Deterministic Transducers over Infinite Terms

Christof Löding
LIAFA (formerly RWTH Aachen)
loeding@liafa.jussieu.fr

joint work with
Thomas Colcombet, Warsaw University
(formerly IRISA, Rennes)
thomas.colcombet@laposte.net

OUTLINE

(1) Basic definitions and terminology

(2) Overview and background

(3) Deterministic top-down tree transducers with rational lookahead

(4) MSO transductions

(5) Main result: comparison of deterministic transducers and MSO transductions

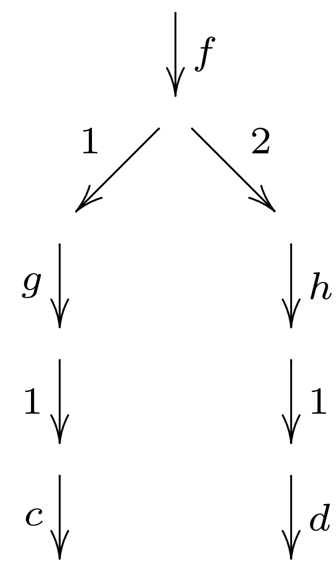
TERMS

- ranked alphabet $\mathcal{F}$ (symbols with arity)
- $|f|$ denotes rank of $f \in \mathcal{F}$
- $|\mathcal{F}|_{\max} = \max\{|f| \mid f \in \mathcal{F}\}$

Terms (possibly infinite) represented as finite edge-labeled trees over the alphabet $\Sigma_{\mathcal{F}} = \mathcal{F} \cup \{1, \ldots, |\mathcal{F}|_{\max}\}$:

Example

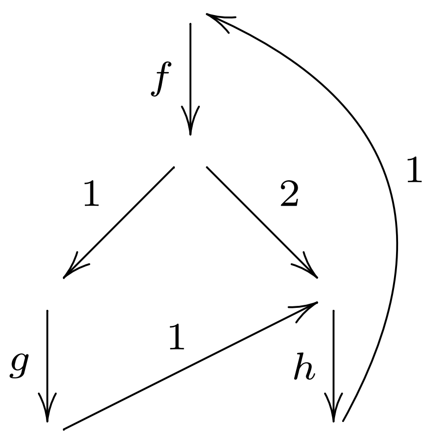
$f(g(c), h(d))$ represented as



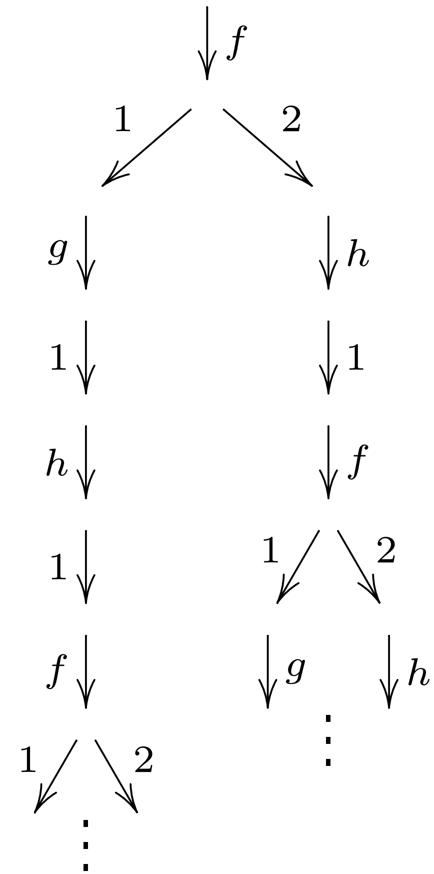
FOLDED TERMS

- rooted graph G (edge labels from $\Sigma_{\mathcal{F}}$)
- unfolding of G from the root denoted by $\mathrm{unfold}(G)$
- G is a folded term if $\mathrm{unfold}(G)$ is a term

Example:



unfolds to



MSO Logic – Rational Sets of Terms

MSO logic over folded terms:

- Signature $(E_a)_{a\in\Sigma_{\mathcal{F}}}$, binary symbols interpreted as the edge relations for each symbol in $\Sigma_{\mathcal{F}}$.
- Quantification over individual vertices.
- Quantification over sets of vertices.

$$\begin{aligned}\phi(x) = \quad & \forall X[x \in X \wedge \forall y, z(y \in X \wedge E(y,z) \rightarrow z \in X) \\ & \rightarrow \exists z', z'' \in X(E_c(z', z''))]\end{aligned}$$

A set of terms is rational

- if it is definable in MSO logic or equivalently
- if it is the set of terms accepted by a Rabin or parity tree automaton or equivalently
- if it is definable in the modal μ-calculus.

