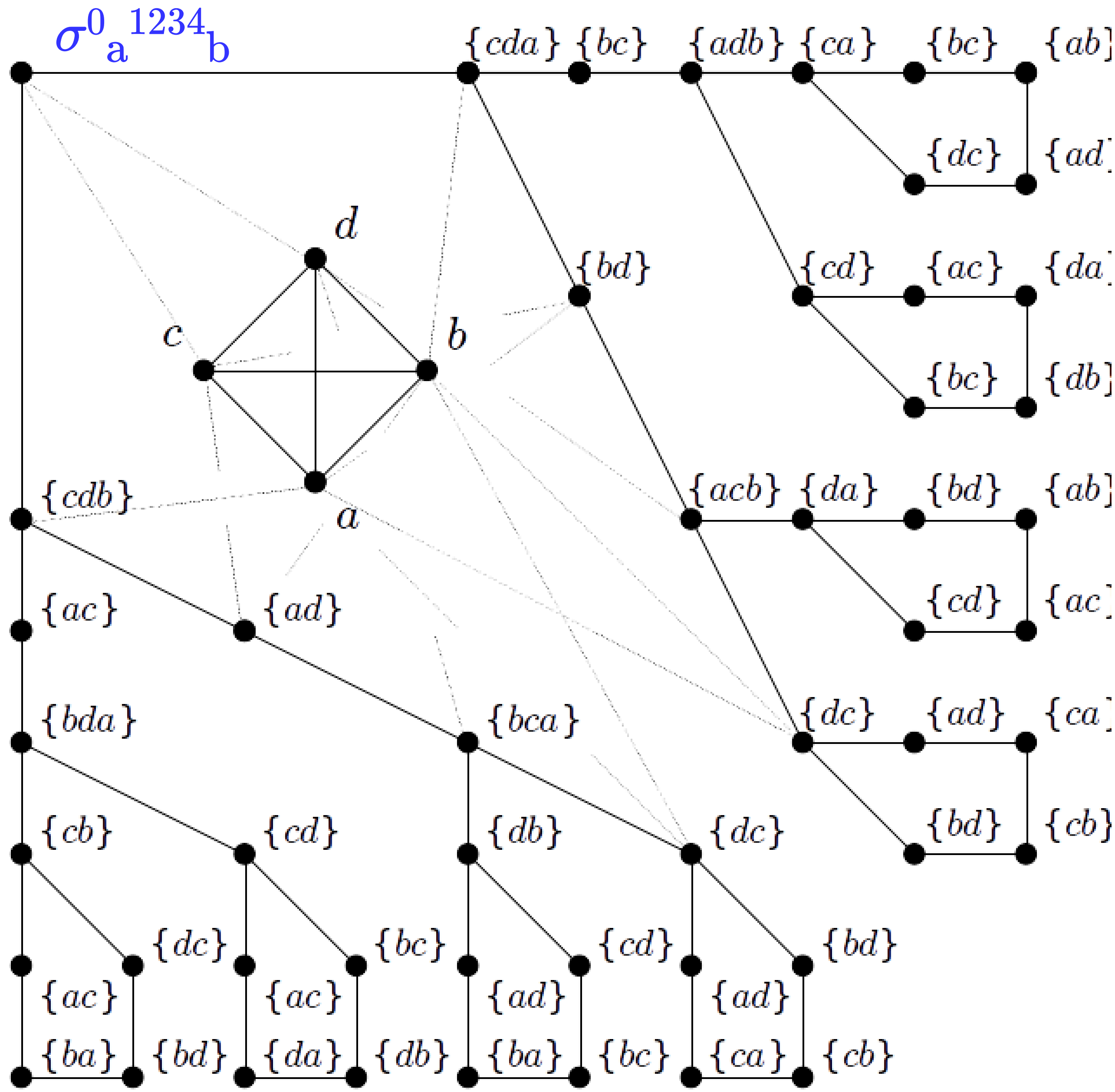
$\sigma^{i,}{}_{a,}{}^{S \cup T}{}_{b}$ is adjacent to $\tau^{i+1,}{}_{c,}{}^{S}{}_{d,}{}^{T}{}_{a}$

$\tau^{i,}{}_{a,}{}^{S}{}_{b,}{}^{T}{}_{c}$ is adjacent to $\sigma^{i+1,}{}_{d,}{}^{S}{}_{a}$ and $\sigma^{i+1,}{}_{d,}{}^{T}{}_{b}$

