Beyond Regular Model Checking

Dana Fisman

joint work with

Amir Pnueli

Minerva Center of Verification of Reactive Systems
Department of Computer Science
The Weizmann Institute of Science

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The problem:

Uniform Verification of Parameterized Systems

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Uniform Verification of Parameterized Systems

A Parameterized System consists of a family of systems $\mathcal{S}(N)$ for every natural $N > 0$.

$$\|_{i=1}^{N} P[i] :: \left(\left[\begin{array}{l} \ell_0\text{: loop forever do} \\ \quad \left[\begin{array}{l} \ell_1\text{: Non-Critical} \\ \ell_2\text{: request } x \\ \ell_3\text{: Critical} \\ \ell_4\text{: release } x \end{array} \right] \end{array} \right] \right)$$

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The System $\mathcal{S}(N)$ is obtained by instantiating the parameter.

$$\mathcal{S}(N) \quad \overbrace{\left(\left[\begin{array}{l} \ell_0\text{: loop forever do} \\ \quad \left[\begin{array}{l} \ell_1\text{: Non-Critical} \\ \ell_2\text{: request } x \\ \ell_3\text{: Critical} \\ \ell_4\text{: release } x \end{array} \right] \end{array} \right] \right)}^{P[1]} \| \overbrace{\left(\left[\begin{array}{l} \ell_0\text{: loop forever do} \\ \quad \left[\begin{array}{l} \ell_1\text{: Non-Critical} \\ \ell_2\text{: request } x \\ \ell_3\text{: Critical} \\ \ell_4\text{: release } x \end{array} \right] \end{array} \right] \right)}^{P[2]} \| \quad \cdots \quad \| \overbrace{\left(\left[\begin{array}{l} \ell_0\text{: loop forever do} \\ \quad \left[\begin{array}{l} \ell_1\text{: Non-Critical} \\ \ell_2\text{: request } x \\ \ell_3\text{: Critical} \\ \ell_4\text{: release } x \end{array} \right] \end{array} \right] \right)}^{P[N]}$$

Uniform Verification $\quad \forall N \; \mathcal{S}(N) \models \varphi$

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The Parameterized Verification Problem

- The parameterized verification problem is undecidable [AK86]
- Approaches
  - Explicit Induction
    [SG92][EN96]
  - Network Invariants
    [WL89][KM95][HLR92][HLR97][KP00]
  - Abstractions and Approximations of Network Invariants
    [BCG86][SG89][CGJ95]
  - Abstractions
    [ID96][EN96]
  - Cutoffs
    [EK00][PRZ01]
  - Regular Model Checking
    [KMMPS97][WB98]

    \+ Completely Automatic

    − Limited to Systems which can be modeled by regular languages

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Goal

uniform verification of
non-regular
parameterized systems

Beyond Regular Model Checking

Outline

- Modeling Systems by Languages
- The Peterson Algorithm
- Symbolic Model Checking and Regular Model Checking
- Adequacy of Classes of Languages for SMC
- We show 3 Methods Beyond Regular Model Checking

Modeling Systems by Languages

A *system* is a quadruple $(\Sigma, \chi, \Theta, \rho)$ consisting of the following components:

- Σ is a finite alphabet.
- $\chi \subseteq \Sigma^*$ is a language over Σ, denoting the set of *states*.
- $\Theta \subseteq \chi$ is a language over Σ, denoting the set of *initial states*.
- $\rho \subseteq \chi \times \chi$ is a bi-language over Σ, denoting the *transition relation*.

$$s = a_1 a_2 \ldots a_n$$

$$s' = b_1 b_2 \ldots b_m$$

$$\rho(s, s')$$

$$w = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \begin{bmatrix} a_3 \\ b_3 \end{bmatrix} \cdots \begin{bmatrix} a_n \\ b_n \end{bmatrix}$$

$$w \in \rho$$

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Modeling Systems by Languages

- Finite-State Systems
- Parameterized Systems
- Systems with Unbounded Variables
- Parameterized Systems with Unbounded Variables

Modeling Parameterized Systems by Languages

The Token-String Example

Σ Local state: $\{0,1\}$ i.e. without / with a token

χ Global states: $\{0,1\}^+$ e.g. 0100

Θ Initial states: 10^*

ρ Transitions: $\left(\begin{bmatrix}0\\0\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}\right)^* \begin{bmatrix}1\\0\end{bmatrix} \begin{bmatrix}0\\1\end{bmatrix} \left(\begin{bmatrix}0\\0\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}\right)^*$

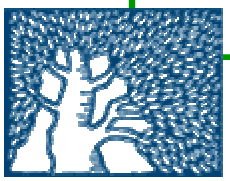
$X\ 1\ 0\ Y \rightarrow X\ 0\ 1\ Y$

Modeling Parameterized Systems with Unbounded Variables by Languages

N : natural initially $N > 1$
y : array $[1..N]$ of $[0..N-1]$ initially $y = 0$
s : array $[1..N-1]$ of $[1..N]$

$$\|_{i=1}^{N} P[i] :: \left[\begin{array}{l} t : \text{integer} \\ \ell_0\text{: loop forever do} \\ \quad \left[\begin{array}{l} \ell_1\text{: Non-Critical} \\ \ell_2\text{: for } t := 1 \text{ to } N-1 \text{ do} \\ \quad \left[\begin{array}{l} \ell_3\text{: } (y[i], s[t]) := (t, i) \\ \ell_4\text{: await } s[t] \neq i \ \vee \ \forall j \neq i : y[j] < y[i] \end{array}\right] \\ \ell_5\text{: Critical} \\ \ell_6\text{: } y[i] := 0 \end{array}\right] \end{array}\right]$$