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BACKGROUND

- (infinite) terms describe (infinite) objects, e.g., graphs or formal languages

Example – Terms Representing Graphs

Representation of vertex-colored graphs

$\mathcal{F} = \{\oplus, \eta_{i,j}, \rho_{i\to j}, \underline{1}, \ldots, \underline{k}, \perp\}$

$\oplus$	disjoint union
$\eta_{i,j}$	add edges between i-vertices and j-vertices
$\rho_{i\to j}$	make i-vertices to j-vertices
$\underline{i}$	single i-vertex
$\perp$	empty graph

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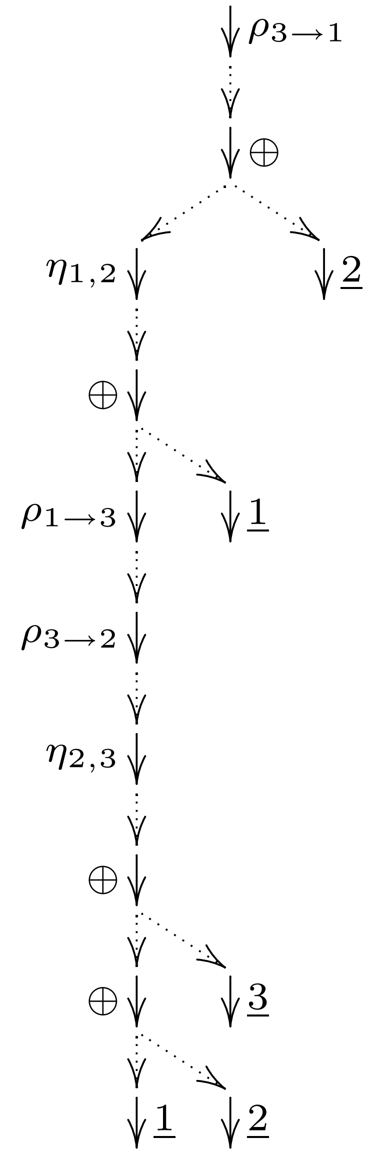

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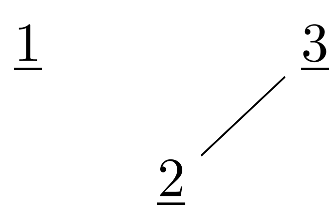

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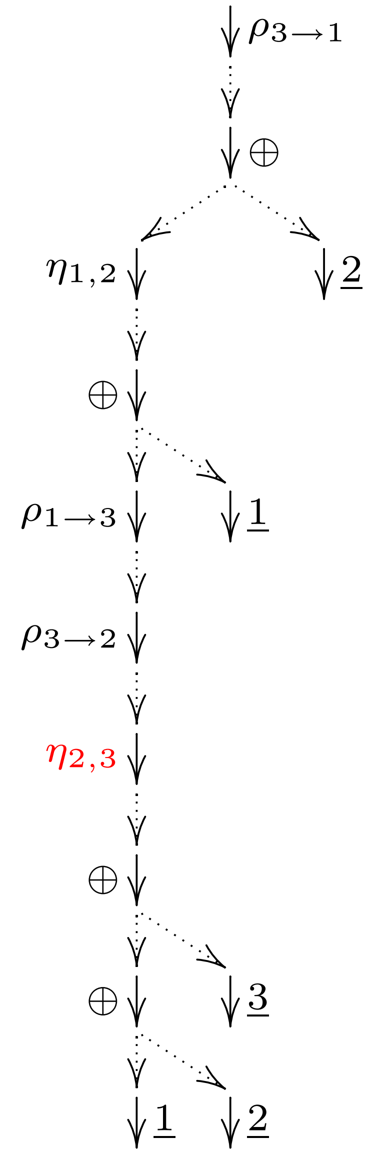

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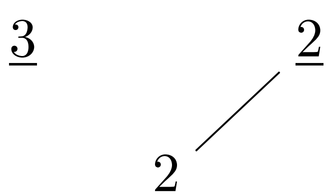