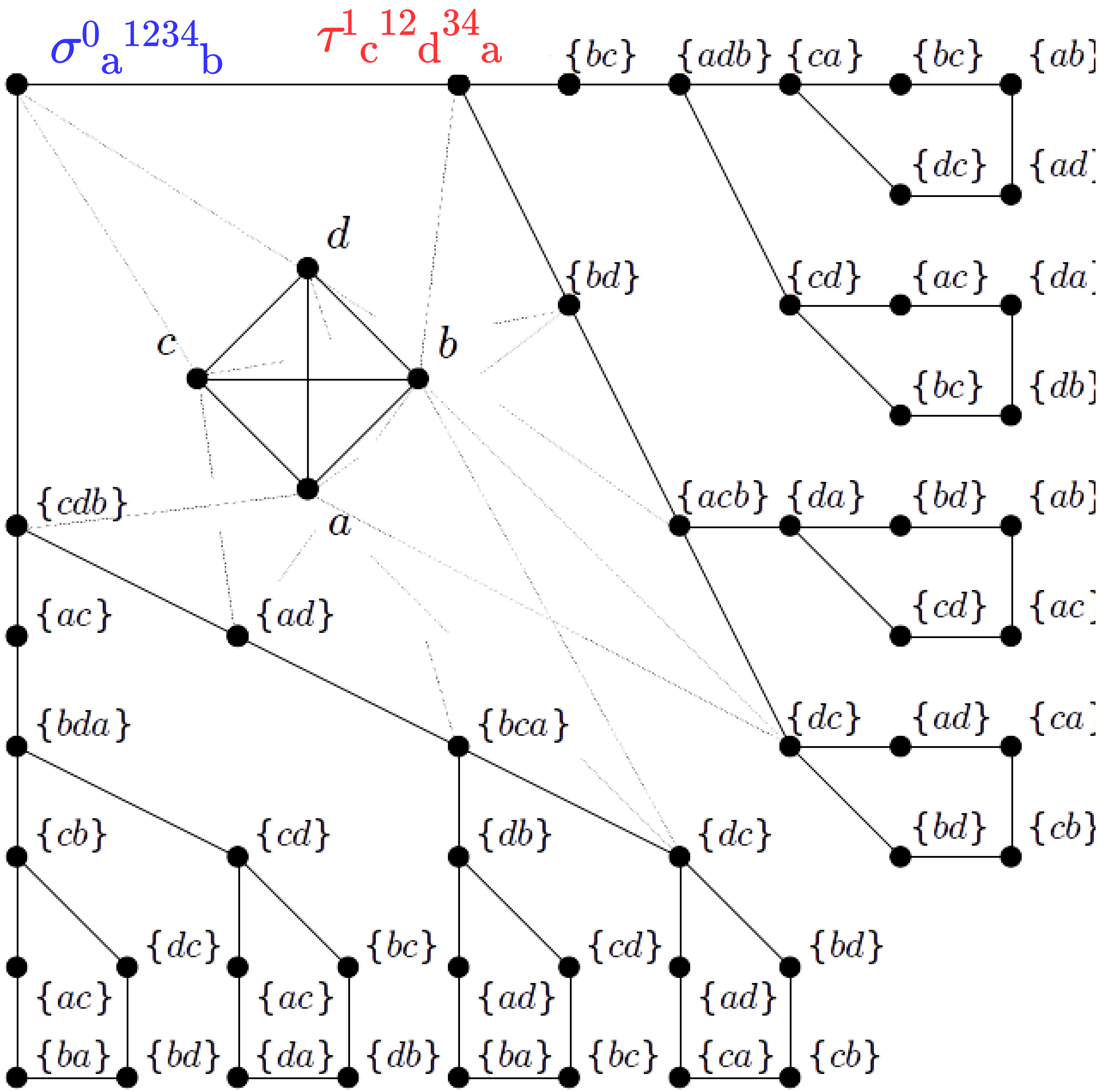
$K_4^{\Omega_{13}(K_8)}$



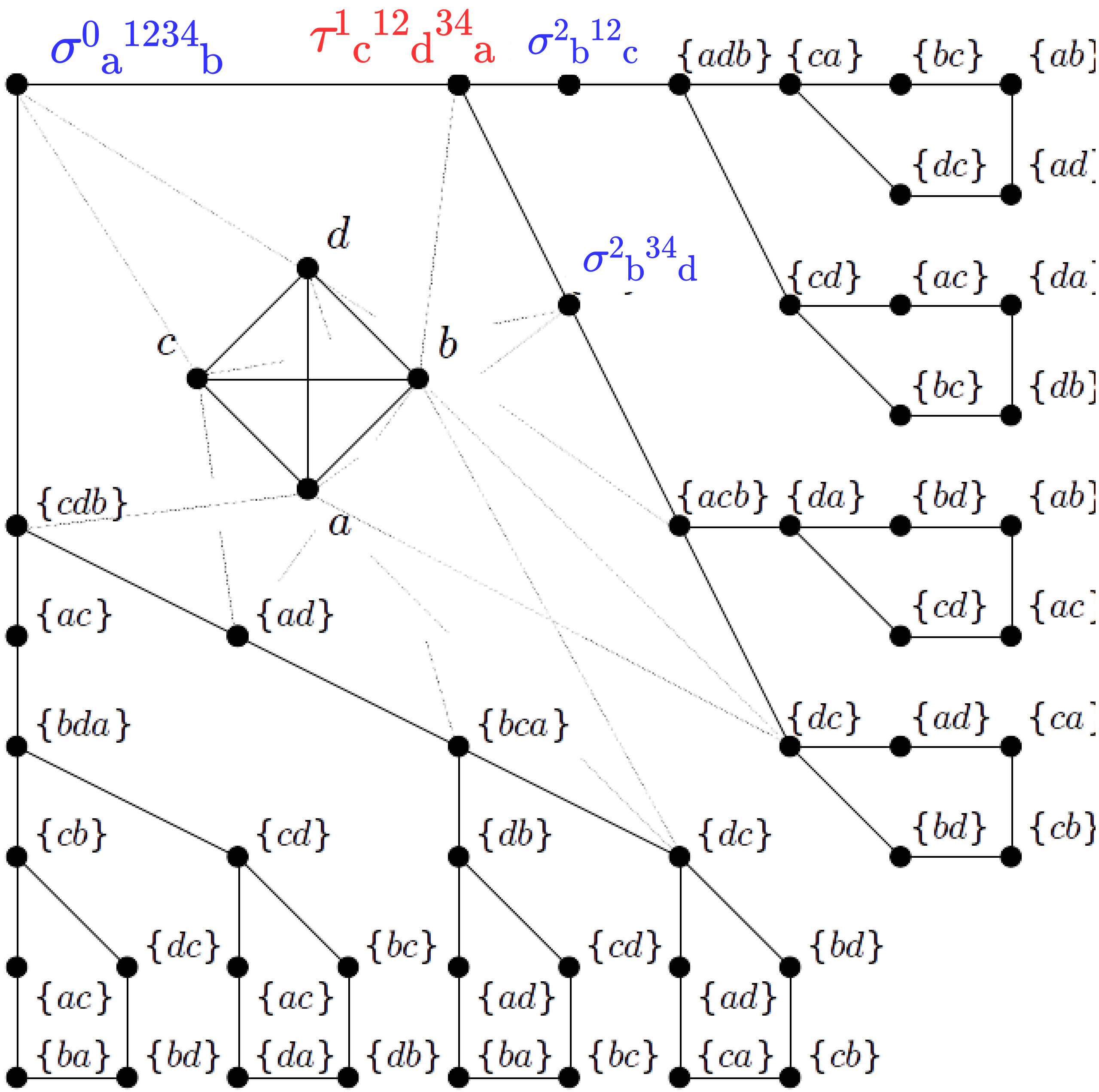
$K_4^{\Omega_{13}(K_8)}$



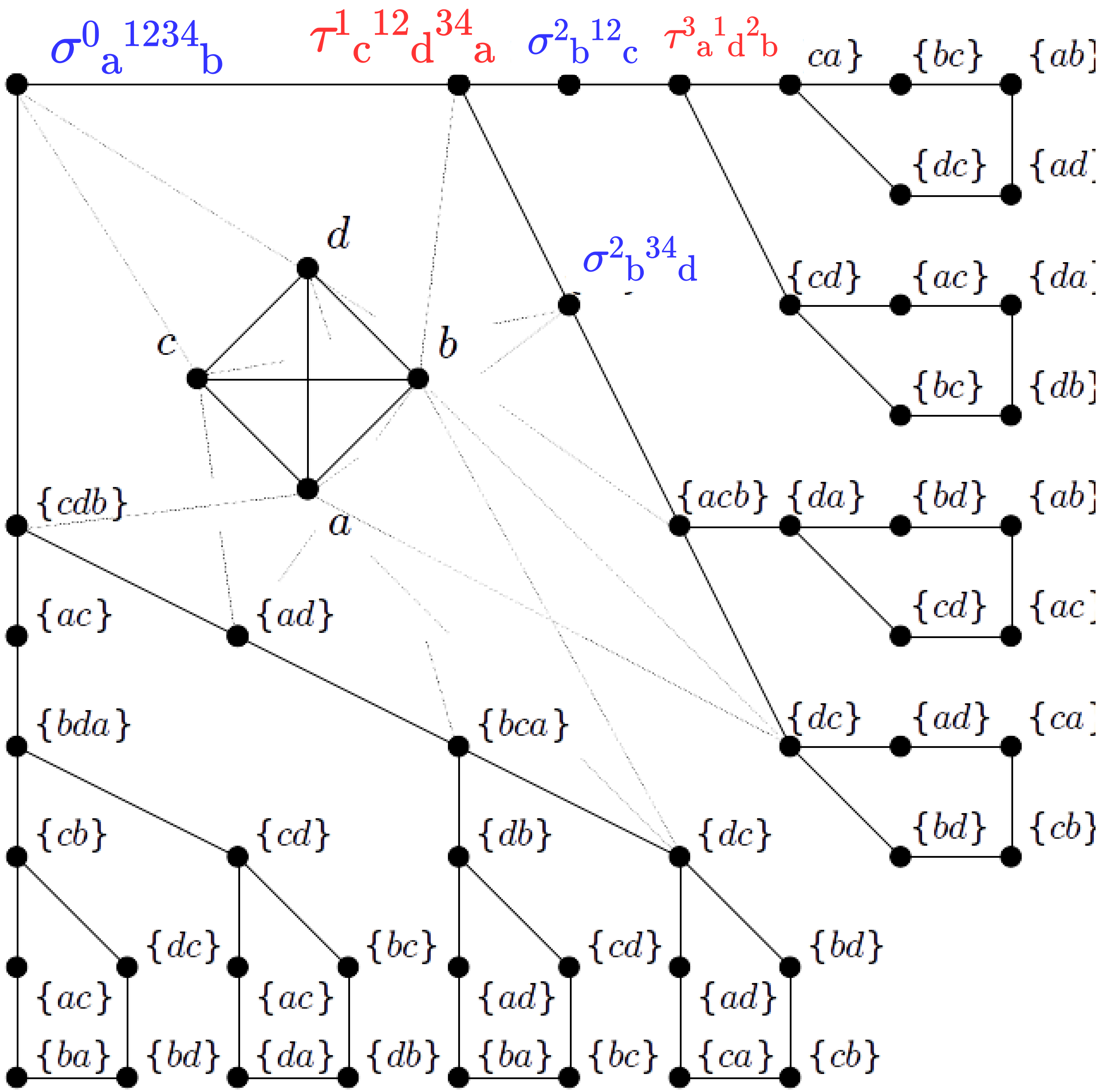
$K_4^{\Omega_{13}(K_8)}$



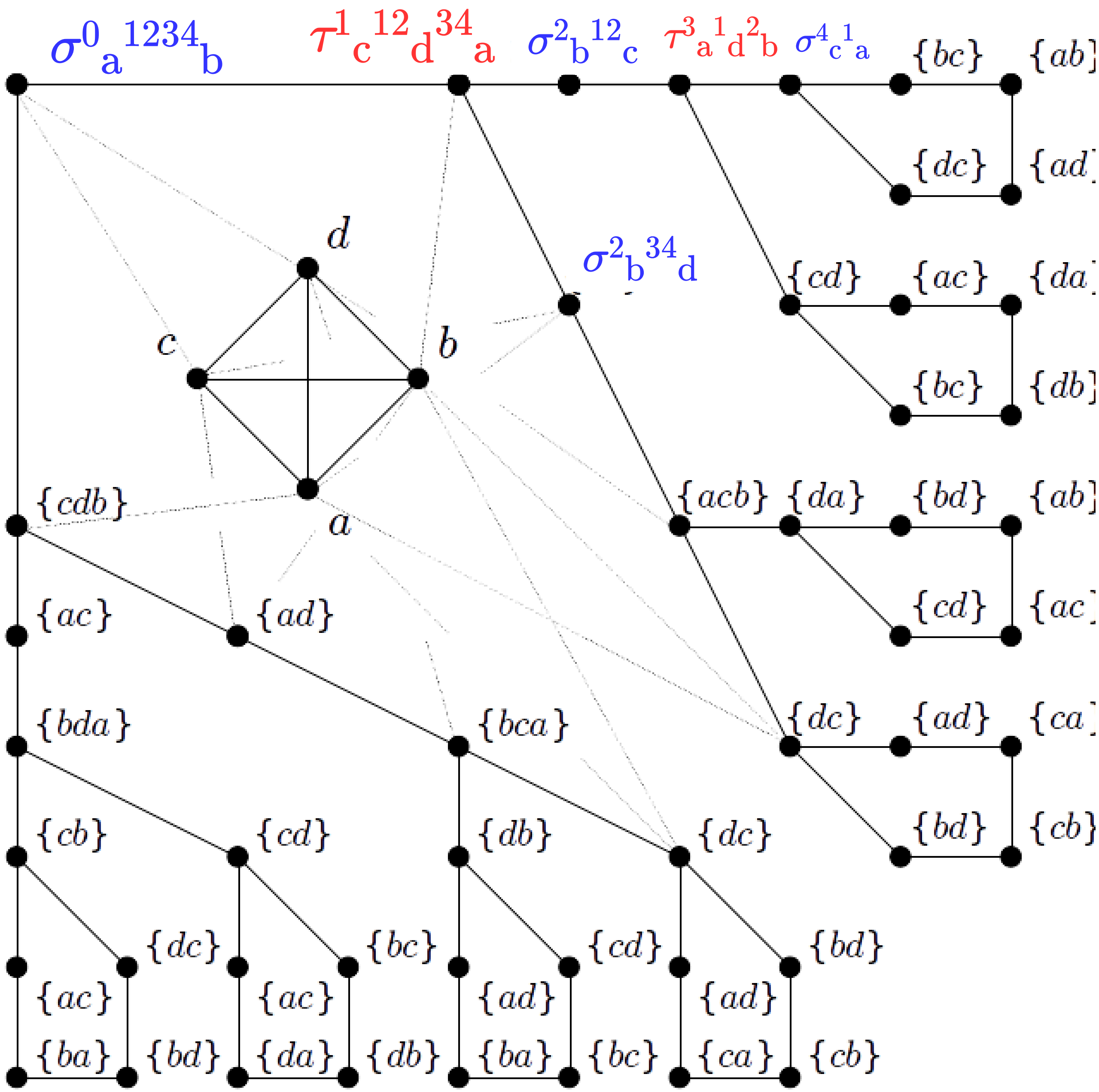
$K_4^{\Omega_{13}(K_8)}$



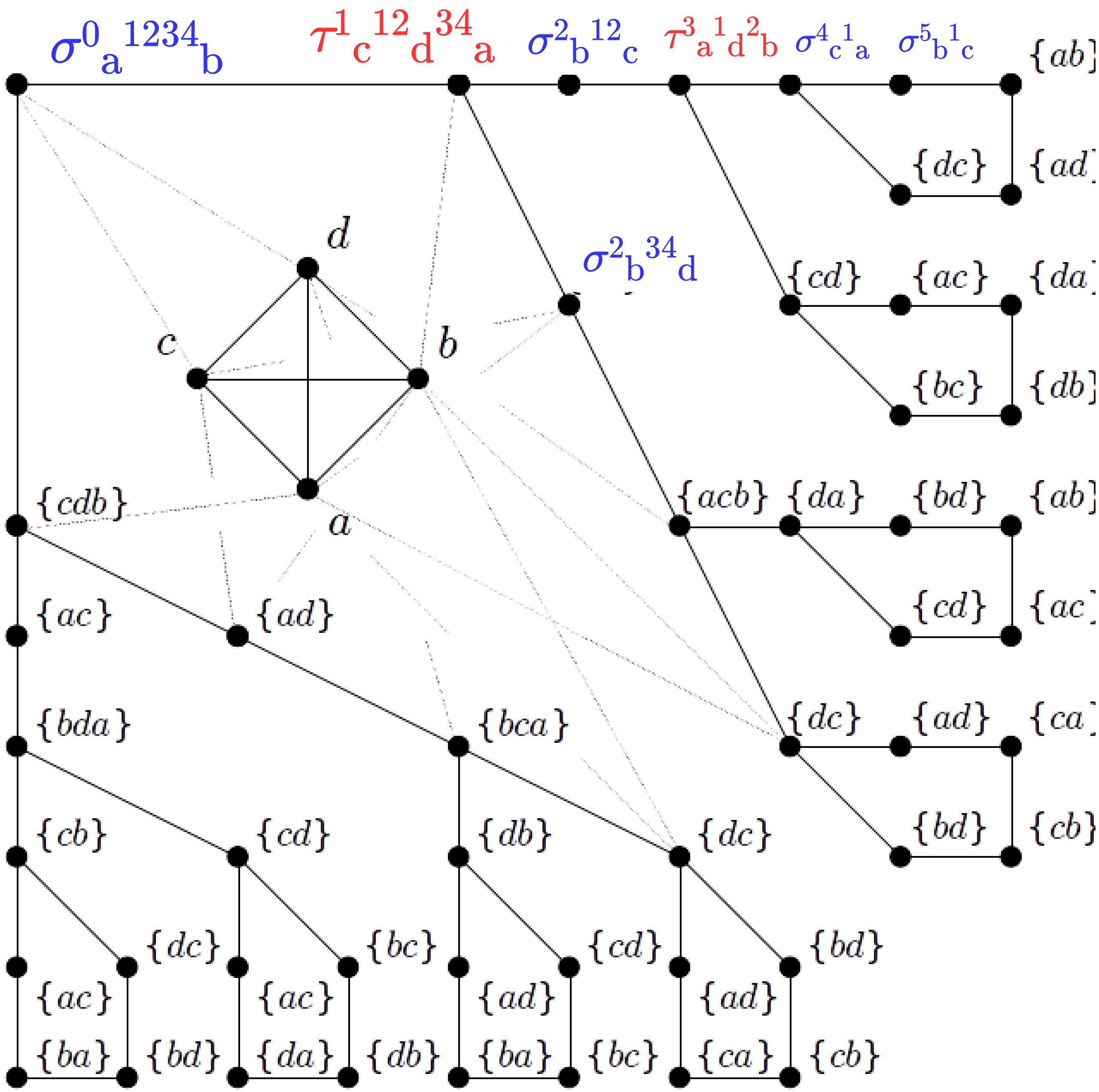
$K_4^{\Omega_{13}(K_8)}$



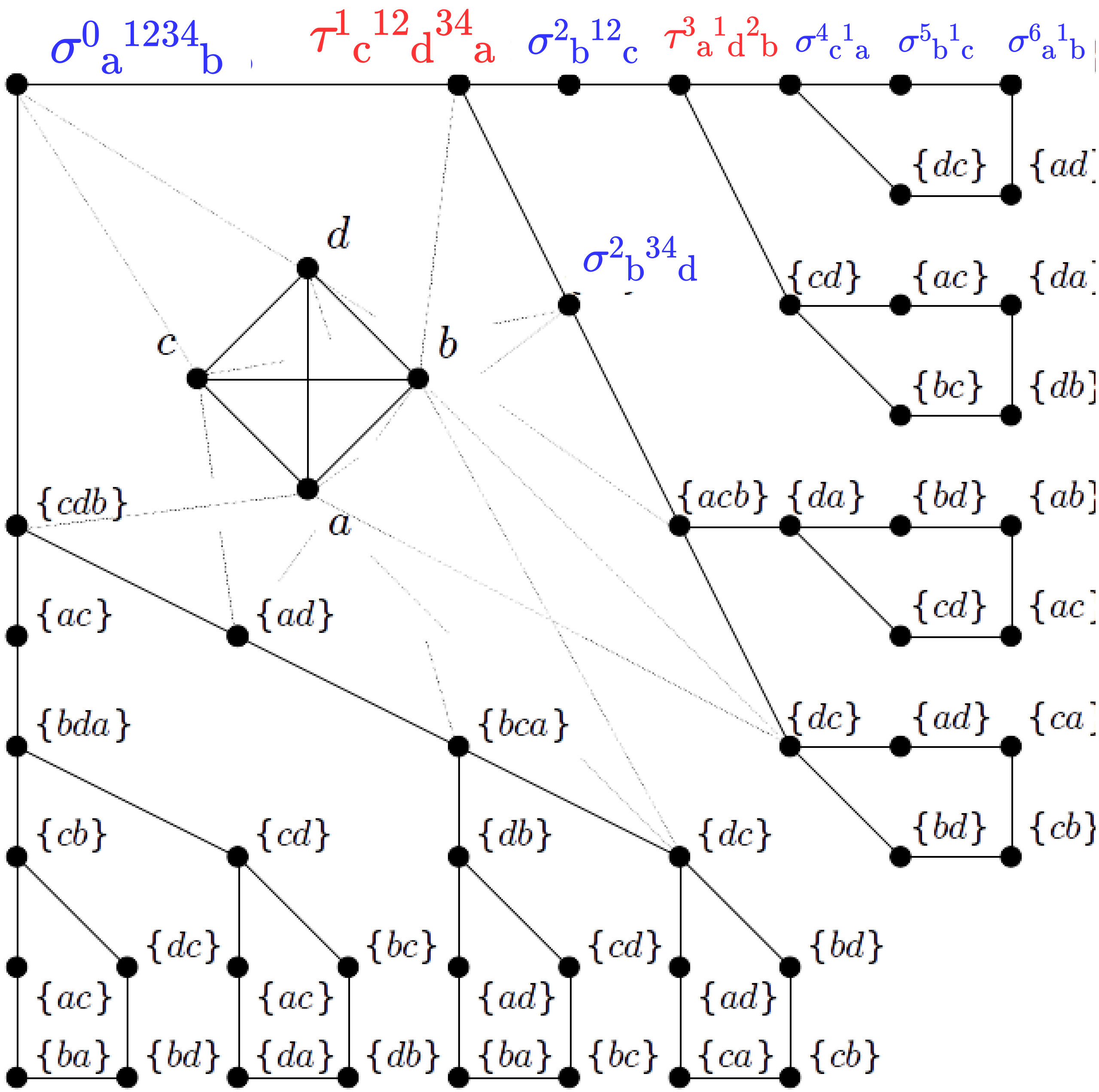
$K_4^{\Omega_{13}(K_8)}$



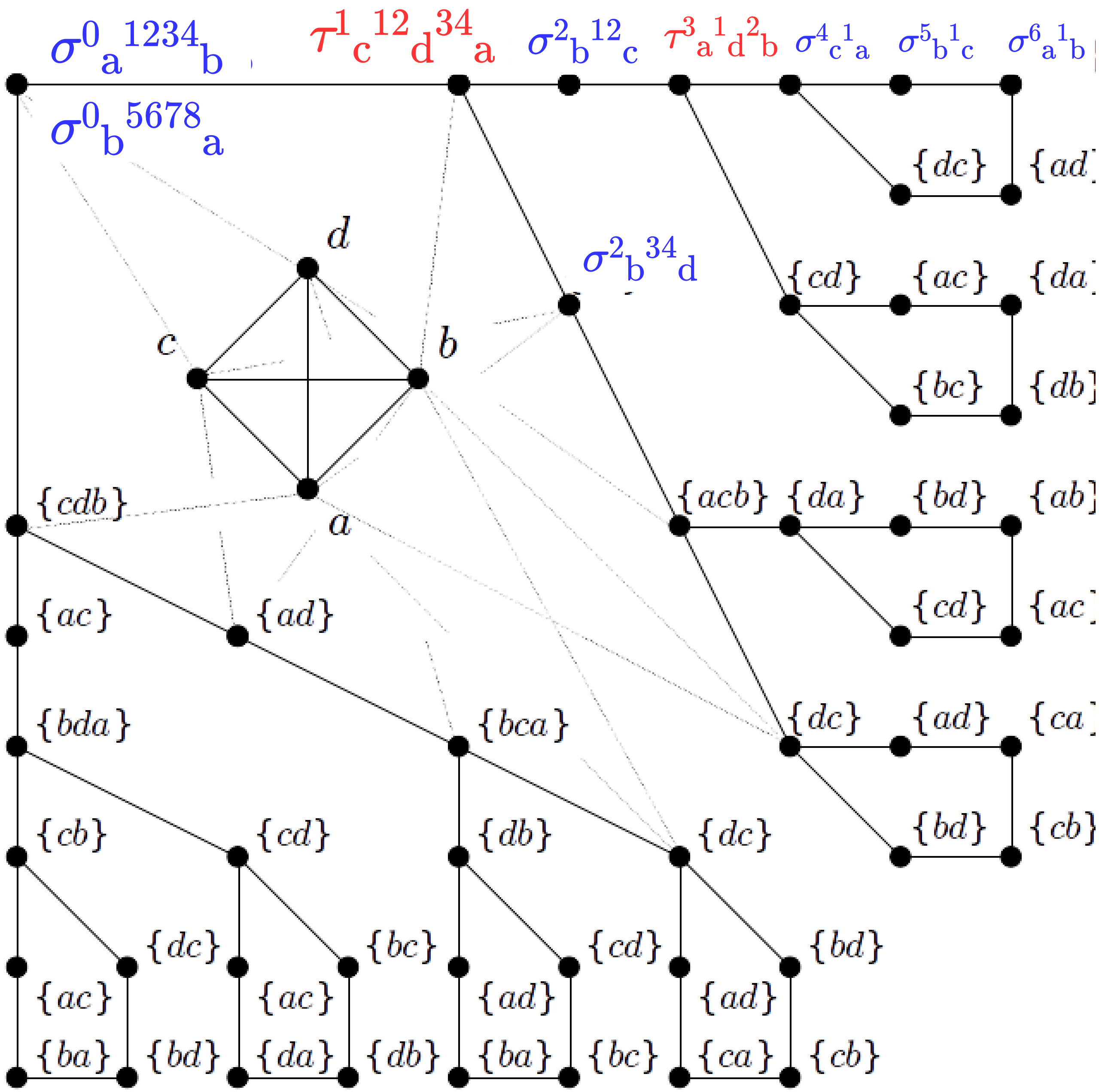
$K_4^{\Omega_{13}(K_8)}$