Peterson Algorithm for Mutual Exclusion among N Processes



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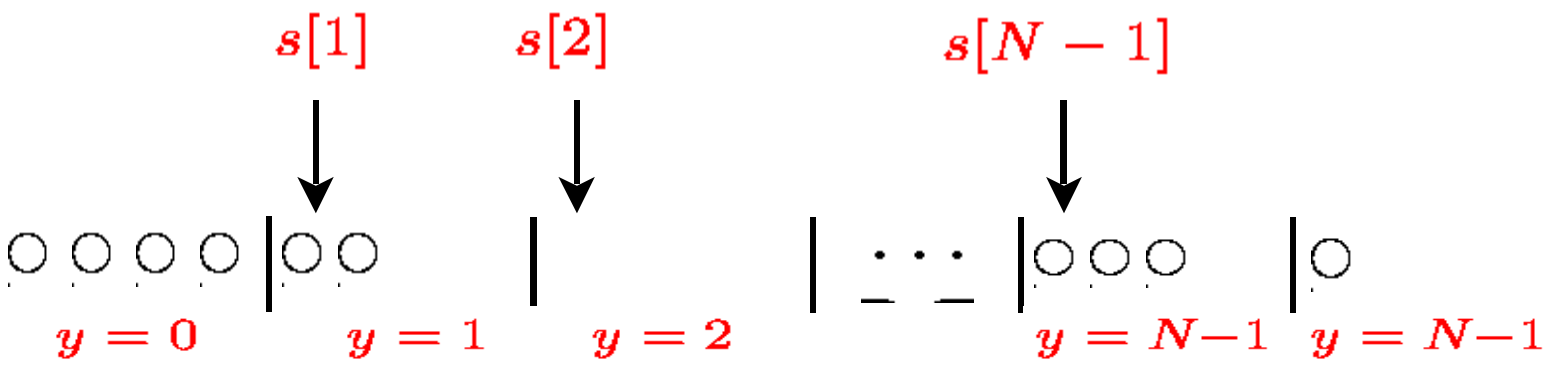
An Encoding for Peterson

We can encode a state of the system as a word of the form

○– processor



Critical

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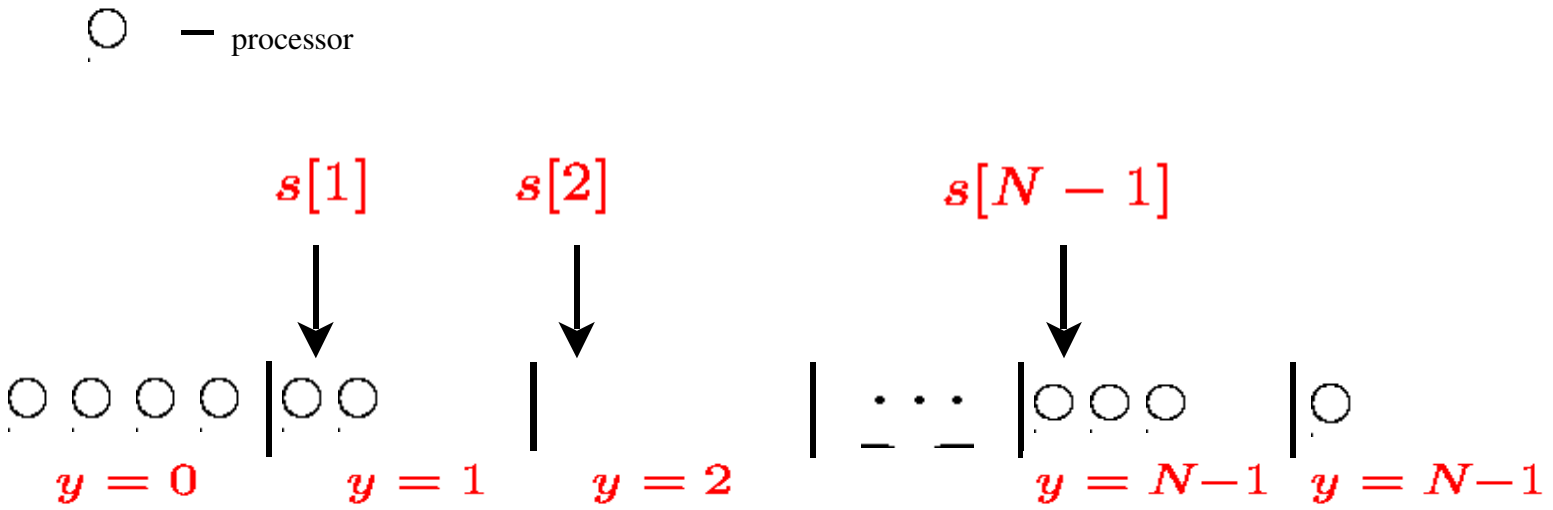

Critical

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An Encoding for Peterson

$$\chi = \left\{ w \in (\bigcirc + |)^* \ : \quad \sharp(|, w) = \sharp(\bigcirc, w) > 1 \right\}$$

$$\Theta = \left\{ \bigcirc^i |^i \ : \quad i > 1 \right\}$$

ρ_1	:	$u \bigcirc \mid v \mapsto u \mid \bigcirc v$	where $u \in \bigcirc^*, v \in (\mid + \bigcirc)^*$
ρ_2	:	$u \mid \bigcirc \mid v \mapsto u \mid \mid \bigcirc v$	where $u \in (\mid + \bigcirc)^*, v \in \mid^*$
ρ_3	:	$u \bigcirc \bigcirc \mid v \mapsto u \bigcirc \mid \bigcirc v$	where $u, v \in (\mid + \bigcirc)^*$
ρ_4	:	$u \bigcirc \mapsto \bigcirc u$	where $u \in (\mid + \bigcirc)^*$
ρ_{id}	:	$u \mapsto u$	where $u \in (\mid + \bigcirc)^*$

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Another Encoding for Peterson

$\Sigma = \{0, 1, 2\}$ Empty / Exactly one / Two or more

$$\chi = \left\{ uavbw \;\middle|\; \begin{array}{l} u, v, w \in \Sigma^*, \quad a, b \in \Sigma, \\ |uavbw| \geq 3 \text{ and } a + b \geq 2 \end{array} \right\}$$

$$\Theta = \left\{ \; 20^i \;\middle|\; i \geq 2 \; \right\}$$

$$\rho_1 : \left\{ \begin{array}{lcl} 2\,0\,v & \mapsto & 2\,1\,v \\ 2\,1\,v & \mapsto & 2\,2\,v \\ 1\,0\,v & \mapsto & 0\,1\,v \\ 1\,1\,v & \mapsto & 0\,2\,v \\ 1\,2\,v & \mapsto & 0\,2\,v \end{array} \right.$$

$\rho_2 : \; u\,1\,0\,v \mapsto u\,0\,1\,v$ where $v \in 0^*$

$\rho_{id} : \; u \mapsto u$

$$\rho_3 : \left\{ \begin{array}{lcl|l} u\,2\,0\,v & \mapsto & u\,1\,1\,v & u\,2\,1\,v \\ u\,2\,1\,v & \mapsto & u\,1\,2\,v & u\,2\,2\,v \\ u\,2\,2\,v & \mapsto & u\,1\,2\,v & \end{array} \right.$$

$$\rho_4 : \left\{ \begin{array}{lcl|l} 0\,u\,2 & \mapsto & 1\,u\,1 & 1\,u\,2 \\ 0\,u\,1 & \mapsto & 1\,u\,0 & \\ 1\,u\,2 & \mapsto & 2\,u\,1 & 2\,u\,2 \\ 1\,u\,1 & \mapsto & 2\,u\,0 & \\ 2\,u\,1 & \mapsto & 2\,u\,0 & \\ 2\,u\,2 & \mapsto & 2\,u\,1 & 2\,u\,2 \end{array} \right.$$

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Symbolic Model checking (SMC) [BCM+92]

Input: a system $\mathcal{S} = (\Sigma, \chi, \Theta, \rho)$
an invariance property φ

Question: Does $\mathcal{S} \models \varphi$?