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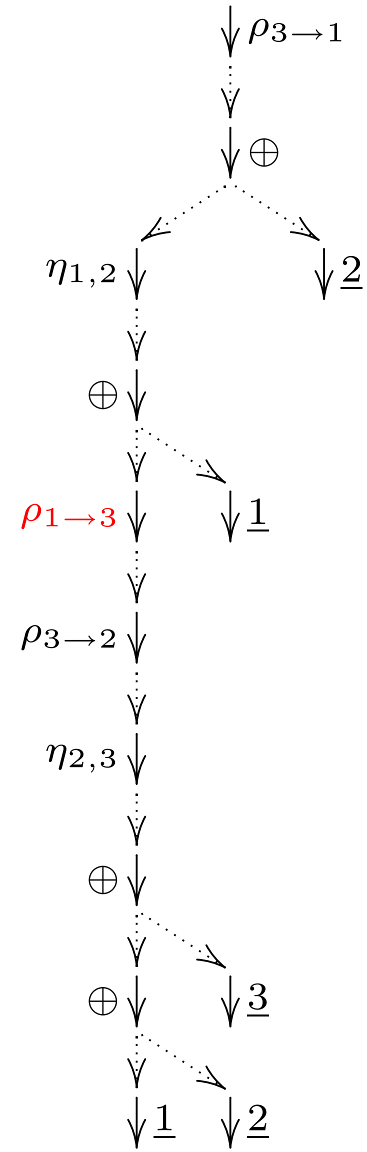

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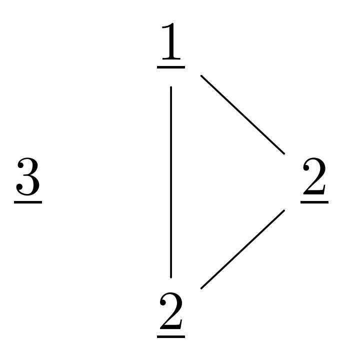

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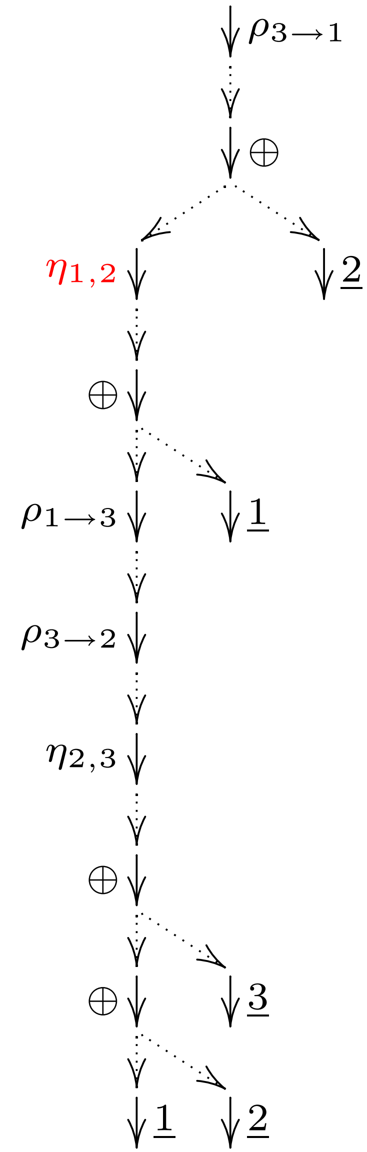

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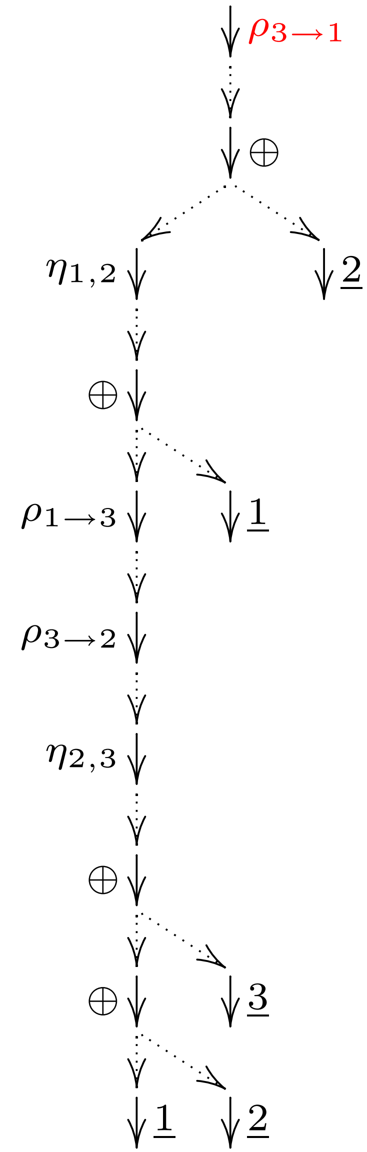