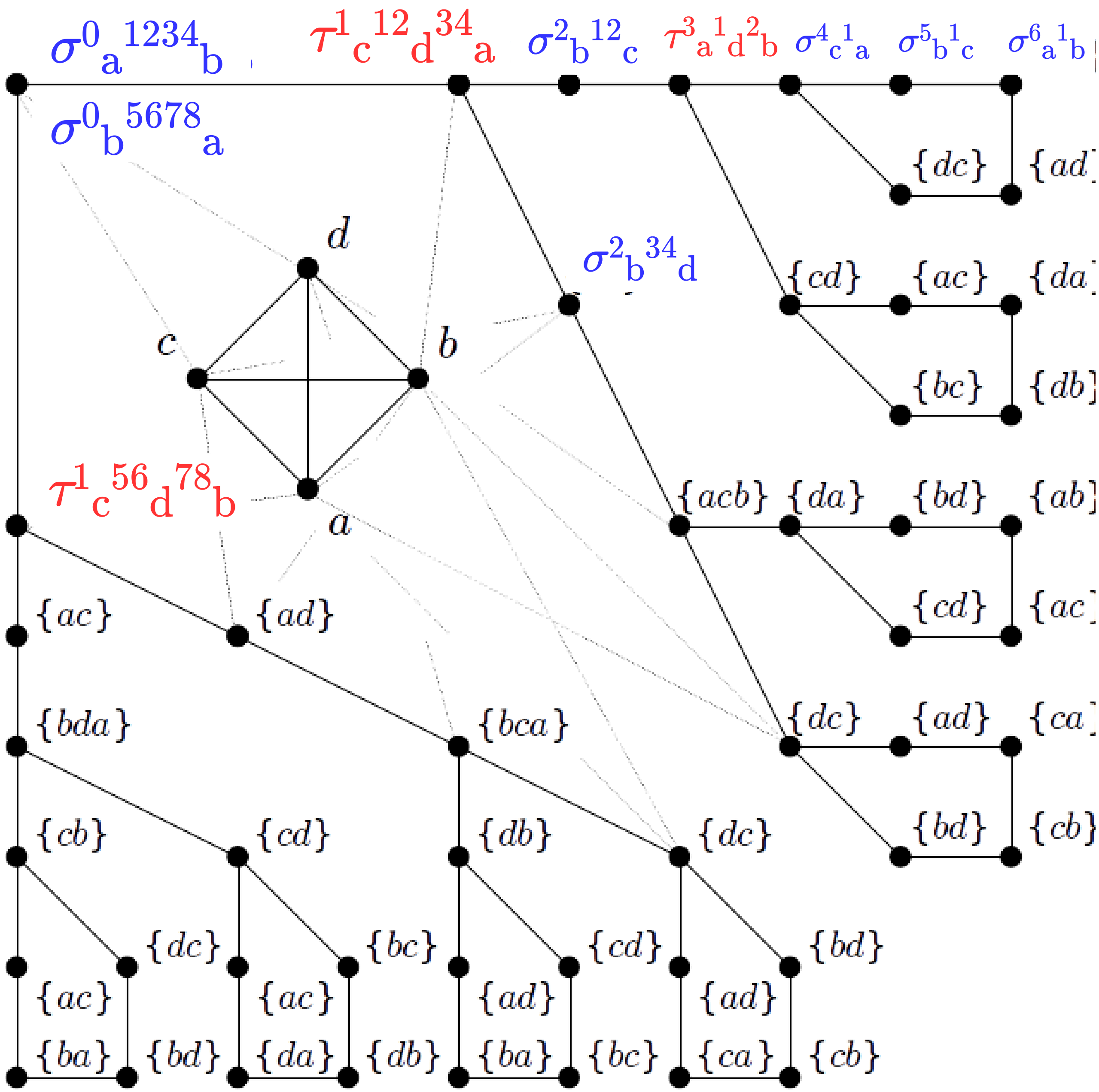
$K_4^{\Omega_{13}(K_8)}$



$K_4^{\Omega_{13}(K_8)}$



$K_4^{\Omega_{13}(K_8)}$



Special functions in $K_n^{\Omega_w(K_m)}$

$\sigma^{i,}{}_{a,}{}^{S}{}_{b}$

$\tau^{i,}{}_{a,}{}^{S}{}_{b,}{}^{T}{}_{c}$

constant functions: a, b, c, ...

Special functions in $K_n^{\Omega_w(K_m)}$

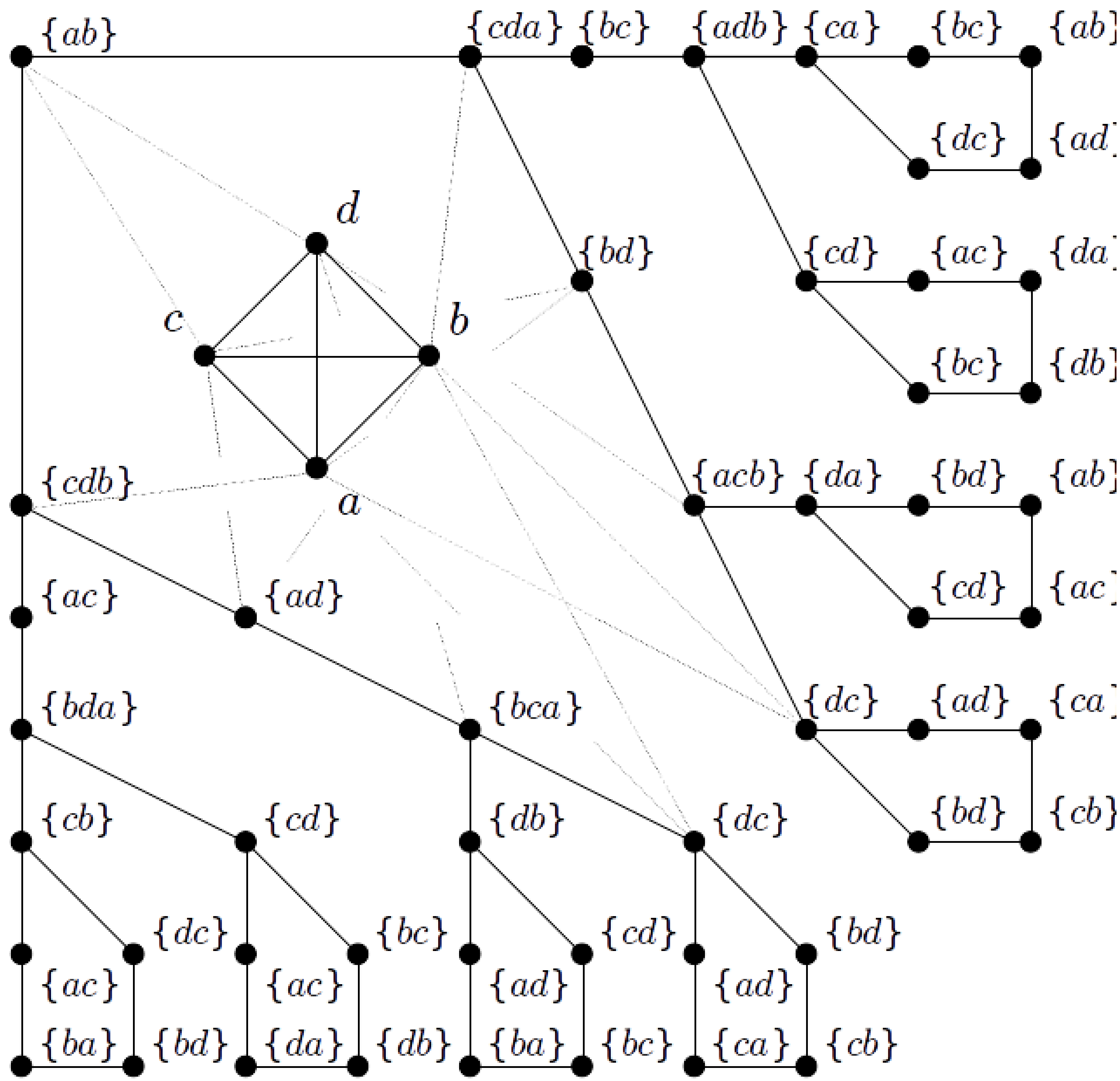
$\sigma^{i,}{}_{a,}{}^{S}{}_{b}$

$\tau^{i,}{}_{a,}{}^{S}{}_{b,}{}^{T}{}_{c}$

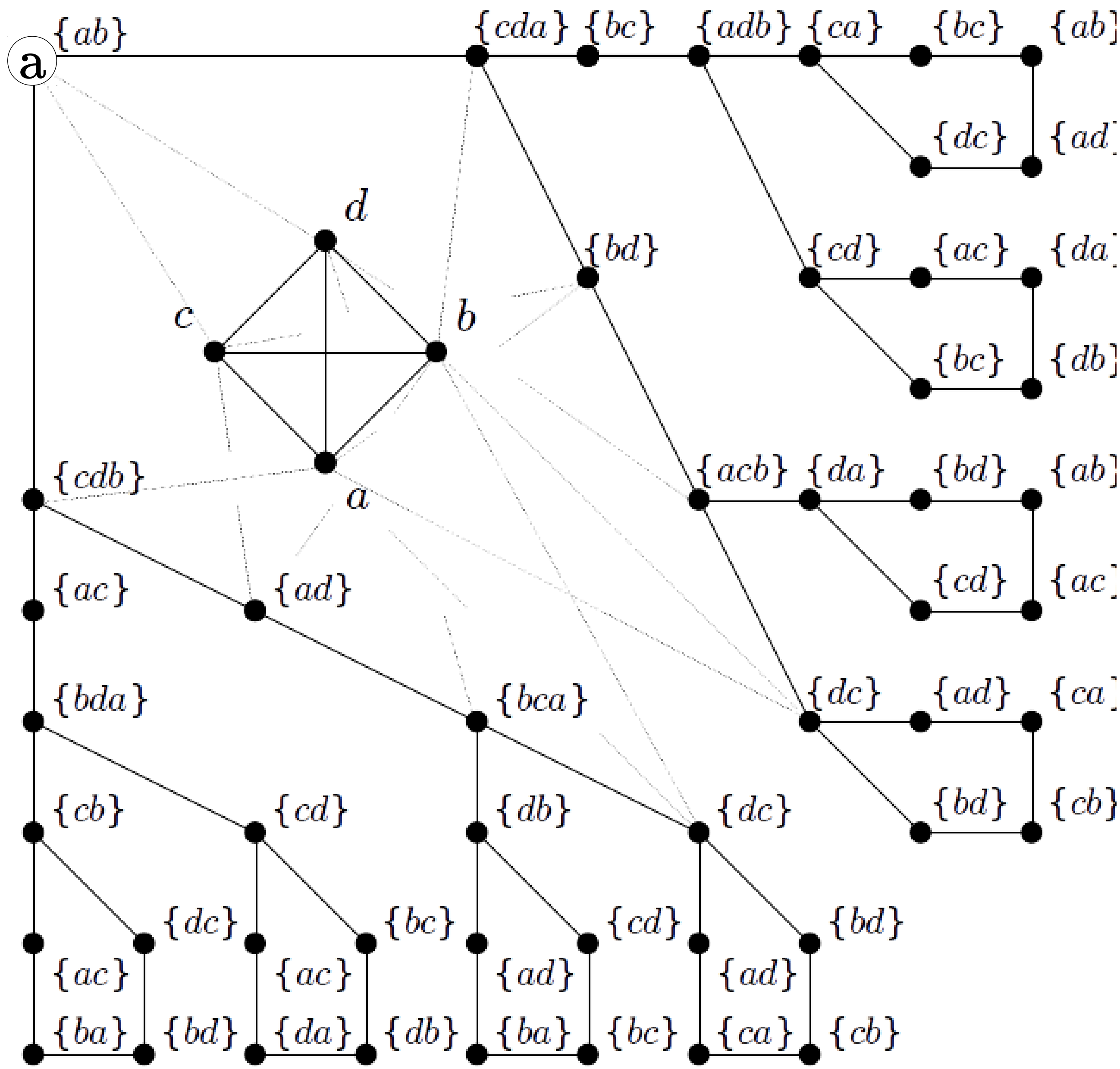
constant functions: a, b, c, ...