Symbolic Model checking (SMC) [BCM+92]

Input: a system $\mathcal{S} = (\Sigma, \chi, \Theta, \rho)$
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Question: Does $\mathcal{S} \models \varphi$?

Procedure ***Forward SMC***
let $\psi_0 := \Theta$
for $i = 0, 1, \ldots$ **repeat**
let $\psi_{i+1} := \psi_i \cup (\psi_i \diamond \rho)$
until $\psi_{i+1} = \psi_i$
return $\psi_i \cap \overline{\varphi} = \emptyset$

where $\psi \diamond \rho$ denotes ρ-successor of φ

$$\psi \diamond \rho = \left\{ s' \;\middle|\; \exists s \in \psi \text{ such that } \begin{bmatrix} s \\ s' \end{bmatrix} \in \rho \right\}$$

Procedure ***Backward SMC***
let $\psi_0 := \overline{\varphi}$
for $i = 0, 1, \ldots$ **repeat**
let $\psi_{i+1} := \psi_i \cup (\rho \diamond \psi_i)$
until $\psi_{i+1} = \psi_i$
return $\psi_i \cap \Theta = \emptyset$

where $\rho \diamond \psi$ denotes ρ-predecessor of φ

$$\rho \diamond \psi = \left\{ s \;\middle|\; \exists s' \in \psi \text{ such that } \begin{bmatrix} s \\ s' \end{bmatrix} \in \rho \right\}$$

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Symbolic Model checking (SMC) [BCM+92]

- In symbolic model checking, some symbolic representation $\mathcal{C}$ is used to represent sets of states.
- The system and the property to be verified are represented as expressions in $\mathcal{C}$.
- The model checking procedures operate by manipulating sets of states (instead of individual states) through operations on the expressions of $\mathcal{C}$.
  - Finite-state systems
    - Symbolic representation: OBDDs
    - termination is guaranteed
  - Infinite-state systems -
    - requires symbolic representation
    - termination is not guaranteed.

Regular Model Checking

Application of symbolic model checking, where regular expressions are used as the (symbolic) assertional language for describing sets of states.

Forward exploration:

$$\begin{array}{lcll} \varphi_0 & = & \Theta & : \quad 10^* \\ \varphi_1 & = & \Theta + \Theta \diamond \rho & : \quad \varphi_0 + 010^* \\ \varphi_2 & = & \Theta + \Theta \diamond \rho + (\Theta \diamond \rho) \diamond \rho & : \quad \varphi_1 + 0010^* \\ & & \cdots & \end{array}$$

Computation doesn't converge!

Backward exploration:

$$\begin{array}{lcll} \varphi_0 & = & \overline{\varphi} & : \quad 0^* + (0+1)^*\ 1\ (0+1)^*\ 1\ (0+1)^* \\ \varphi_1 & = & \overline{\varphi} + \rho \diamond \overline{\varphi} & : \quad 0^* + (0+1)^*\ 1\ (0+1)^*\ 1\ (0+1)^* \end{array}$$

Computation terminates in one iteration step.

Adequacy of Classes of Languages for SMC

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Adequacy of Classes of Languages for SMC

Let $\mathcal{M}$, $\mathcal{R}$ and $\mathcal{A}$ be three classes of languages.

Let $A_\varphi, A_{Init} \in \mathcal{A}$, $M_{Init}, M_\varphi \in \mathcal{M}$, $R \in \mathcal{R}$ and $M_0, M_1, M_2, \ldots \in \mathcal{M}$.

Procedure *Forward MC*

$M_0 := M_{Init}$

For $i = 0, 1, \ldots$ **repeat**

$M_{i+1} := ((M_i \times \Sigma^*) \cap R)\Downarrow_2$

until $M_{i+1} = M_i$

return $M_i \cap \overline{A_\varphi} = \emptyset$

Procedure *Backward MC*

$M_0 := \overline{M_\varphi}$

For $i = 0, 1, \ldots$ **repeat**

$M_{i+1} := ((\Sigma^* \times M_i) \cap R)\Downarrow_1$

until $M_{i+1} = M_i$

return $M_i \cap A_{Init} = \emptyset$

ρ-**successor** of φ

$$\varphi \diamond \rho = \left\{ s' \;\middle|\; \exists s \in \psi \text{ such that } \begin{bmatrix} s \\ s' \end{bmatrix} \in \rho \right\}$$

$$\varphi \diamond \rho = ((\psi \times \Sigma^*) \cap \rho)\Downarrow_2$$

ρ-**predecessor** of φ

$$\rho \diamond \varphi = \left\{ s \;\middle|\; \exists s' \in \psi \text{ such that } \begin{bmatrix} s \\ s' \end{bmatrix} \in \rho \right\}$$

$$\rho \diamond \varphi = \rho \diamond \psi = ((\Sigma^* \times \psi) \cap \rho)\Downarrow_1$$

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \begin{bmatrix} a_3 \\ b_3 \end{bmatrix} \cdots \begin{bmatrix} a_n \\ b_n \end{bmatrix}$$

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$$\rho \diamond \varphi = \rho \diamond \psi = ((\Sigma^* \times \psi) \cap \rho)\Downarrow_1$$

$b_1 \quad b_2 \quad b_3 \quad \cdots \quad b_n$

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Adequacy of Classes of Languages for SMC

The classes $\mathcal{M}$, $\mathcal{R}$ and $\mathcal{A}$ are adequate for SMC if:

1. $\mathcal{M}$ and $\mathcal{A}$ are adequate for specifying Θ and φ respectively, and $\mathcal{R}$ is adequate for representing ρ.
2. $\mathcal{A}$ is effectively closed under complementation.
3. $\mathcal{M}$ is effectively closed under lifting.
4. $\mathcal{M}$ is effectively closed under intersection with $\mathcal{R}$.
5. $\mathcal{M}$ is effectively closed under projection.
6. Either $\mathcal{M}$ is effectively closed under intersection with $\mathcal{A}$ and emptiness is decidable for $\mathcal{M}$, or $\mathcal{A}$ is effectively closed under intersection with $\mathcal{M}$ and emptiness is decidable for $\mathcal{A}$.
7. Equivalence of two languages in $\mathcal{M}$ is decidable.

Procedure *Forward MC*

$M_0 := M_{Init}$

For $i = 0, 1, \ldots$ **repeat**

$M_{i+1} := ((M_i \times \Sigma^*) \cap R)\Downarrow_2$

until $M_{i+1} = M_i$

return $M_i \cap \overline{A_\varphi} = \emptyset$

Adequacy of Classes of Languages for SMC - Meeting the Requirements

1. $\mathcal{M}$- Θ, $\mathcal{A}$ - φ and $\mathcal{R}$ - ρ.
2. $\mathcal{A}$ - complementation.
3. $\mathcal{M}$ - lifting.
4. $\mathcal{M}$ - intersection with $\mathcal{R}$.
5. $\mathcal{M}$ - projection.
6. • $\mathcal{M}$ - intersection with $\mathcal{A}$ and emptiness
   • $\mathcal{A}$ - intersection with $\mathcal{M}$ and emptiness
7. $\mathcal{M}$ - Equivalence

Regular Model Checking

- $\mathcal{M}$, $\mathcal{R}$ and $\mathcal{A}$ are taken to be class $\mathcal{L}_{FA}$.

Beyond Regular Model Checking

- $\mathcal{A}$ is the class $\mathcal{L}_{DPDA}$ and $\mathcal{M}$, $\mathcal{R}$ and are taken to be class $\mathcal{L}_{FA}$. ✓
- $\mathcal{A}$ is the class $\mathcal{L}_{PDA}$ and $\mathcal{M}$, $\mathcal{R}$ and are taken to be class $\mathcal{L}_{FA}$. ✓
- $\mathcal{M}$ is the class $\mathcal{L}_{DPDA}$ and $\mathcal{A}$, $\mathcal{R}$ and are taken to be class $\mathcal{L}_{FA}$. ✗ projection

Beyond Regular Model checking

overview of the 3 methods

- Method 1: Computing reachable set by regular means and **then** intersecting with a CFL.
- Method 2: Computing with cascade product $1\mathrm{DPDA} \circ \mathrm{DFA}$.
- Method 3: Computing with product $\mathrm{DPDA} \times \mathrm{DFA}$.

Method 1: Computing reachable set by regular means and then intersecting with a CFL.

Method 1: Computing reachable set by regular means and then intersecting with a CFL.

- Start with a regular language (M_φ / M_{Init}, R)
- Compute the regular fix point
- Intersect with a context-free language ($A_{Init} / \overline{A_\varphi}$)

Procedure *Forward MC*

$M_0 := M_{Init}$

For $i = 0, 1, \ldots$ **repeat**

$M_{i+1} := ((M_i \times \Sigma^*) \cap R)\Downarrow_2$

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Procedure *Backward MC*

$M_0 := \overline{M_\varphi}$

For $i = 0, 1, \ldots$ **repeat**

$M_{i+1} := ((\Sigma^* \times M_i) \cap R)\Downarrow_1$

until $M_{i+1} = M_i$

return $M_i \cap A_{Init} = \emptyset$

Beyond Regular

Method 1: Computing reachable set by regular means and then intersecting with a CFL.

Applying Regular Model Checking

Globals: $\chi = \left\{ uavbw \;\middle|\; \begin{array}{l} u,v,w \in \Sigma^*, \quad a,b \in \Sigma, \\ |uavbw| \geq 3 \text{ and } a+b \geq 2 \end{array} \right\}$

Initials: $\Theta = \left\{ \; 20^i \;\middle|\; i \geq 2 \; \right\}$

Reachables: $\psi = \chi \cap (\Theta \cup L_1 \cup L_2)$ where

$L_1 = \left\{ \; au2v \;\middle|\; a \in \{0,1,2\} \;\; u \in \{1,2\}^* \;,\; v \in 0^* \; \right\}$

$L_2 = \left\{ \; auv1w \;\middle|\; a \in \{0,1,2\} \;\; u \in \{1,2\}^* \;,\; v,w \in 0^* \; \right\}$

Property: $\varphi^c = \left\{ \; x2 \;\middle|\; x \in \{0,1,2\}^* \; \right\}$

Intersection: $\psi \cap \varphi^c = \left\{ \; au2 \;\middle|\; a \in \{0,1,2\} \;\; u \in \{1,2\}^* \; \right\} \neq \emptyset.$

Thus, we were unable to verify the mutual exclusion property for Peterson(N), using regular model checking.

Beyond Regular

Method 1: Computing reachable set by regular means and then intersecting with a CFL.

Applying SMC Using Context-Free Languages

Globals: $\widetilde{\chi} = \{\ w\ :\ \ \sharp(|, w) = \sharp(\bigcirc, w) \geq 2\ \}$

Initials: $\widetilde{\Theta} = \left\{\ \bigcirc^i|^i\ :\ \ i \geq 2\ \right\}$

Reachables: $\widetilde{\psi} = \widetilde{\chi} \cap (\widetilde{\Theta} \cup \widetilde{L_1} \cup \widetilde{L_2})$ where

$$\widetilde{L_1} = \bigcirc^*| \left(\bigcirc\ \bigcirc^*|\right)^* \bigcirc\ \bigcirc\ \bigcirc^* \left(\epsilon + |\,|^*\right)$$

$$\widetilde{L_2} = \bigcirc^*| \left(\bigcirc\ \bigcirc^*|\right)^* |^* \bigcirc \left(\epsilon + |\,|^*\right)$$

Property: $\widetilde{\varphi^c} = (\bigcirc + |)^* \bigcirc\ \bigcirc$

Intersection: $\widetilde{\psi} \cap \widetilde{\varphi^c} = \emptyset$

Thus, using this method we have proved, what we failed to do using regular model checking, that Peterson(N) satisfied mutual exclusion.

Method 2:

Computing with cascade product 1DPDA∘DFA.