If $K_n^{\Omega_w(K_m)}$ is coloured with n colours, we can suppose that each constant function is coloured with its own colour.

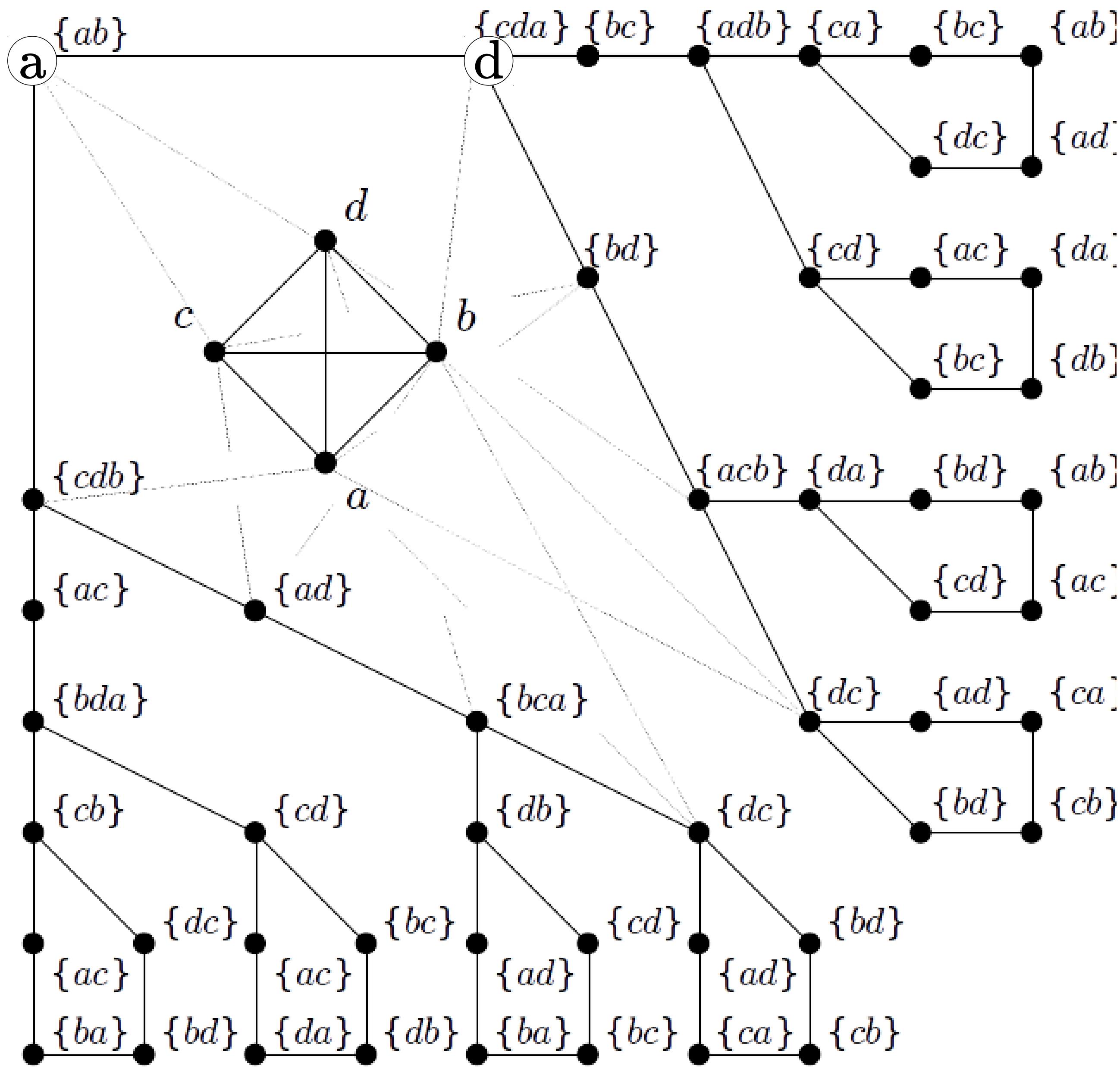
$K_4^{\Omega_{13}(K_8)}$



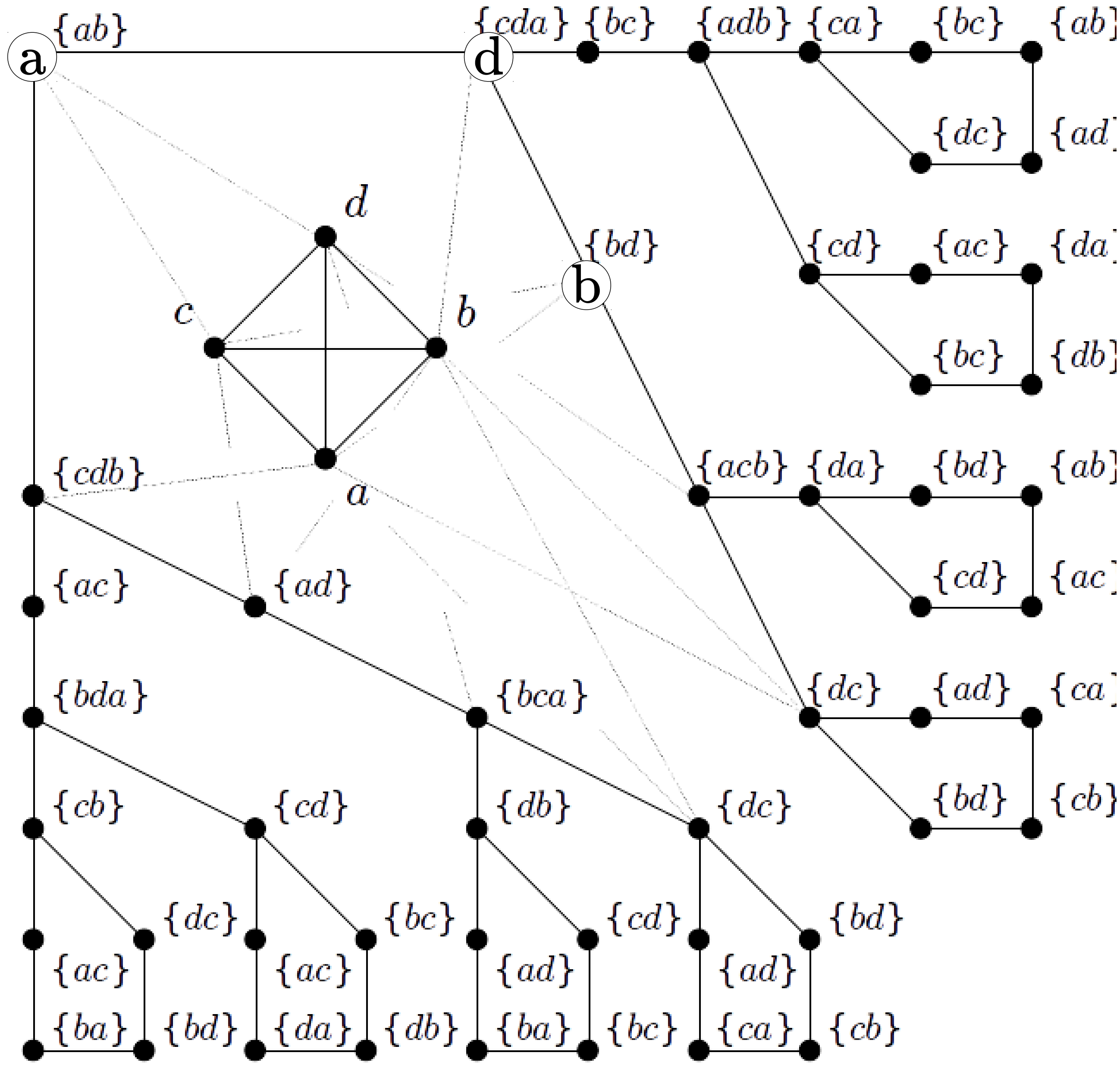
$K_4^{\Omega_{13}(K_8)}$



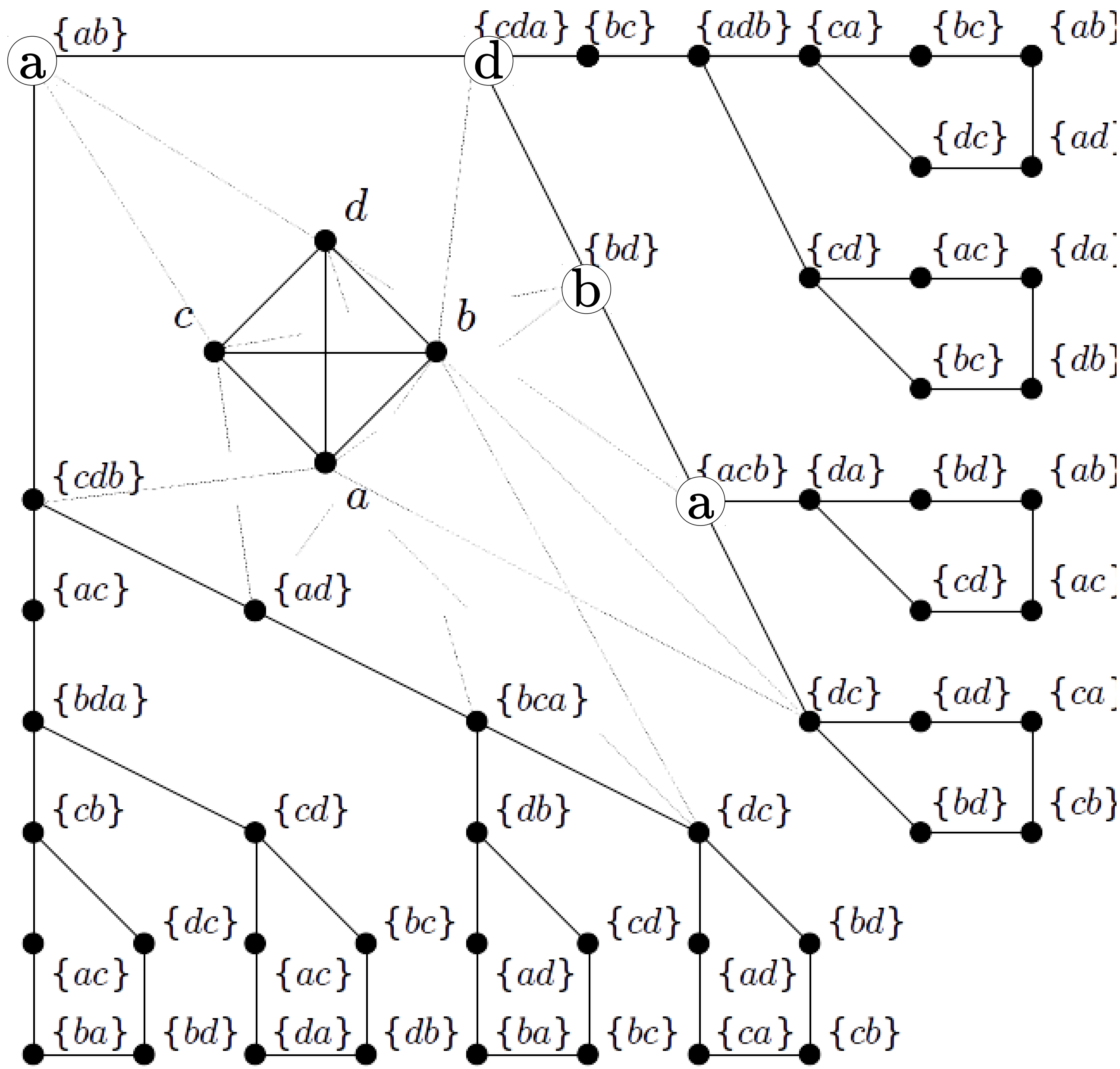
$K_4^{\Omega_{13}(K_8)}$



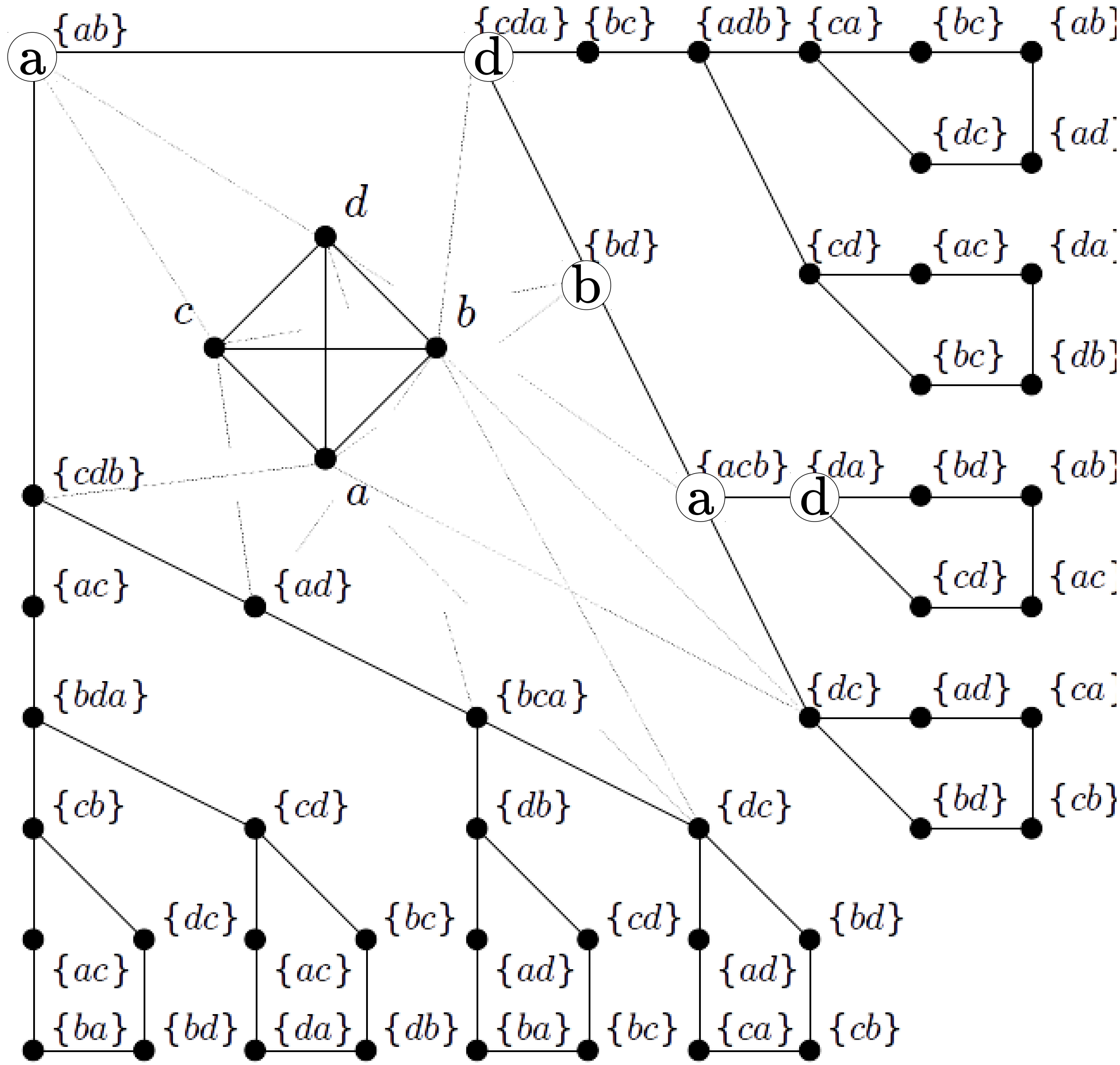
$K_4^{\Omega_{13}(K_8)}$



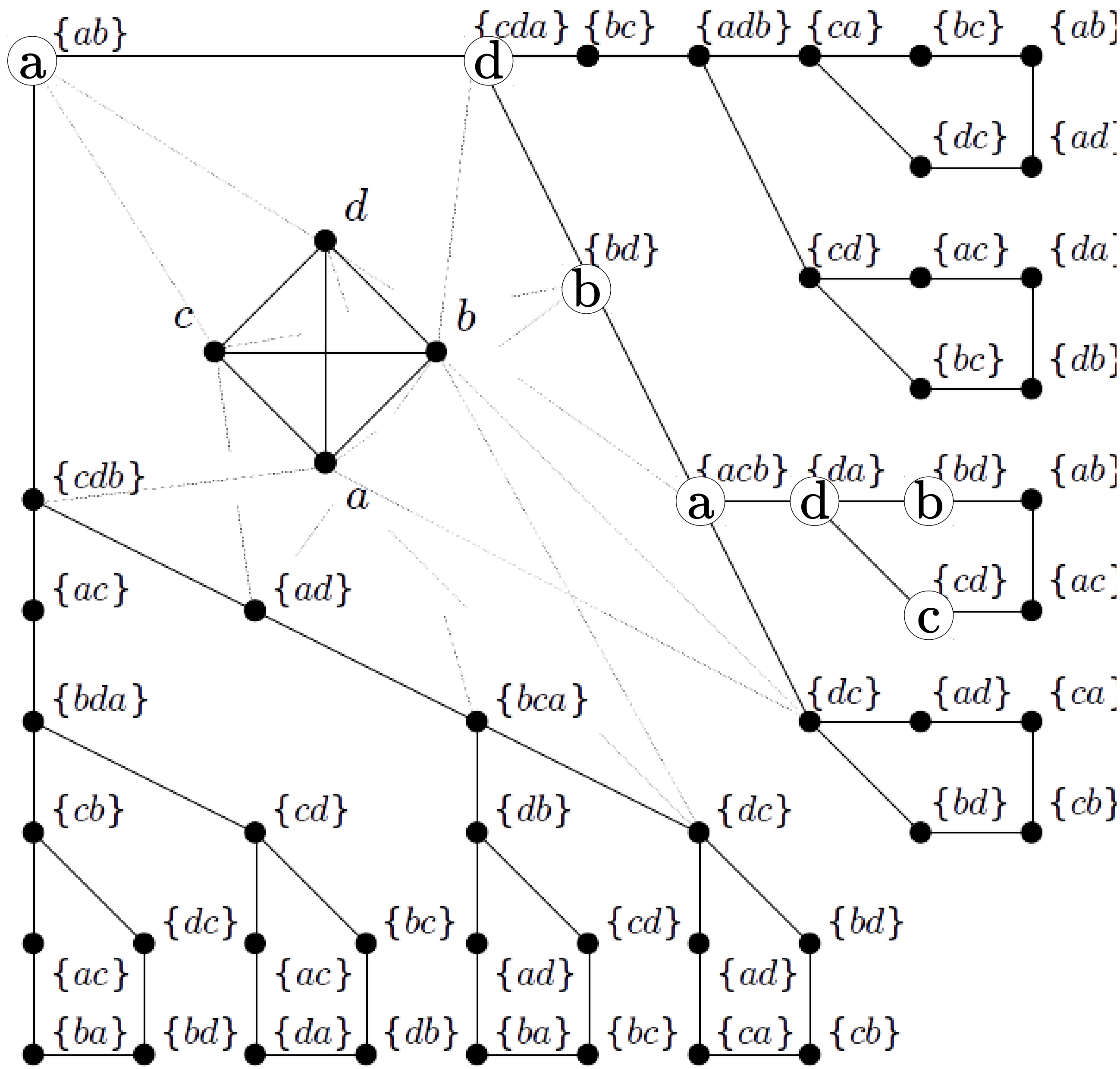
$K_4^{\Omega_{13}(K_8)}$



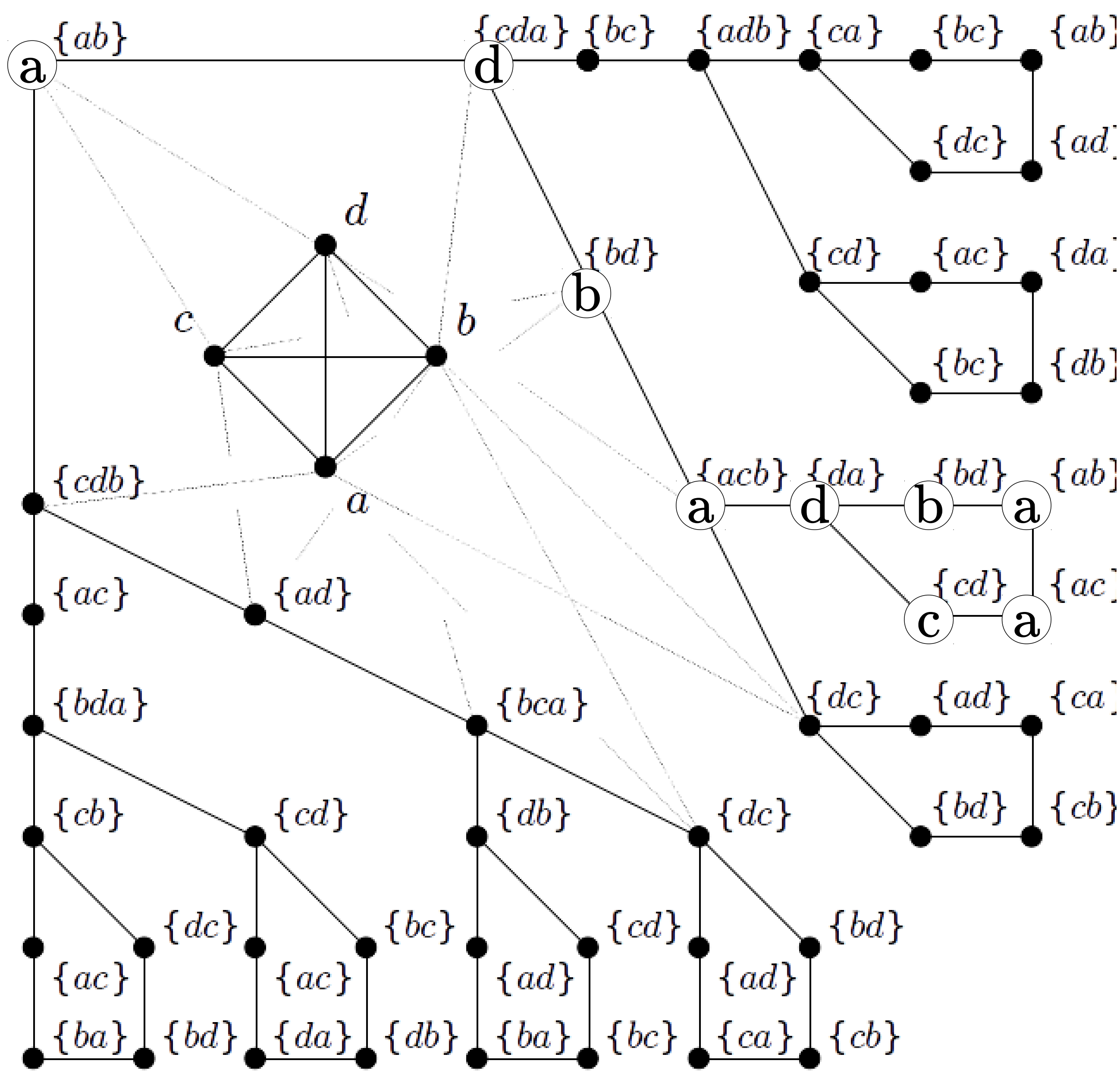
$K_4^{\Omega_{13}(K_8)}$



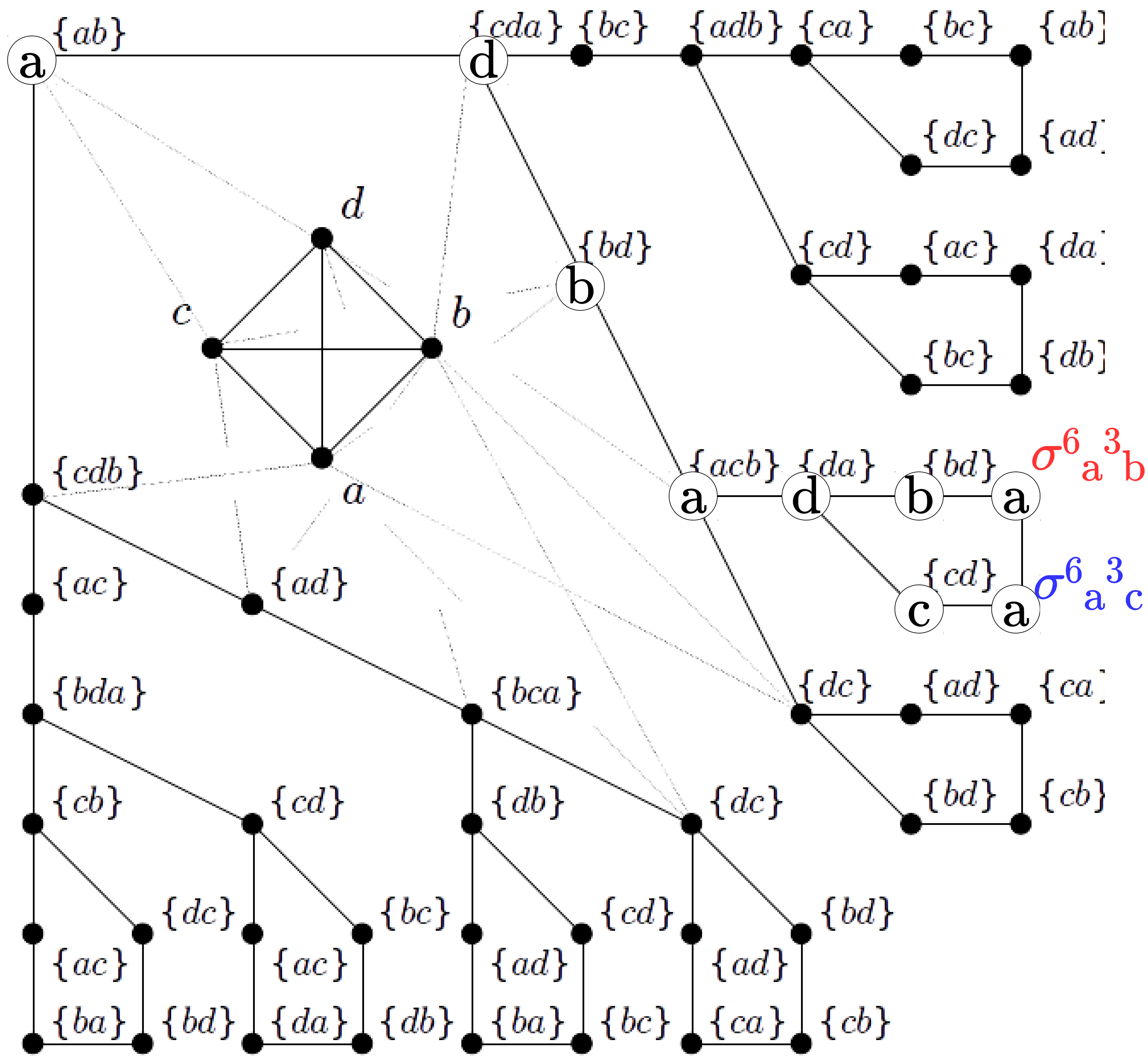
$K_4^{\Omega_{13}(K_8)}$



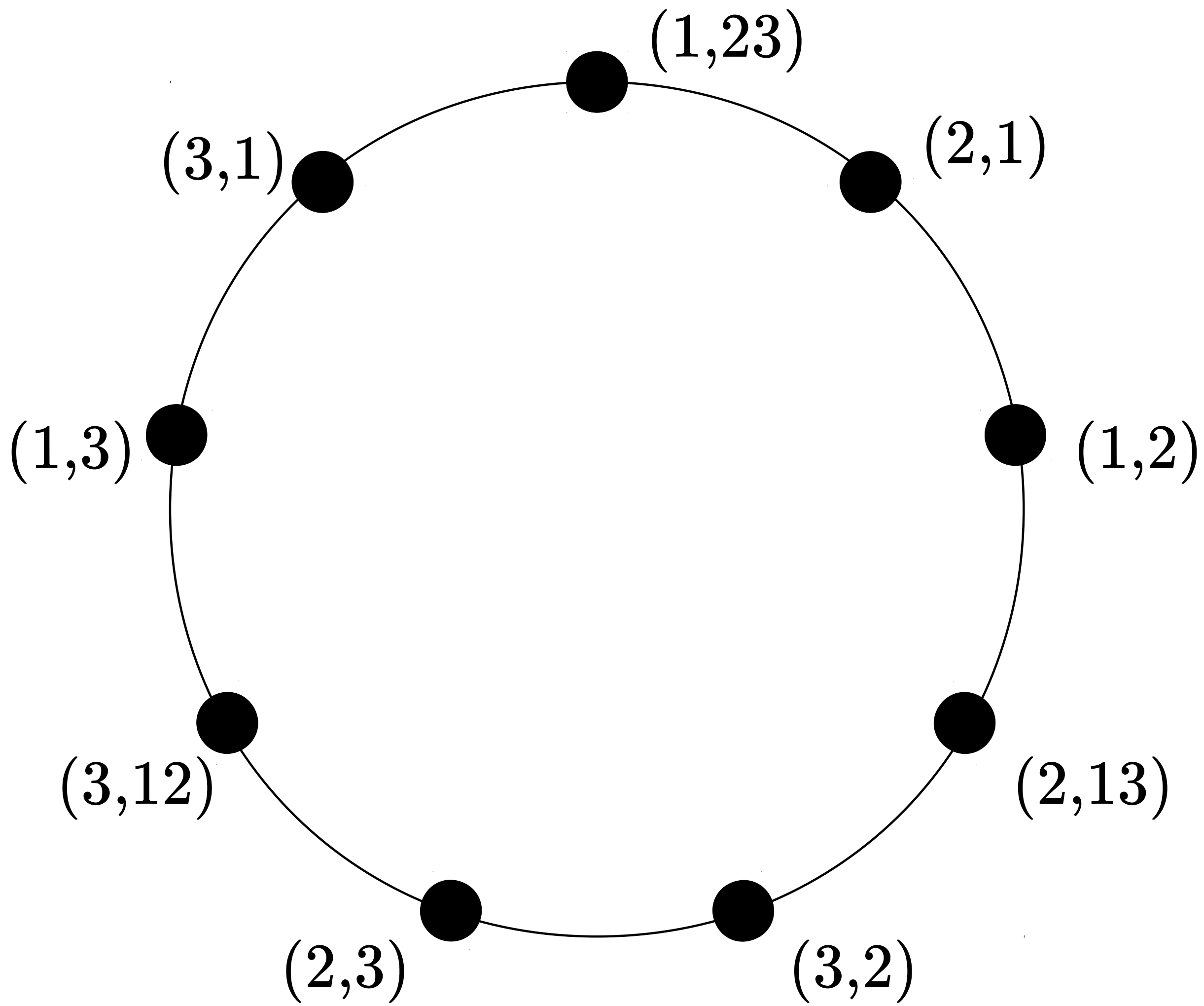
$K_4^{\Omega_{13}(K_8)}$



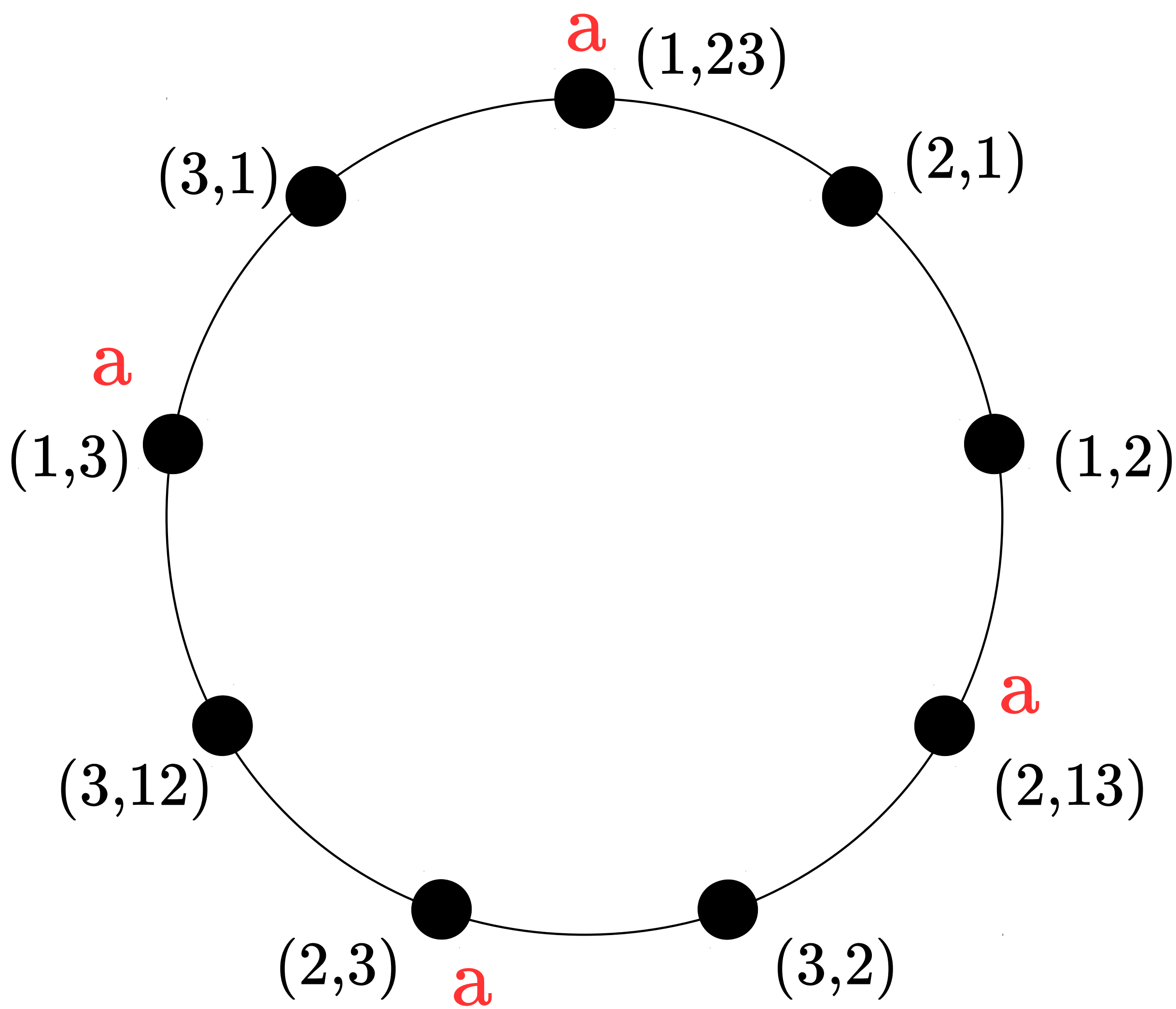
$K_4^{\Omega_{13}(K_8)}$



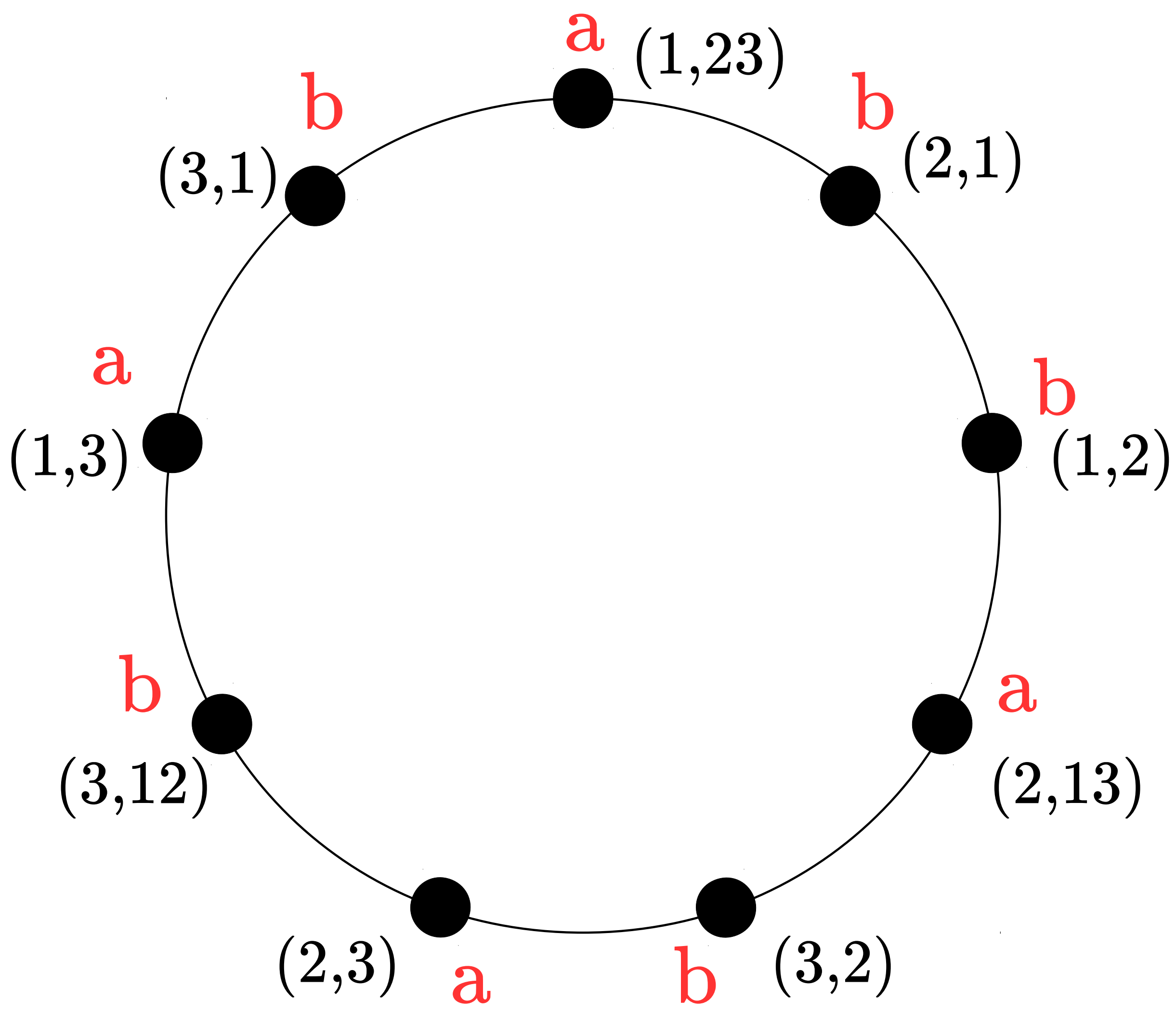
$\Omega_3(K_3)$: $\sigma^{1,}{}_{a,}{}^{\{3\}}{}_{b}$