1DPDA - single state DPDA with no accepting states

DFA - the DFA looks at the top of the stack of the 1DPDA

Method 2:

Computing with cascade product $\mathrm{lDPDA} \circ \mathrm{DFA}$.

lDPDA

$\sharp(\mid, w) = \sharp(\bigcirc, w)$

$\Delta(0, \bot) = +a \quad \Delta(0, a) = +a \quad \Delta(0, b) = -b$
$\Delta(1, \bot) = +b \quad \Delta(1, a) = -a \quad \Delta(1, b) = +b$

Input Letter Stack symbol Stack operation

Method 2:

Computing with cascade product $\mathrm{lDPDA} \circ \mathrm{DFA}$.

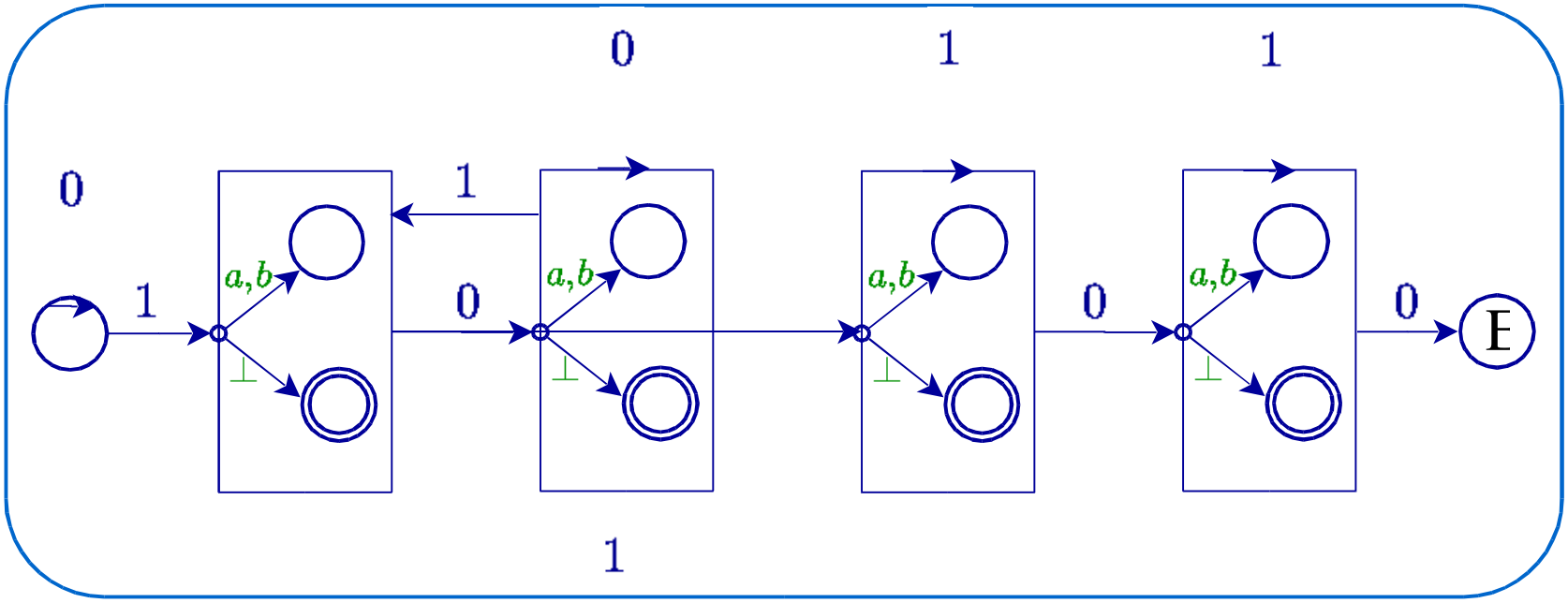
lDPDA

$\sharp(|, w) = \sharp(\bigcirc, w)$

$\Delta(0, \bot) = +a \qquad \Delta(0, a) = +a \qquad \Delta(0, b) = -b$
$\Delta(1, \bot) = +b \qquad \Delta(1, a) = -a \qquad \Delta(1, b) = +b$

DFA $\quad \bigcirc^* \left(| \bigcirc^+ \right)^* \left(|^* + \; ||^+ \bigcirc |^* \right)$

to the right of the first empty partition there is at most one processor



Method 2:

Computing with cascade product $\mathrm{lDPDA} \circ \mathrm{DFA}$.

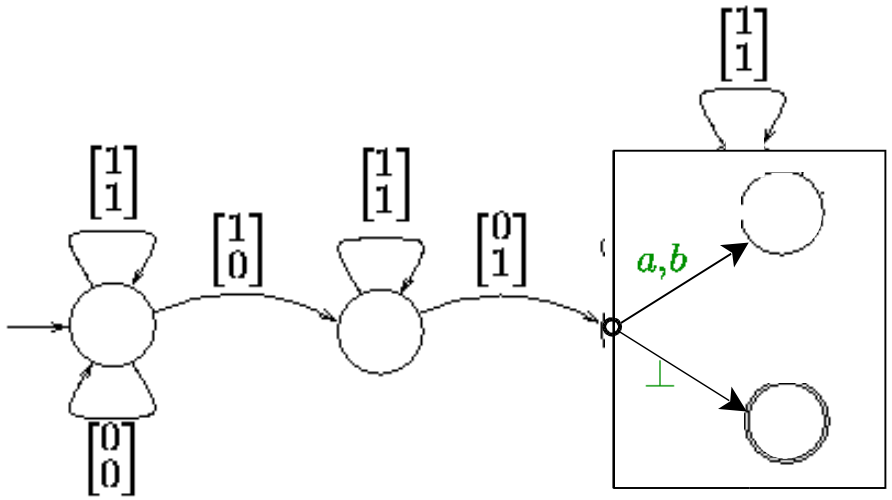
lDPDA

$\sharp(\vert, w) = \sharp(\bigcirc, w)$

$$\Delta(0,\perp) = +a \qquad \Delta(0,a) = +a \qquad \Delta(0,b) = -b$$
$$\Delta(1,\perp) = +b \qquad \Delta(1,a) = -a \qquad \Delta(1,b) = +b$$

DFA $\qquad X\ 1\ Y\ 0\ Z \mapsto X\ 0\ Y\ 1\ Z$

where $X \in \{1,0\}^*$ and $Y, Z \in 1^*$



Beyond Regular

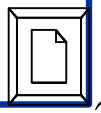
Method 2:

Computing with cascade product $\mathrm{DPDA} \circ \mathrm{DFA}$.