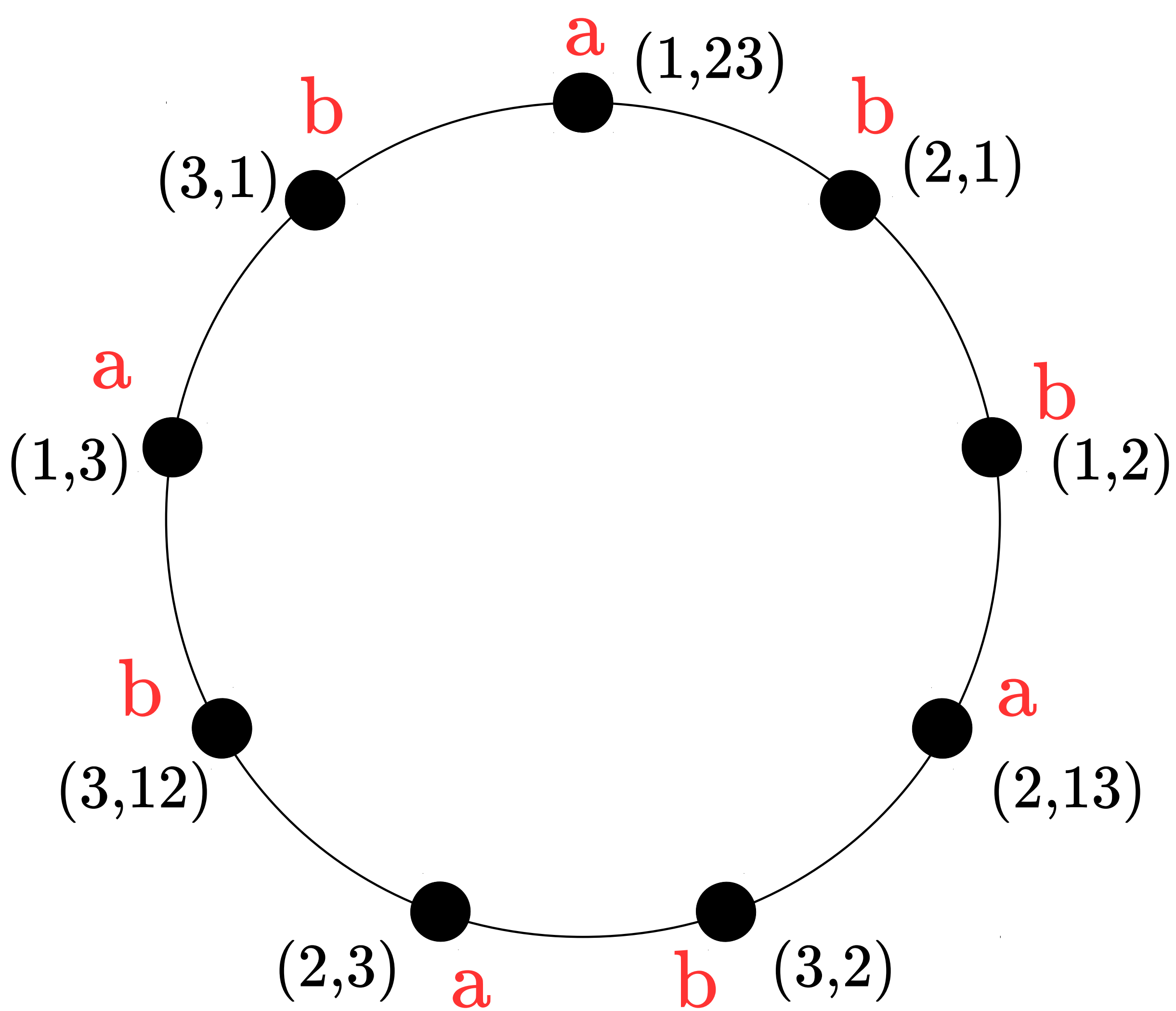
$\Omega_3(K_3)$: $\sigma^{1,}{}_{a,}{}^{\{3\}}{}_{b}$



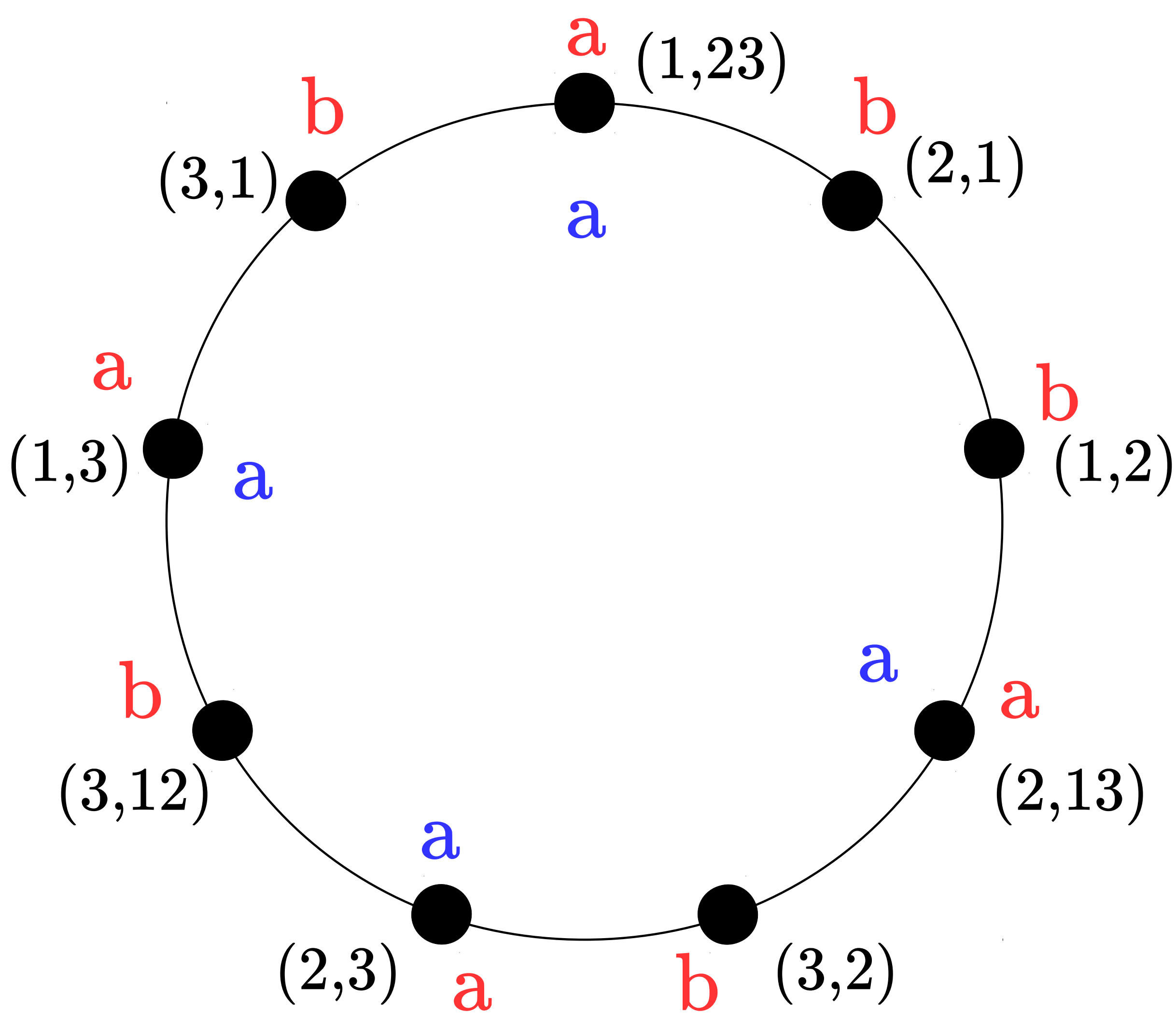
$\Omega_3(K_3)$: $\sigma^{1,}{}_{a,}{}^{\{3\}}{}_{b}$



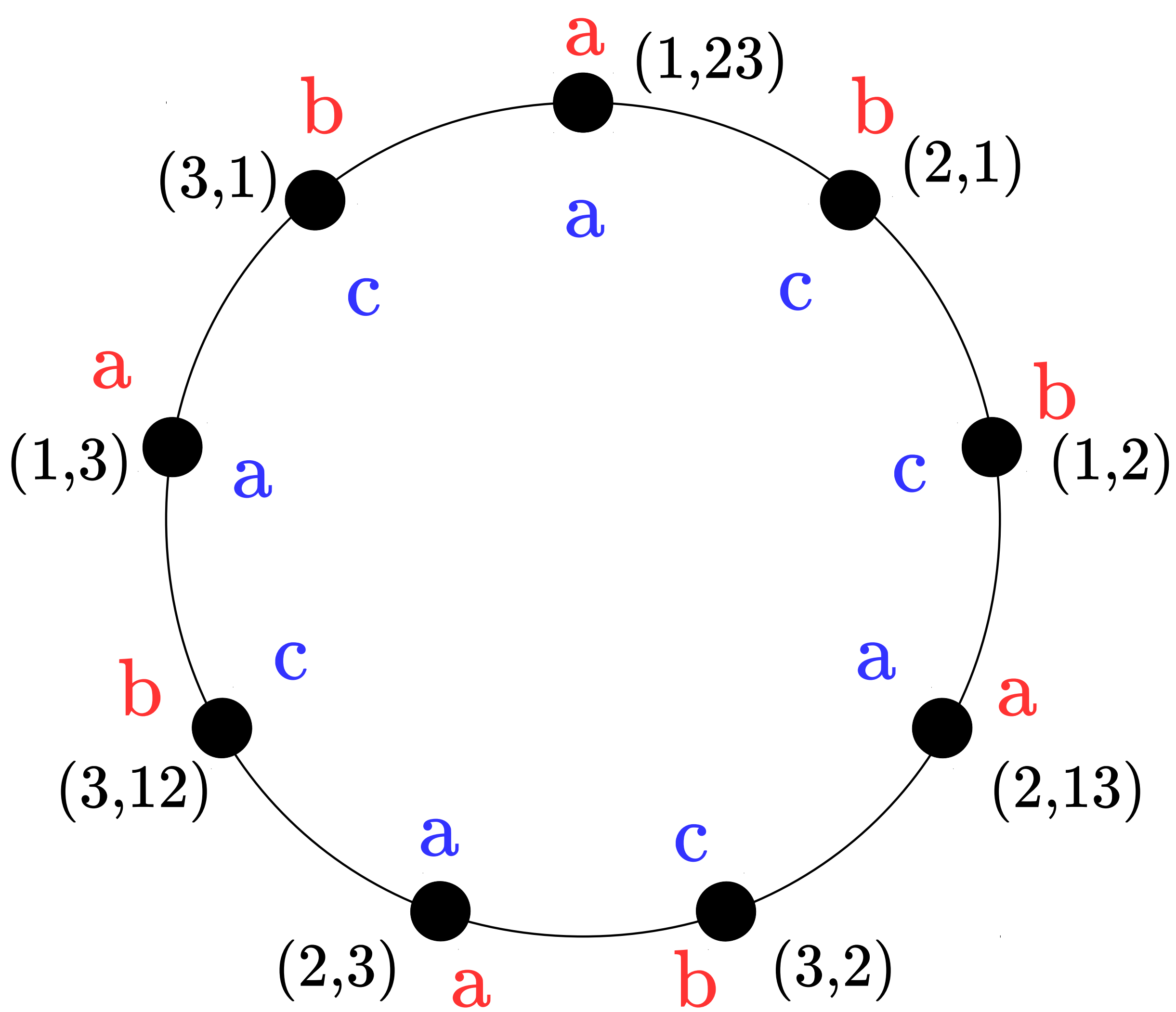
$\Omega_3(K_3)$: $\sigma^{1,}{}_{a,}{}^{\{3\}}{}_{b}$ $\sigma^{1,}{}_{a,}{}^{\{3\}}{}_{c}$



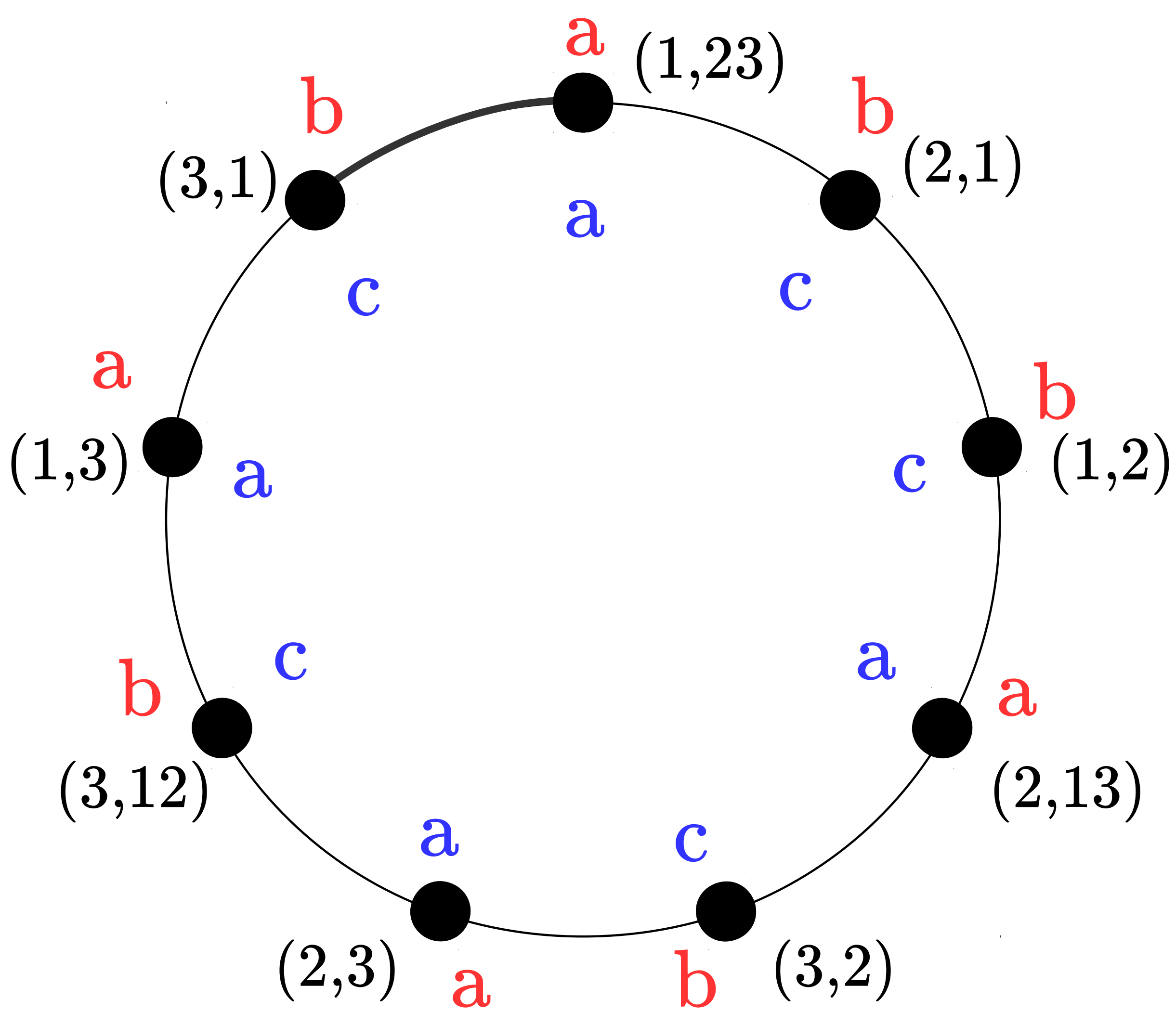
$\Omega_3(K_3)$: $\sigma^{1,}{}_{a,}{}^{\{3\}}{}_{b}$ $\sigma^{1,}{}_{a,}{}^{\{3\}}{}_{c}$



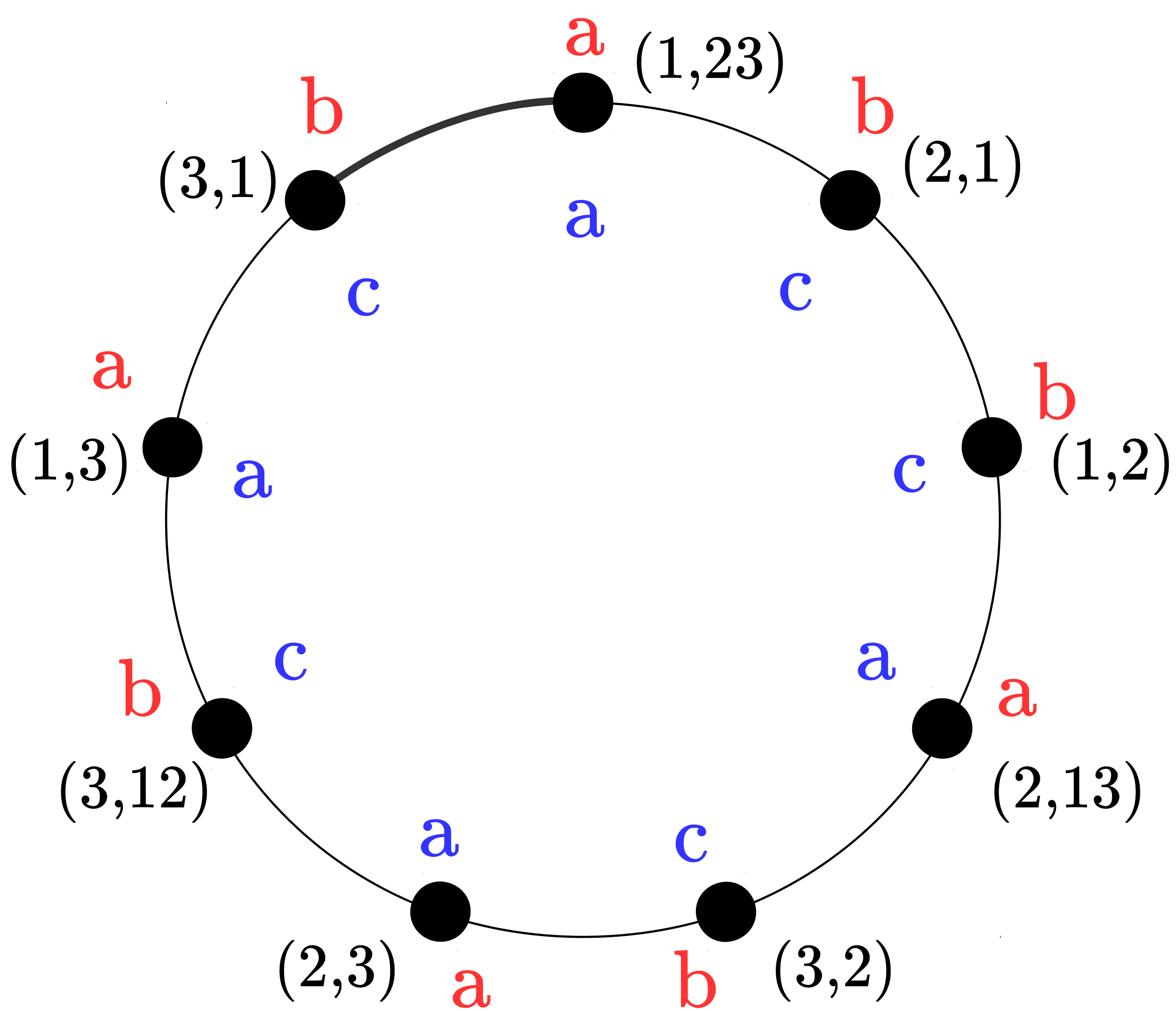
$\Omega_3(K_3)$: $\sigma^{1,}{}_{a,}{}^{\{3\}}{}_{b}$ $\sigma^{1,}{}_{a,}{}^{\{3\}}{}_{c}$



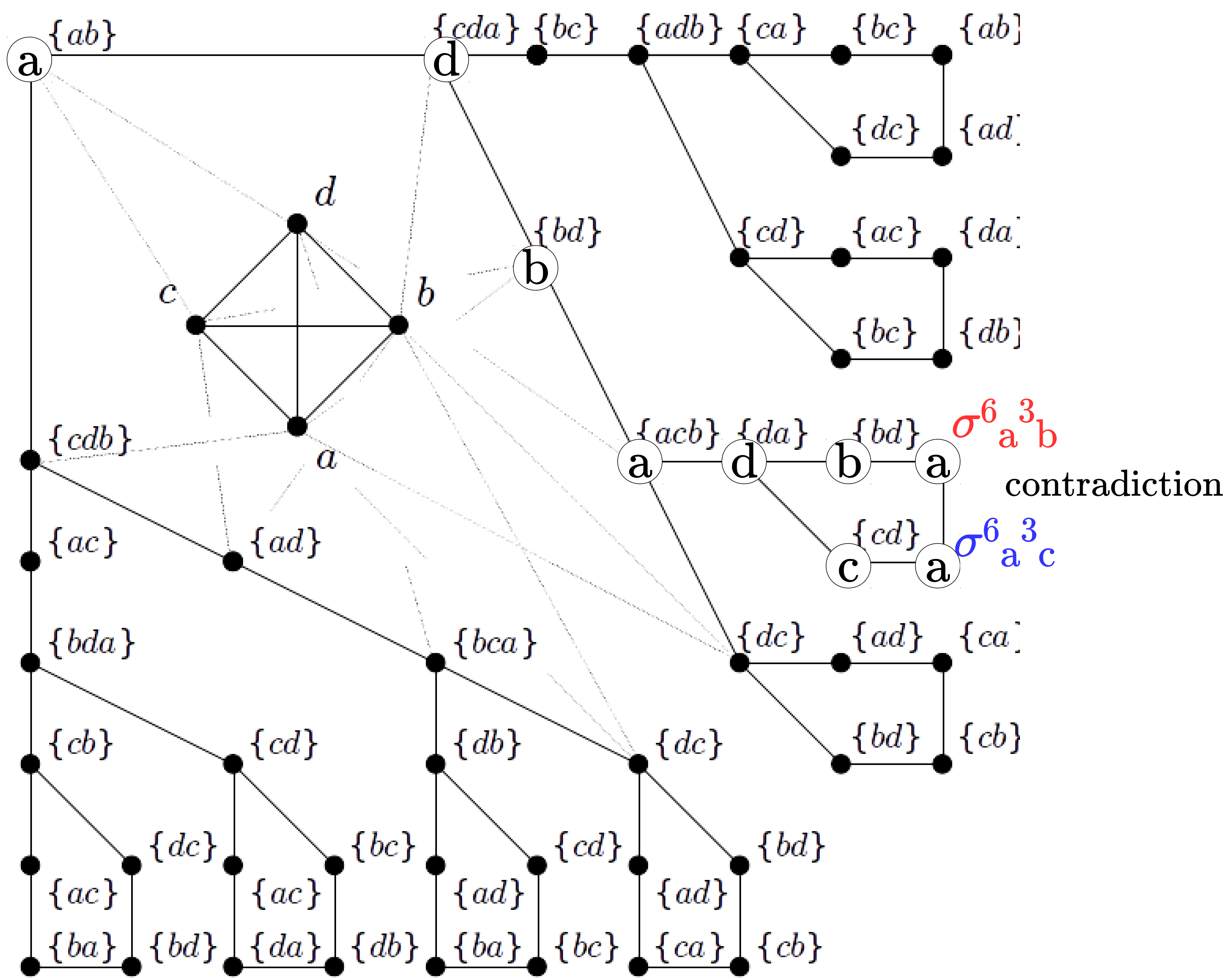
$\Omega_3(K_3)$: $\sigma^{1,}{}_{a,}{}^{\{3\}}{}_{b}$ $\sigma^{1,}{}_{a,}{}^{\{3\}}{}_{c}$



$\Omega_3(K_3)$: $\sigma^{1,}{}_{a,}{}^{\{3\}}{}_{b}$ ——— $\sigma^{1,}{}_{a,}{}^{\{3\}}{}_{c}$ in $K_n{}^{\Omega_w(K_m)}$



$K_4^{\Omega_{13}(K_8)}$



Today: $\chi(\Omega_{13}(K_8) \times {K_4}^{\Omega_{13}(K_8)}) = 4$

$\Omega_{13}(K_8)$: 8-chromatic, 4348856 vertices

${K_4}^{\Omega_{13}(K_8)}$: $4^{4348856}$ vertices, $\chi \geq 5$

Today: $\chi(\Omega_{13}(K_8) \times {K_4}^{\Omega_{13}(K_8)}) = 4$

$\Omega_{13}(K_8)$: 8-chromatic, 4348856 vertices

${K_4}^{\Omega_{13}(K_8)}$: $4^{4348856}$ vertices, $\chi \geq 5$

4-colouring of $\Omega_{13}(K_8) \times {K_4}^{\Omega_{13}(K_8)}$:

colour of (u,f) is f(u).

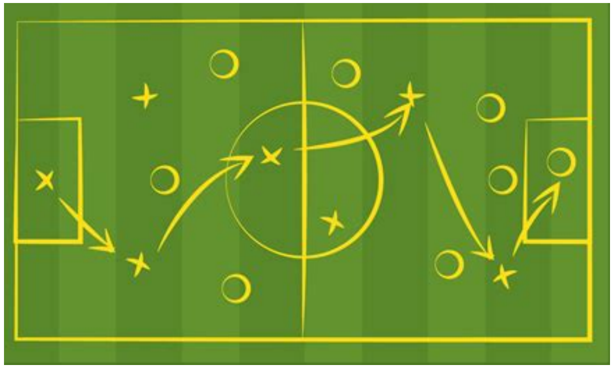
Thank you!

Thank you!

… but I'm not finished:

- Questions?
- Next I ramble on with my comments on the proof

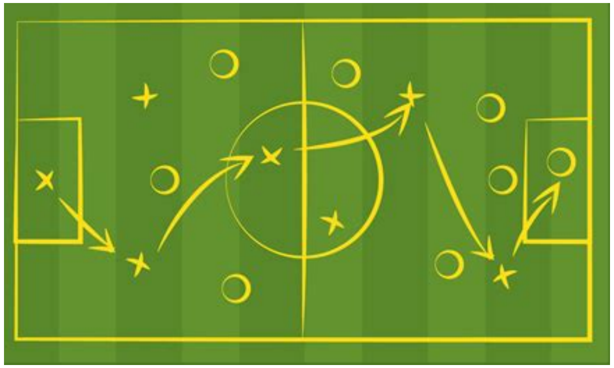
My comments on the proofs

the football	the game
	

My comments on the proofs

the football

the game



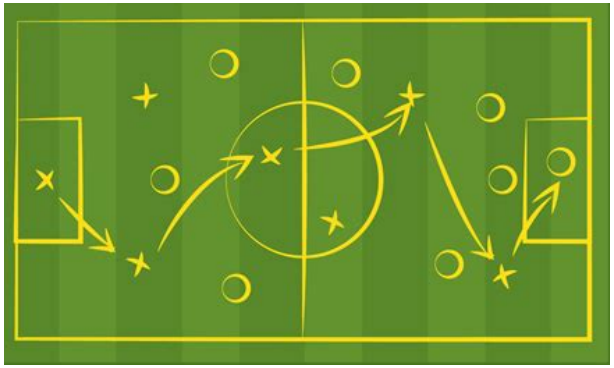
Shitov: $G_{\text{Erdős}}[K_q]$

$K_n^{G_{\text{Erdős}}[K_q]}$

My comments on the proofs

the football

the game



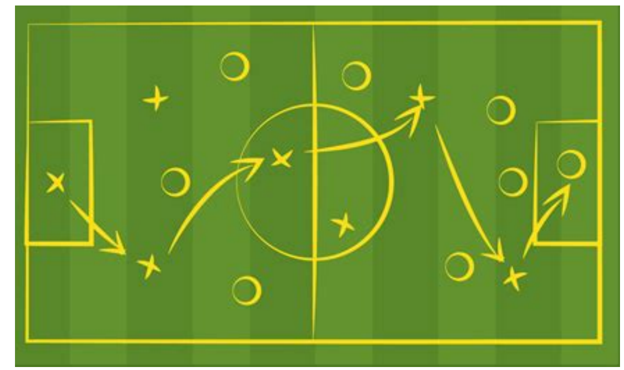
Shitov: $G_{\text{Erdős}}[K_q]$

Xhu: $G_{\text{Exoo}}[K_q]$

${K_n}^{G_{\text{Erdős}}[K_q]}$

${K_n}^{G_{\text{Exoo}}[K_q]}$

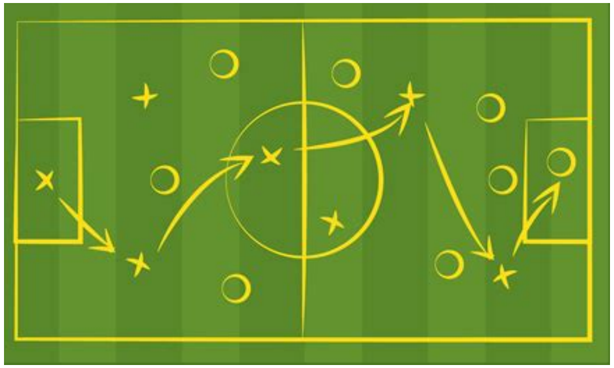
My comments on the proofs

the football	the game
	
Shitov: $G_{\text{Erdős}}[K_q]$	${K_n}^{G_{\text{Erdős}}[K_q]}$
Xhu: $G_{\text{Exoo}}[K_q]$	${K_n}^{G_{\text{Exoo}}[K_q]}$
Me: $\Omega_5(K_9)[T]$	${K_{13}}^{\Omega_5(K_9)[T]}$

My comments on the proofs

the football

the game



Shitov: $G_{\text{Erdős}}[K_q]$

$K_n^{G_{\text{Erdős}}[K_q]}$

Xhu: $G_{\text{Exoo}}[K_q]$

$K_n^{G_{\text{Exoo}}[K_q]}$

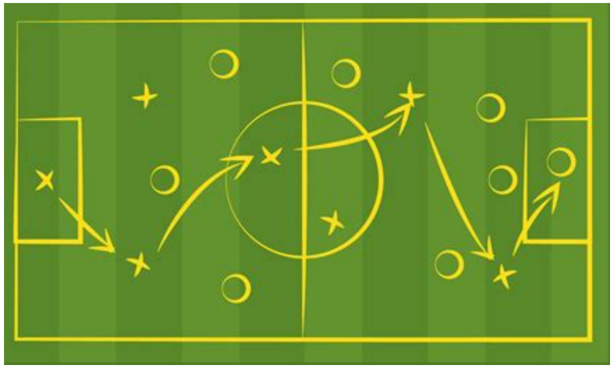
Me: $\Omega_5(K_9)[T]$

$K_{13}^{\Omega_5(K_9)[T]}$

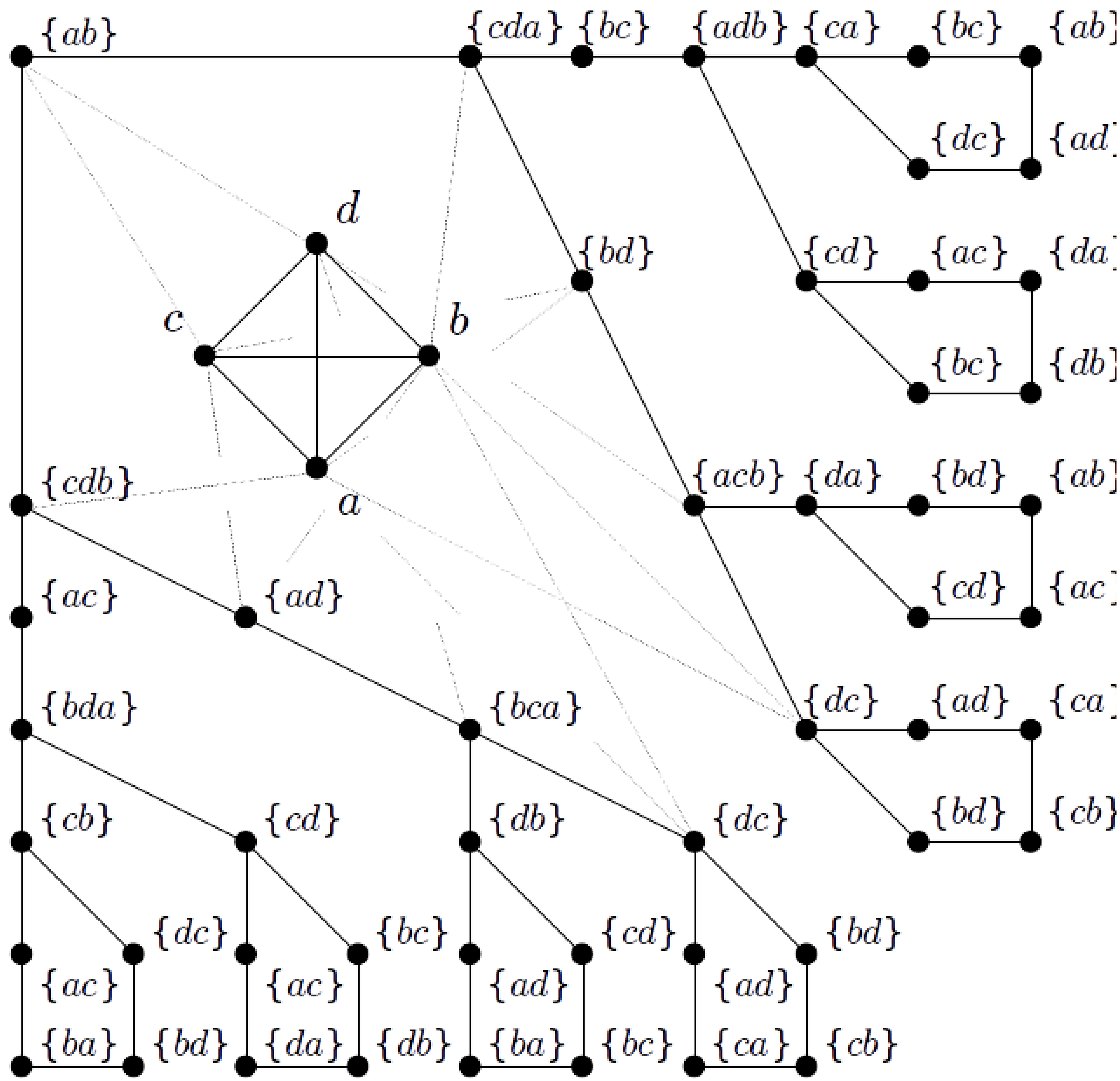
Wrochna: $\Omega_7(K_6)$

$K_5^{\Omega_7(K_6)}$

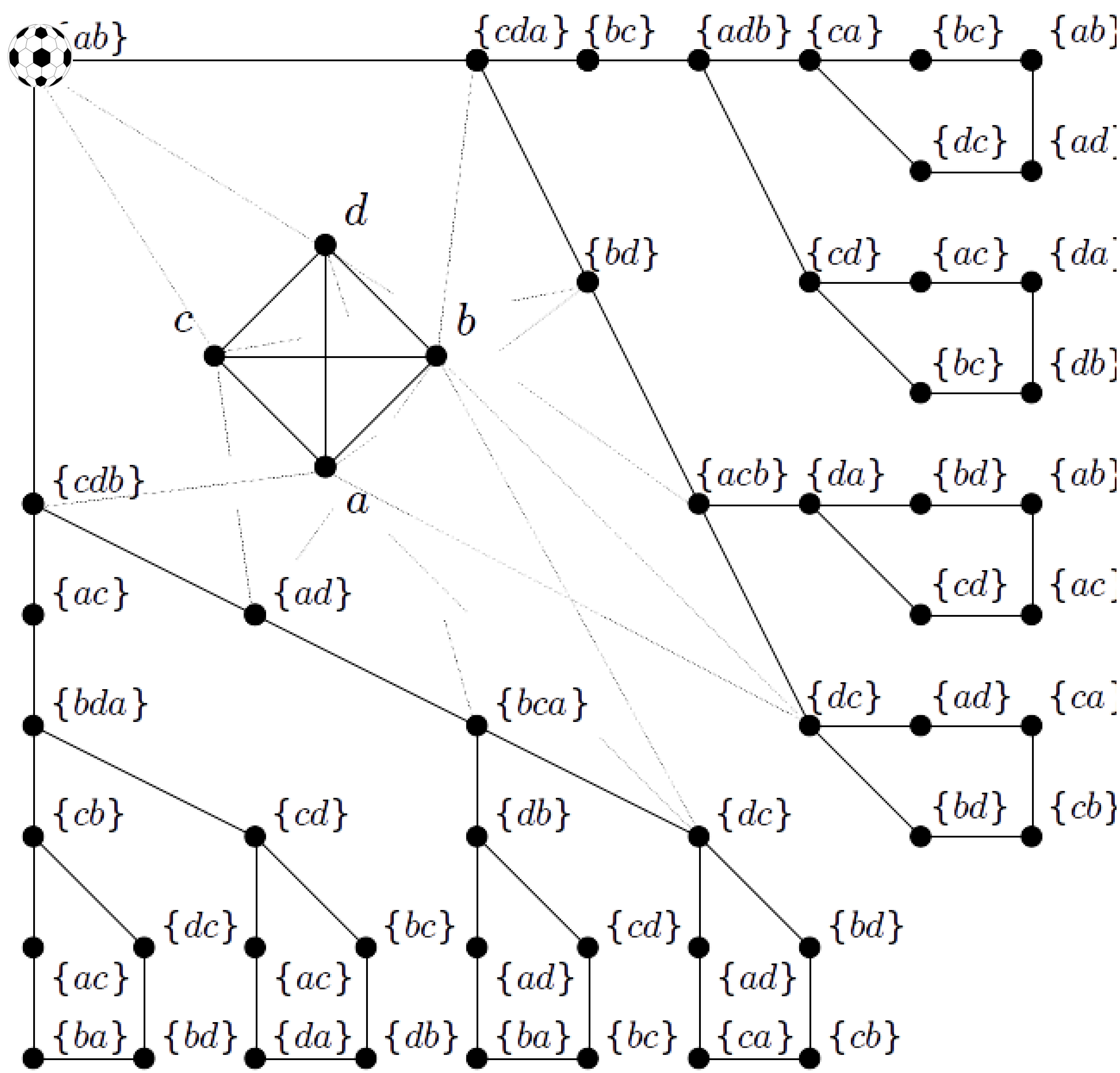
My comments on the proofs

the football	the game
	
Shitov: $G_{\text{Erdős}}[K_q]$	$K_n^{G_{\text{Erdős}}[K_q]}$
Xhu: $G_{\text{Exoo}}[K_q]$	$K_n^{G_{\text{Exoo}}[K_q]}$
Me: $\Omega_5(K_9)[T]$	$K_{13}^{\Omega_5(K_9)[T]}$
Wrochna: $\Omega_7(K_6)$	$K_5^{\Omega_7(K_6)}$
Now: $\Omega_{13}(K_8)$	$K_4^{\Omega_{13}(K_8)}$