The class $\mathcal{L}_{DPDA-M}$

A_1

A_2



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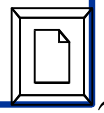

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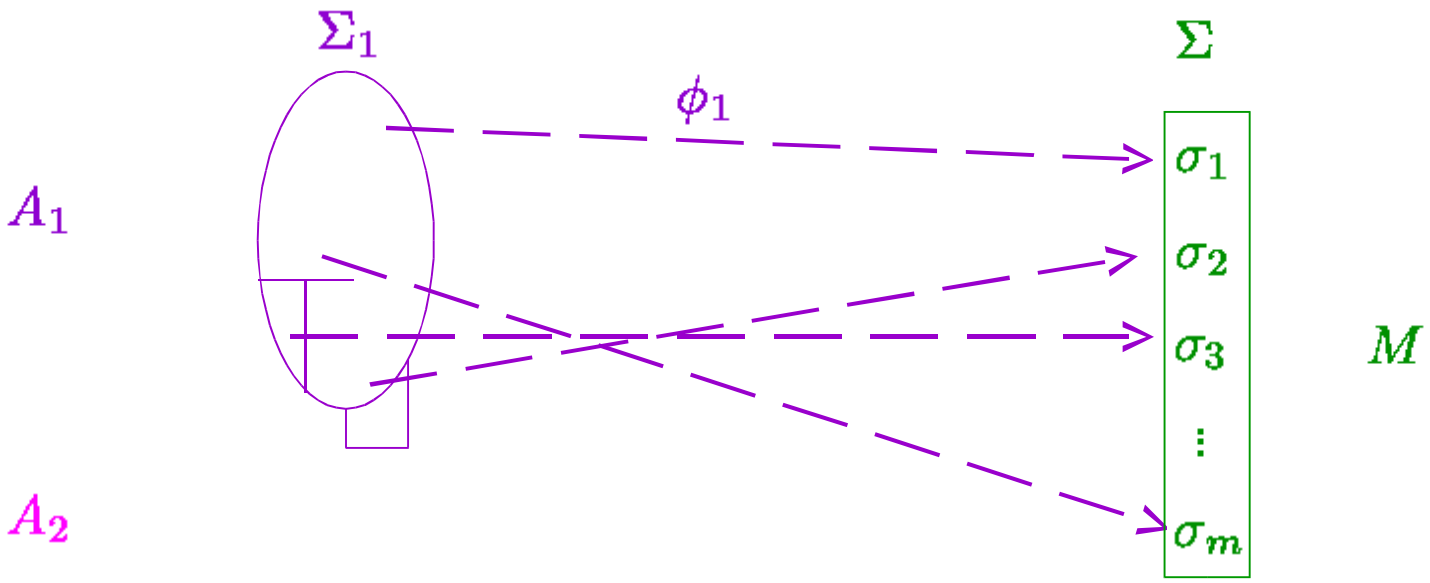
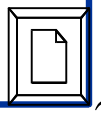
Computing with cascade product $\mathrm{DPDA} \circ \mathrm{DFA}$.

The class $\mathcal{L}_{DPDA-M}$



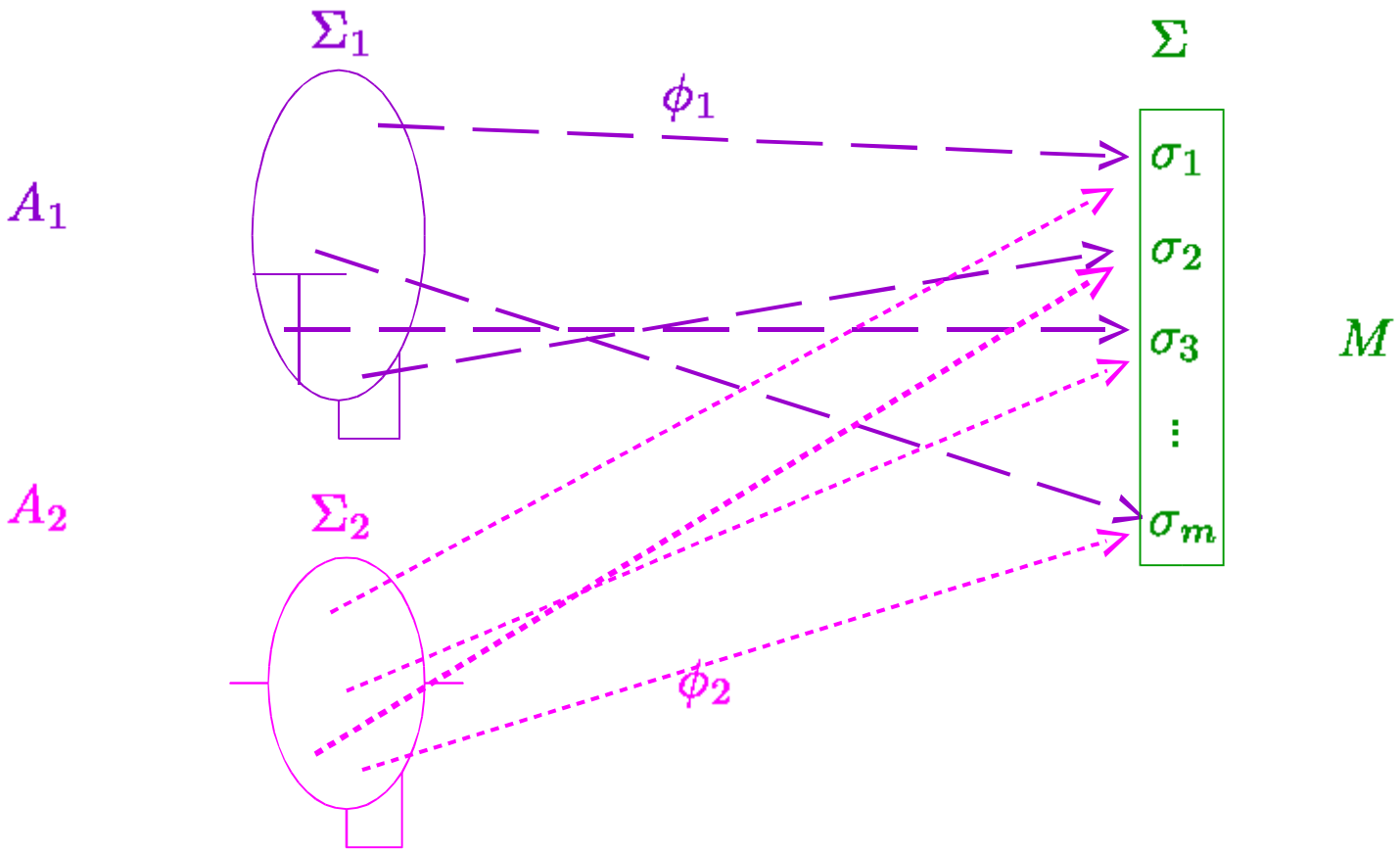


Beyond Regular

Method 2:

Computing with cascade product $\mathrm{DPDA} \circ \mathrm{DFA}$.

The class $\mathcal{L}_{DPDA-M}$



Method 2:

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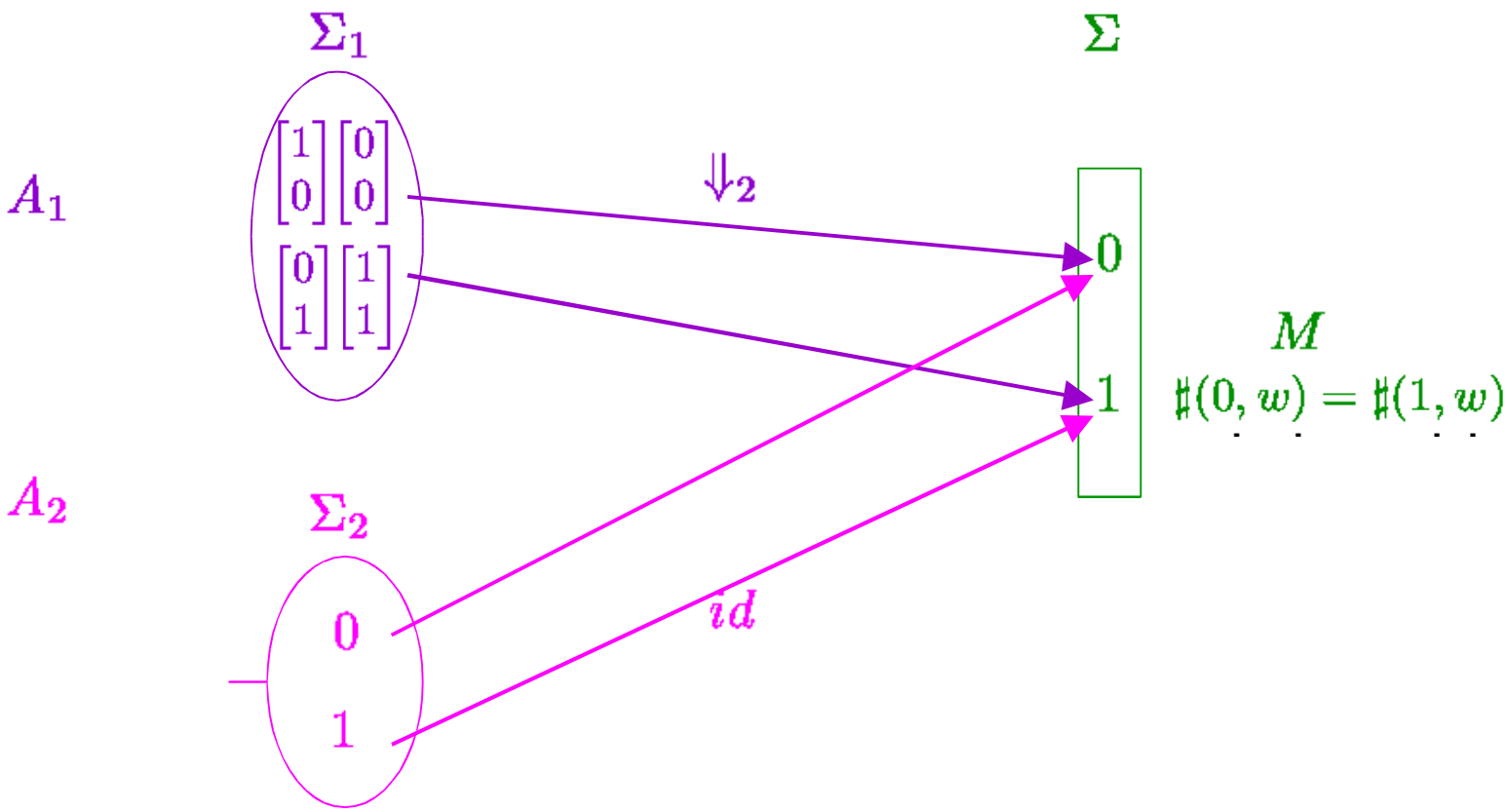
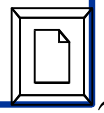
The class $\mathcal{L}_{DPDA-M}$





Beyond Regular

Method 2:

Computing with cascade product $\mathrm{DPDA} \circ \mathrm{DFA}$.

Computing Projection

Input: $A = M \circ_{\Downarrow_2} R$ over $\Sigma \times \Sigma$

Output: $\widetilde{A} = M \circ_{id} \widetilde{R}$ over Σ

Phase 1: *Reachability Analysis*

- Compute for the initial state the language of stack content

Phase 2: *Annotation*

- Guess stack operations when looking at the unprimed coordinate
- Compute stack difference between primed and unprimed
- Compute stack language of prime for all states

Phase 3: *Projection*

- Project on the unprimed coordinate

Phase 4: *Determinization*

- Compute subset construction

Beyond Regular

Method 2: Computing with cascade product $\mathrm{DPDA} \circ \mathrm{DFA}$

The class $\mathcal{L}_{DPDA-M}$

a DPDA A is in the class $\mathcal{L}_{DPDA-M}$ if $A = M \circ_\phi R$ for some DFA R and some substitution ϕ

Method 2: Computing with cascade product $\mathrm{DPDA} \circ \mathrm{DFA}$

The class $\mathcal{L}_{DPDA-M}$

Claim

- The class $\mathcal{L}_{DPDA-M}$ is closed under complementation.
- The class $\mathcal{L}_{DPDA-M}$ is closed under lifting.
- The class $\mathcal{L}_{DPDA-M}$ is closed under intersection with a regular language.
- The Emptiness question is decidable for the class $\mathcal{L}_{DPDA-M}$.
- The Equivalence question is decidable for the class $\mathcal{L}_{DPDA-M}$.

Theorem

There exists a procedure in RE for calculating the projection of languages in $\mathcal{L}_{DPDA-M}$.