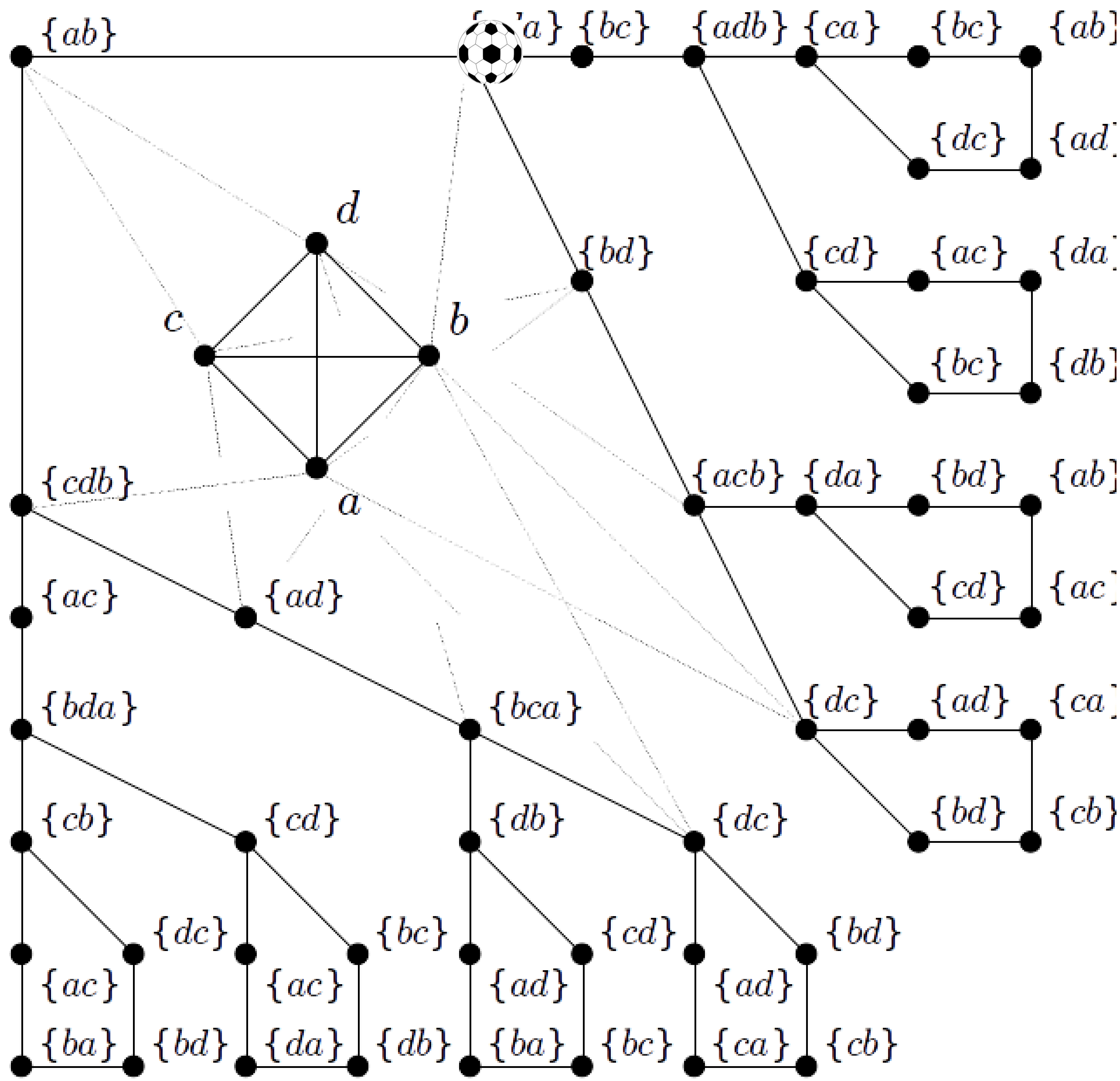
$$K_4^{\Omega_{13}(K_8)}$$



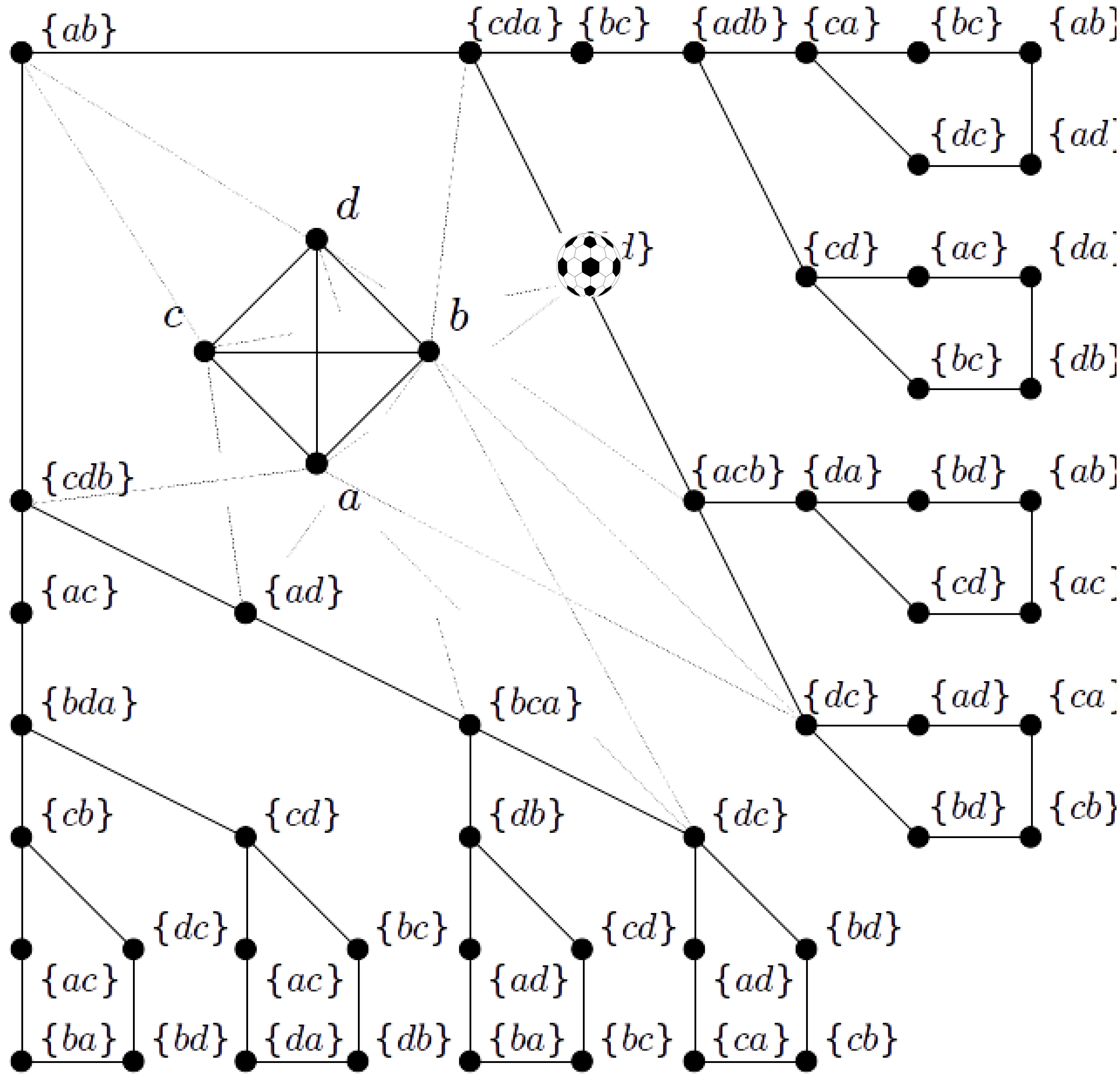
$K_4^{\Omega_{13}(K_8)}$



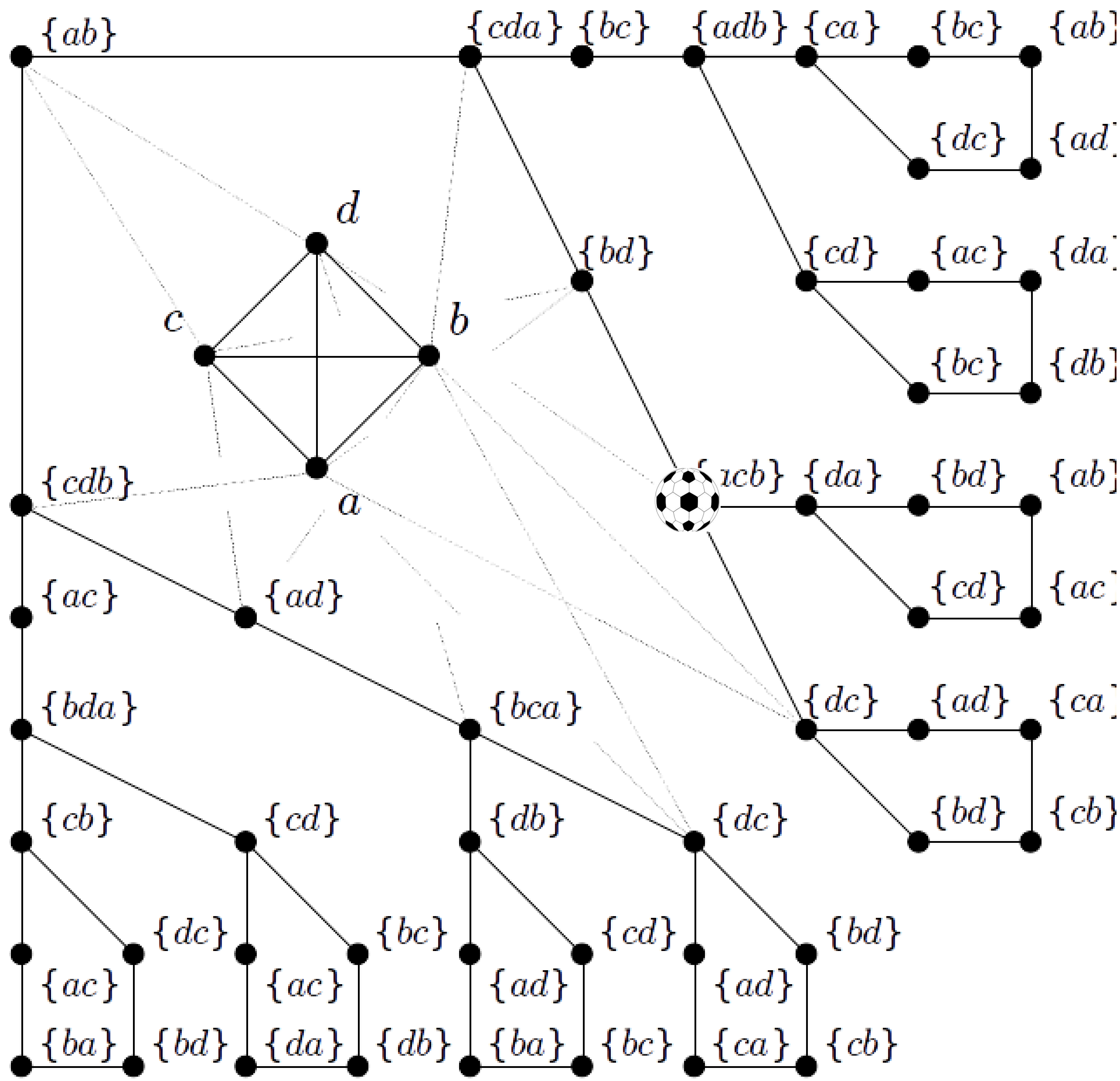
$K_4^{\Omega_{13}(K_8)}$



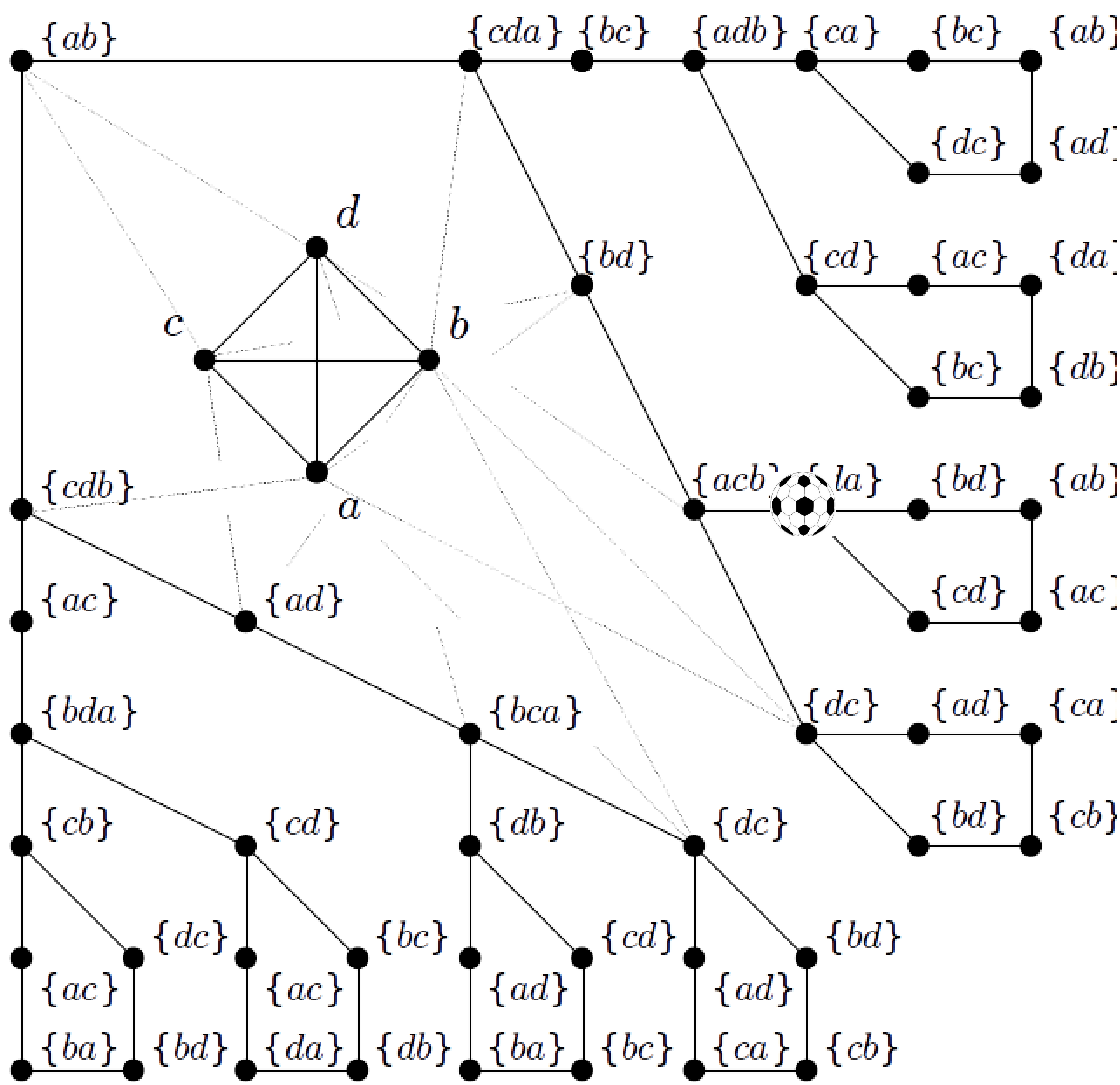
$K_4^{\Omega_{13}(K_8)}$



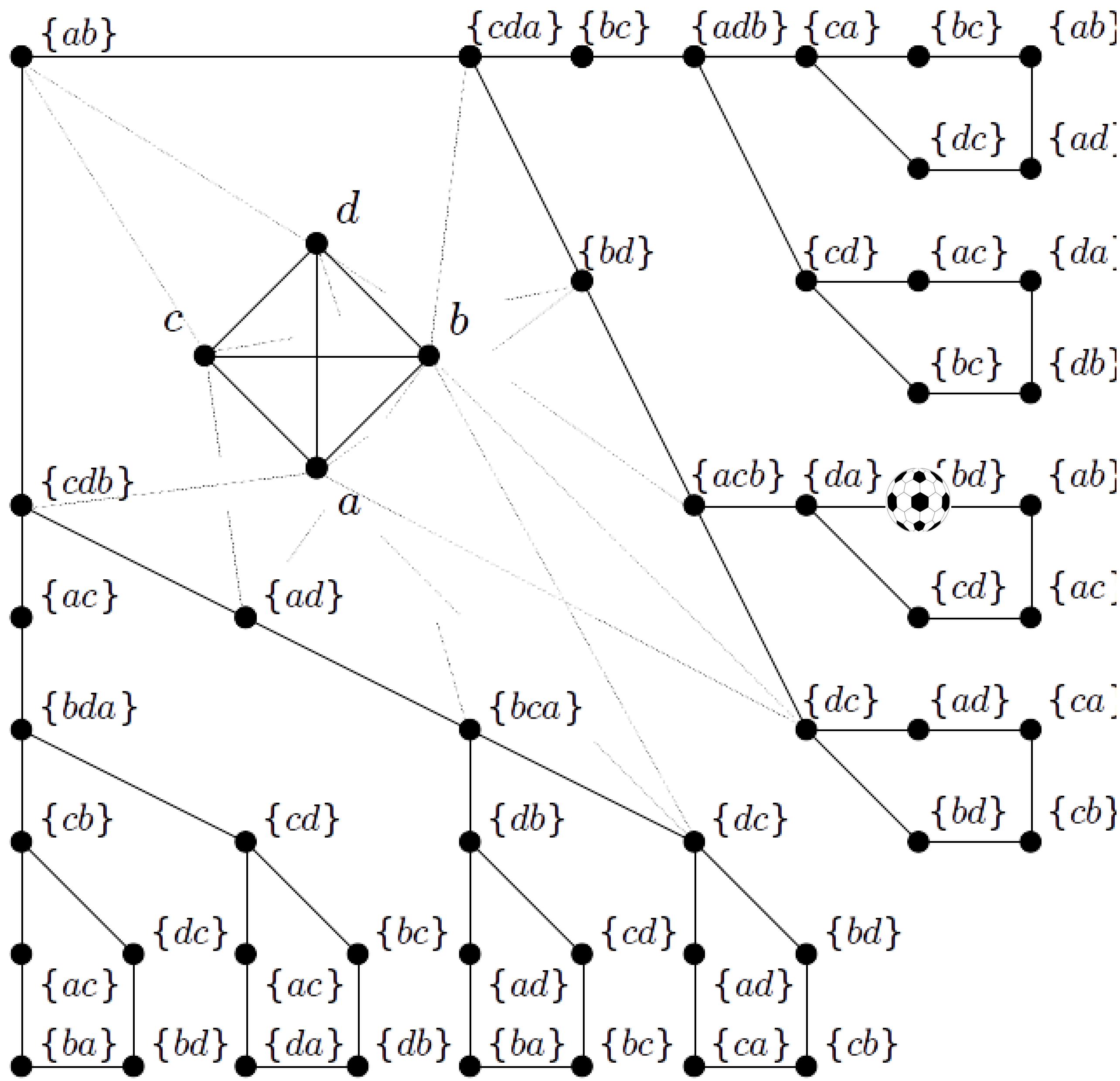
$K_4^{\Omega_{13}(K_8)}$



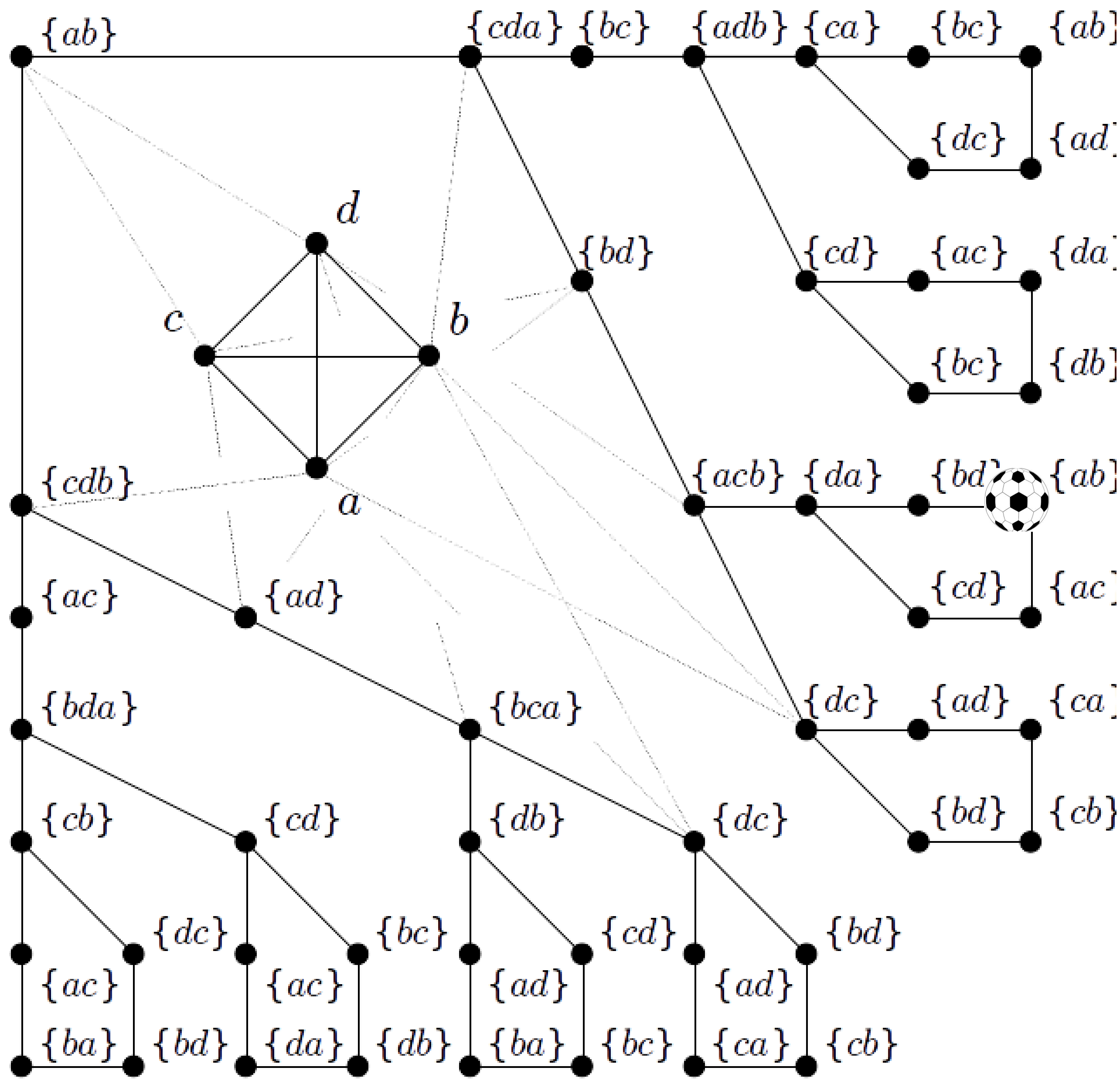
$K_4^{\Omega_{13}(K_8)}$



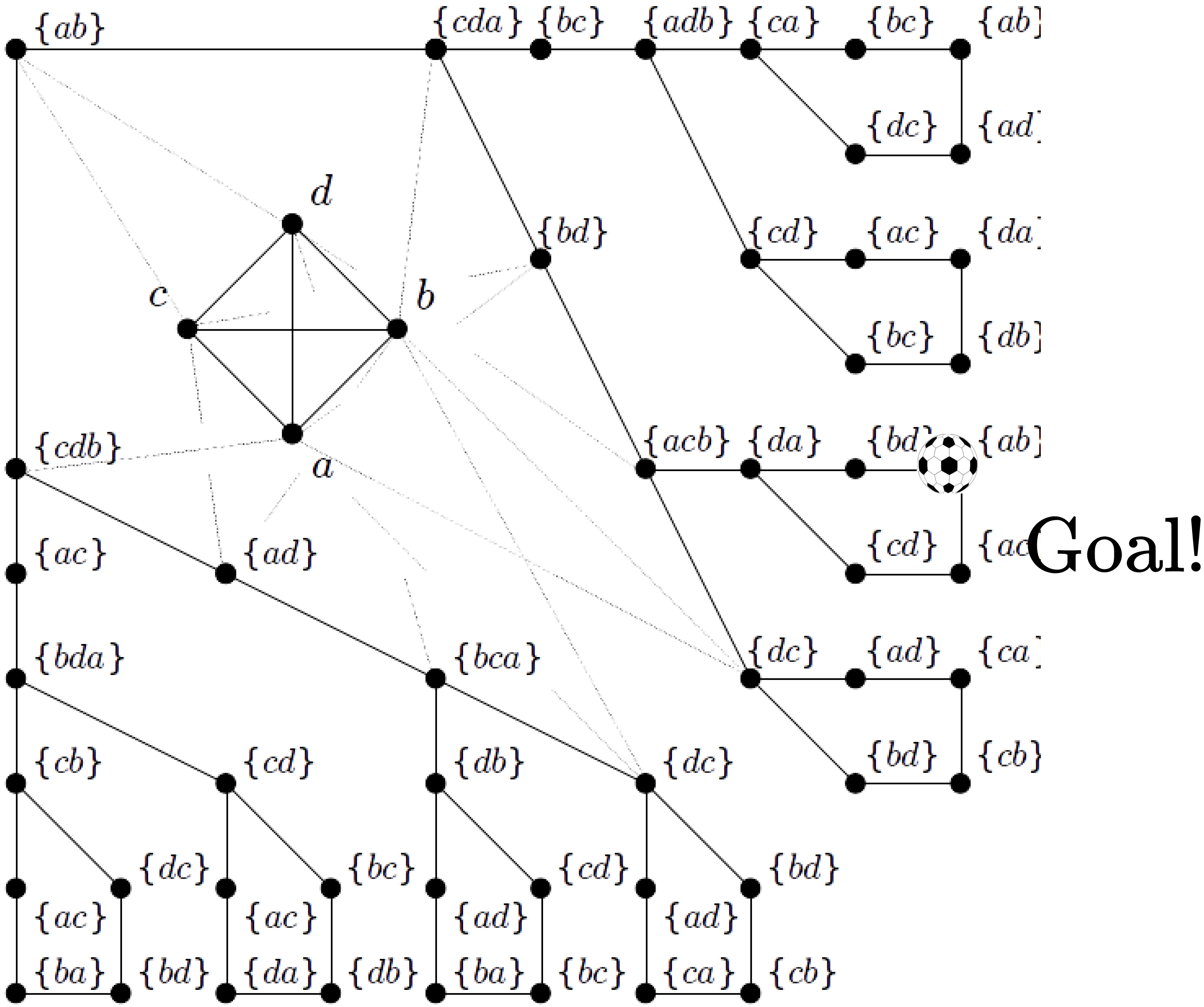
$$K_4^{\Omega_{13}(K_8)}$$



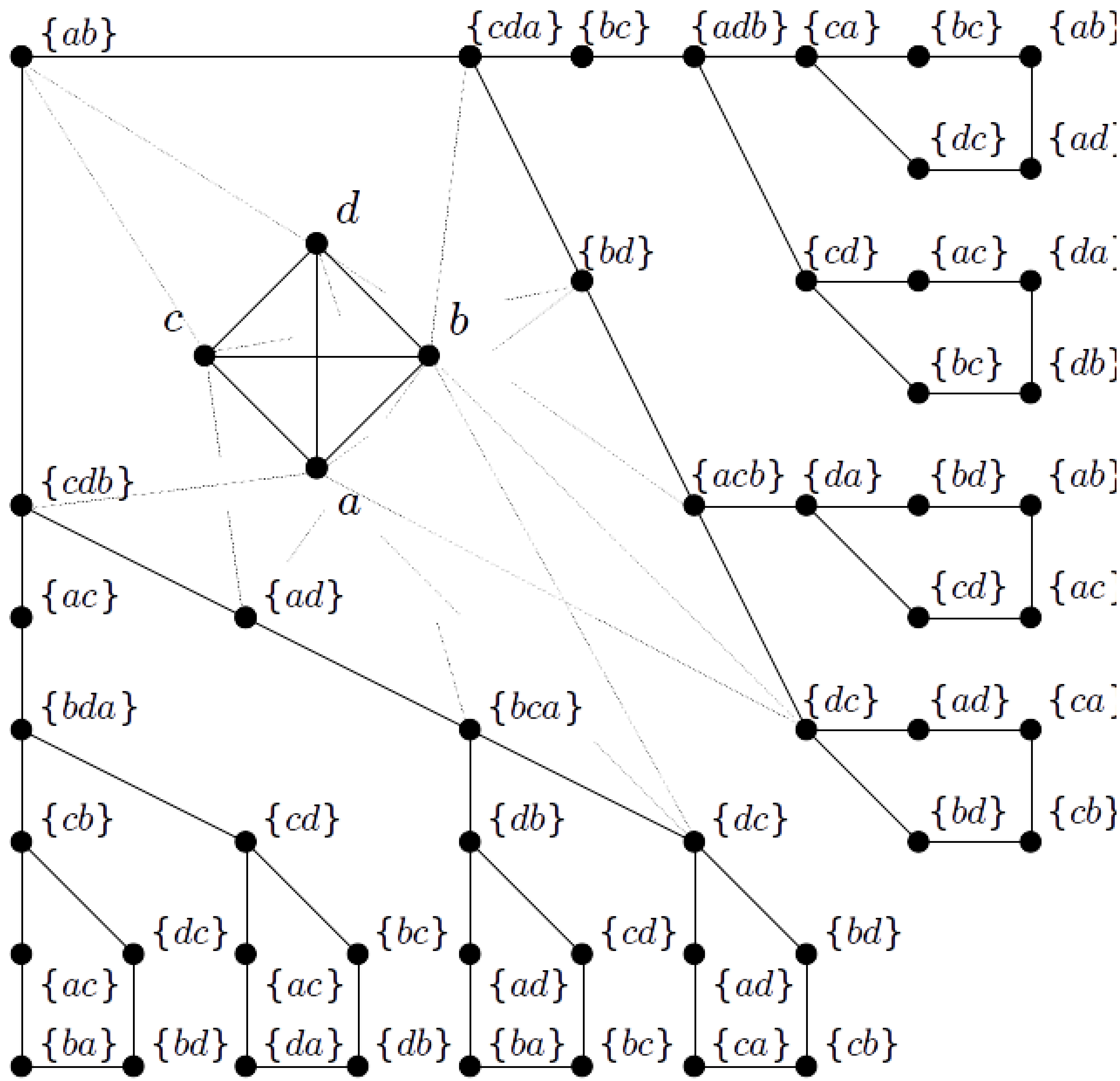
$K_4^{\Omega_{13}(K_8)}$



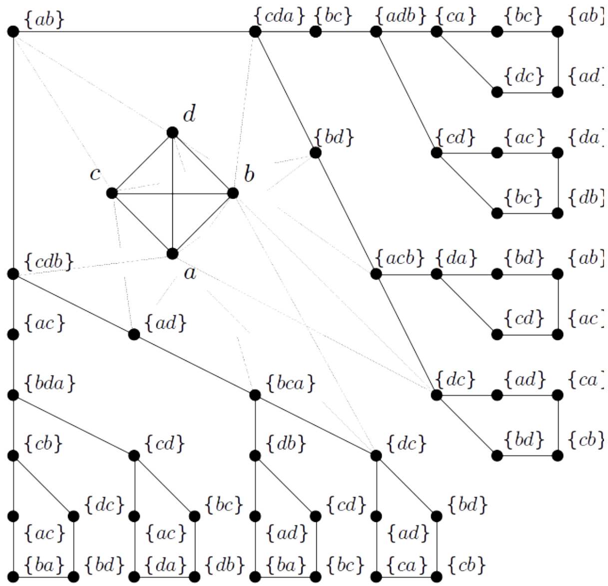
$K_4^{\Omega_{13}(K_8)}$



$$H \subseteq K_4^{\Omega_{13}(K_8)}$$



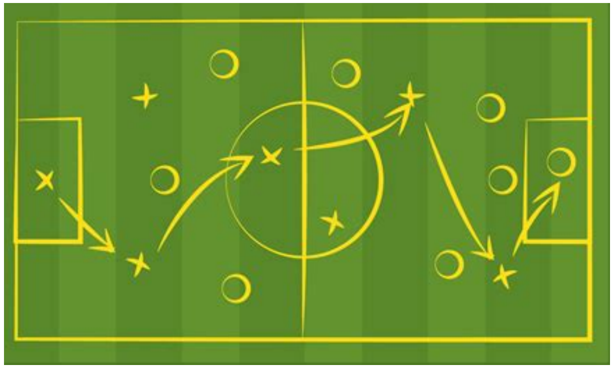
H = the football?



My comments on the proofs

the football

the game



Shitov: $G_{Erdős}[K_q]$

Xhu: $\Omega_5(K_9)[T]$

Me: $G_{Exoo}[K_q]$

Wrochna: $M_{13}(K_8)$

Now: (K_6)

K_n $G_{Exoo}[K_q]$

K_n $G_{Erdős}[K_q]$

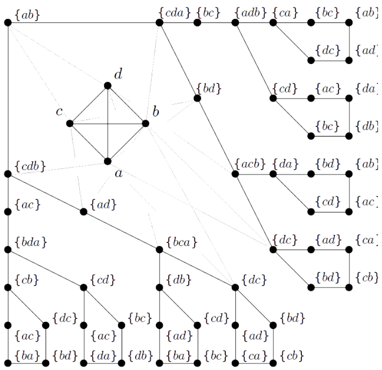
K_n $\Omega_5(K_9)[T]$

K_{13} $M_{13}(K_8)$

K_5

K_4

H $\approx$

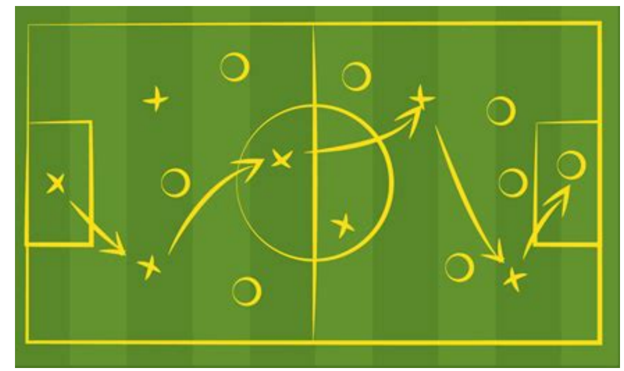


K_n^H using topological bound

My comments on the proofs

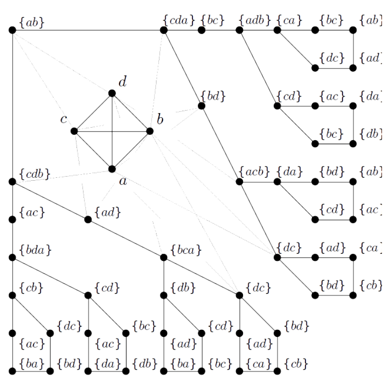
the football

the game



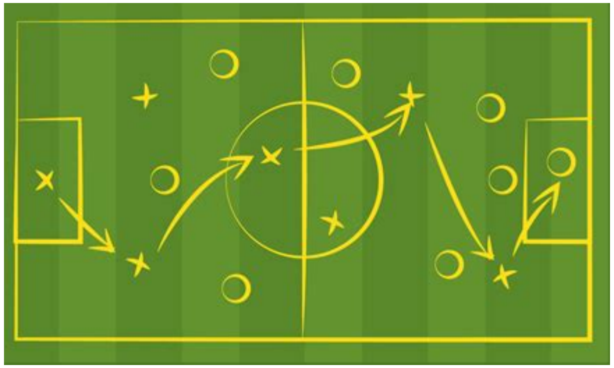
Shitov: $G_{Erdős}[K_q]$
Xhu: $G_{Exoo}[K_q]$
Me: $\Omega_5(K_9)$
Wrochna: $\Omega_7(K_6)$
Now: $M_{13}(K_8)$

$K_n^{G_{Exoo}[K_q]}$
$K_n^{G_{Erdős}[K_q]}$
$K_n^{\Omega_5(K_9)[T]}$
$K_{13}^{\Omega_7(K_6)}$