Corollary

The class $\mathcal{L}_{DPDA-M}$ is adequate for symbolic model checking (in collaboration with the $\mathcal{L}_{FA}$ class).

Method 3:

Computing with product DPDA × DFA

Method 3:
Computing with product DPDA×DFA

- Start with a product of a DPDA L and a DFA R_0
- Check that L is preserved under R $\qquad L = ((L \times \Sigma^*) \cap R)\Downarrow_2$
- Compute projection only on the DFA part.

Procedure *Forward MC*

$M_0 := L \cap R_0$

For $i = 0, 1, \ldots$ **repeat**

$M_{i+1} := L \cap ((R_i \times \Sigma^*) \cap R)\Downarrow_2$

until $M_{i+1} = M_i$

return $M_i \cap \overline{A_\varphi} = \emptyset$

Procedure *Backward MC*

$M_0 := L \cap R_0$

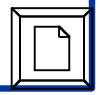
For $i = 0, 1, \ldots$ **repeat**

$M_{i+1} := ((\Sigma^* \times R_i) \cap R)\Downarrow_1 \cap L$

until $M_{i+1} = M_i$

return $M_i \cap A_{Init} = \emptyset$

Beyond Regular



Method 3:
Computing with product DPDA × DFA

Any acceleration technique developed for regular model checking can be applied here as well.

This class is therefore *practically* adequate for symbolic model checking.

Method 3: Computing with product DPDA × DFA

Peterson Example

$$\chi = \left\{ w \in (\bigcirc + |)^* \ : \ \sharp(|, w) = \sharp(\bigcirc, w) > 1 \right\}$$

$$\Theta = \left\{ \bigcirc^i |^i \ : \ i > 1 \right\}$$

ρ_1	:	$u \bigcirc \, \vert v \mapsto u \vert \bigcirc v$	where $u \in \bigcirc^*, v \in (\vert + \bigcirc)^*$
ρ_2	:	$u \vert \bigcirc \, \vert v \mapsto u \vert \, \vert \bigcirc v$	where $u \in (\vert + \bigcirc)^*, v \in \vert^*$
ρ_3	:	$u \bigcirc \bigcirc \, \vert v \mapsto u \bigcirc \vert \bigcirc v$	where $u, v \in (\vert + \bigcirc)^*$
ρ_4	:	$u \bigcirc \mapsto \bigcirc u$	where $u \in (\vert + \bigcirc)^*$
ρ_{id}	:	$u \mapsto u$	where $u \in (\vert + \bigcirc)^*$

Θ is right preserved by ρ thus it is enough to project the DFA part.

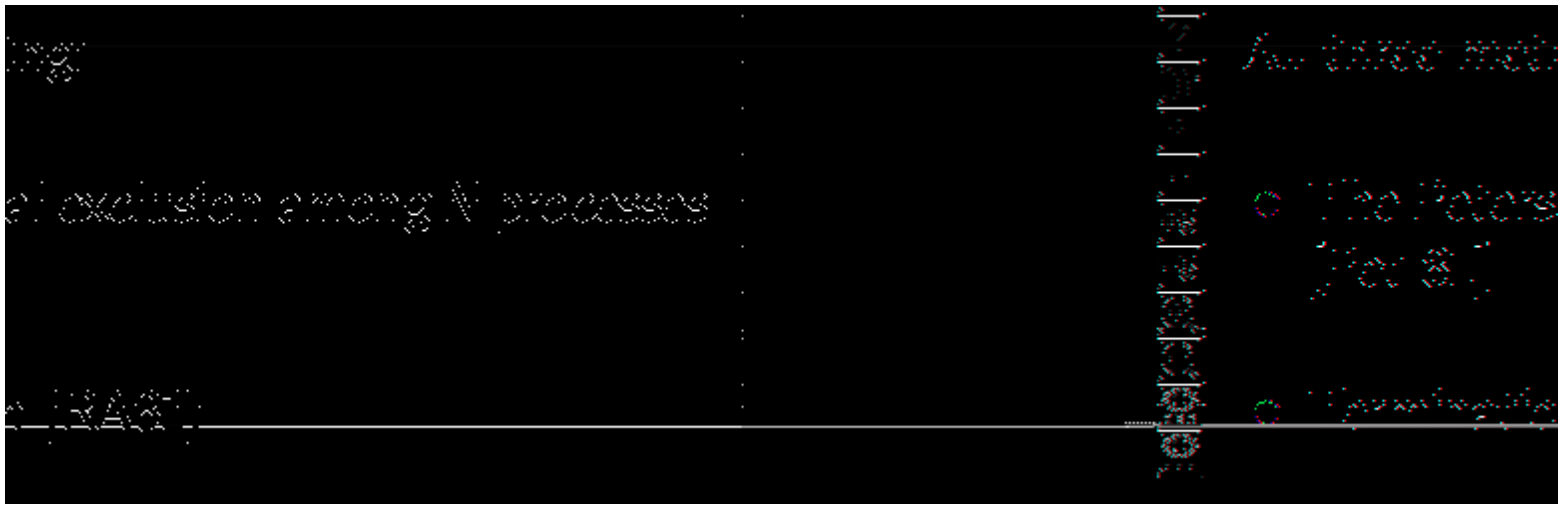
Beyond Regular

Summary

We aimed to find automatic methods for the uniform verification of non-regular parameterized systems.

- Method 1: Computing reachable set by regular means and **then** intersecting with a CFL.
- Method 2: Computing with cascade product $1\mathrm{DPDA} \circ \mathrm{DFA}$.
- Method 3: Computing with product $\mathrm{DPDA} \times \mathrm{DFA}$.

Summary



Related Work

- PushDown Systems

  [BS9596] [WB9292] [BG96] [BEM97] [JES00] [BM96] [BGWW97] [Wal97] [FWW97]

- Systems with Counters

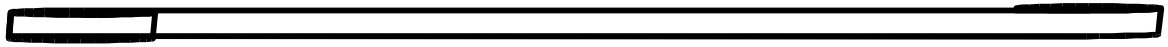
  [Boigelot and Wolper 94][Comon and Jurski 98]

- Combining restricted finite automata and linear constrains on the number of occurrences of symbols

  [Bouajjani and Halbermehl 97]

Future Work

- Verification of the correctness of encoding.
- An equivalent assertional language.
- Use of the $\mathcal{L}_{DPDA-M}$ class in deductive verification.
- Find other classes, stronger than regular languages, that are adequate for SMC.



Beyond Regular