$K_5^{M_{13}(K_8)}$
K_4

$H \approx$ 

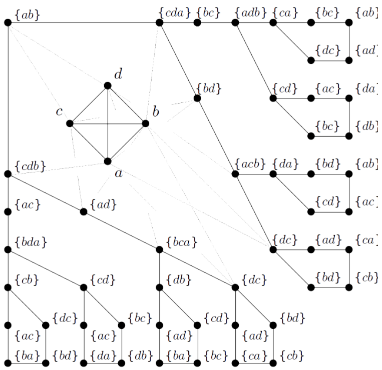
K_n^H using
topological bound
fractional chromatic number

My comments on the proofs

the football	the game
	

Shitov: $G_{Erdős}[K_q]$
Zhu: $\Omega_5(K_9)$
Me: $G_{Exoo}[K_q]$
Wrochna: $\Omega_{13}(K_8)$
Now: (K_7) (K_6)

K_n $G_{Exoo}[K_q]$
K_n $G_{Erdős}[K_q]$
K_n $\Omega_5(K_9)[T]$
K_{13} $\Omega_{13}(K_8)$
K_5
K_4

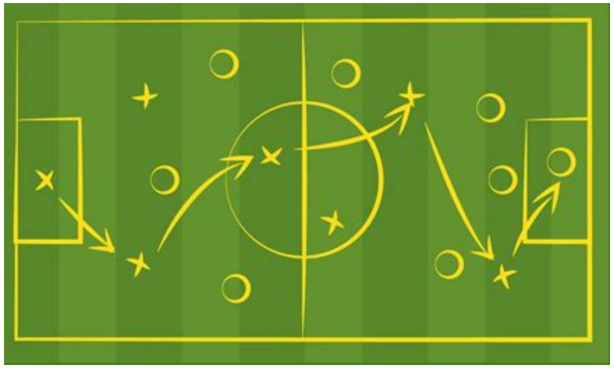
$H \approx$ 

K_n^H using
topological bound
fractional chromatic number
Lovász parameter?

My comments on the proofs

the football

the game



Shitov: $G_{Erdős}[K_q]$

Now: $M_{13}(K_8)$ (K_6)

K_n $G_{Exoo}[K_q]$

K_n $G_{Erdős}[K_q]$

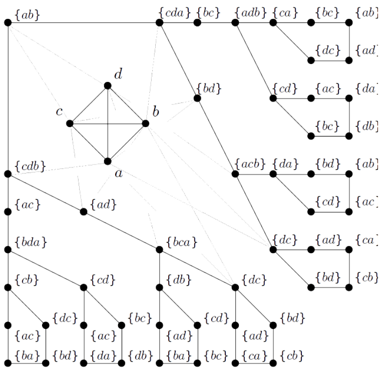
K_n $\Omega_5(K_9)[T]$

K_{13} $M_{13}(K_8)$

K_5

K_4

H $\approx$



$K_n{}^H$ using

topological bound

fractional chromatic number

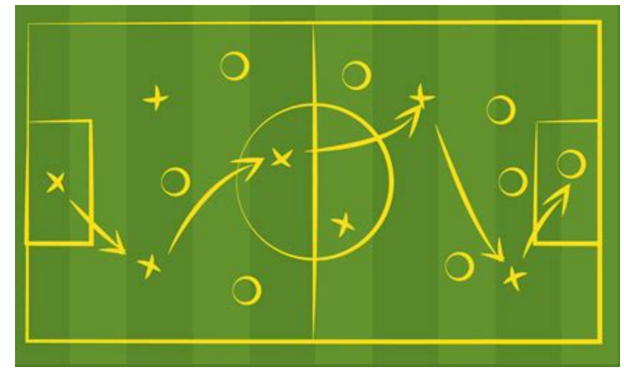
Lovász parameter?

altermatic number?

My comments on the proofs

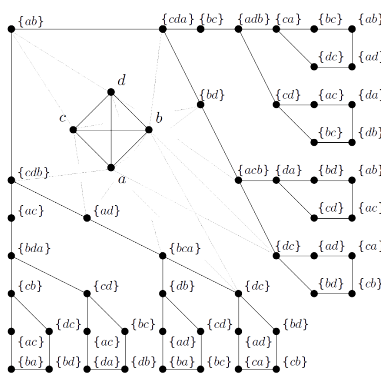
the football

the game



Shitov: $G_{\text{Erdős}}[K_q]$
[illegible] $G_{\text{Exoo}}[K_q]$ [illegible]
[illegible] $\Omega_{13}(K_8)$ [illegible] (K_6)

$K_n^{G_{\text{Exoo}}[K_q]}$
$K_n^{G_{\text{Erdős}}[K_q]}$
$K_n^{\Omega_5(K_9)[T]}$
$K_{13}^{\Omega_{13}(K_8)}$
K_5 [illegible] K_4

H $\approx$ 

K_n^H using
topological bound
fractional chromatic number
Lovász parameter?
altermatic number?
other?

Thank you!

(now I'm really done.)