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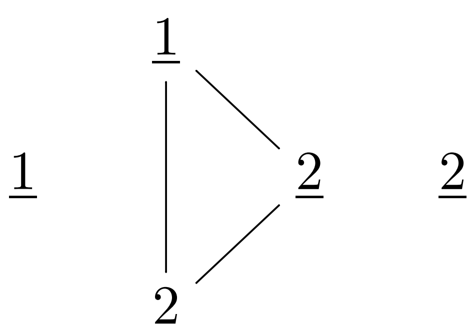
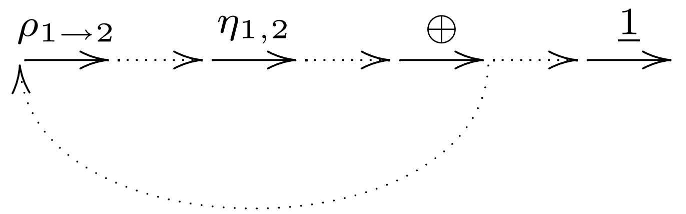


Background

- (infinite) terms describe (infinite) objects, e.g., graphs or formal languages
- another way of describing objects is via equational systems
- equational systems can be represented by folded terms

$$G = \rho_{1\to 2}(\eta_{1,2}(\underline{1} \oplus G))$$



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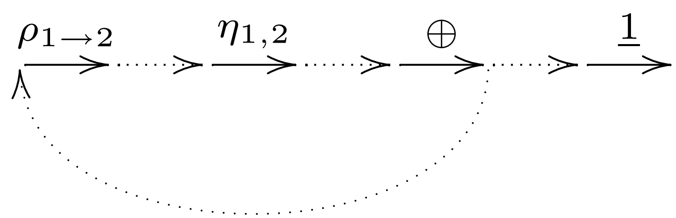

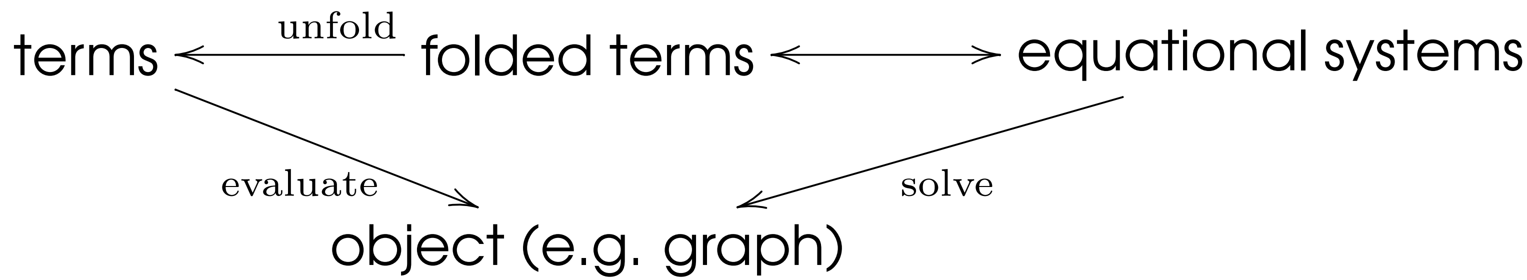

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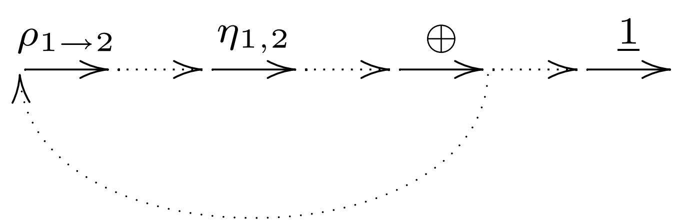

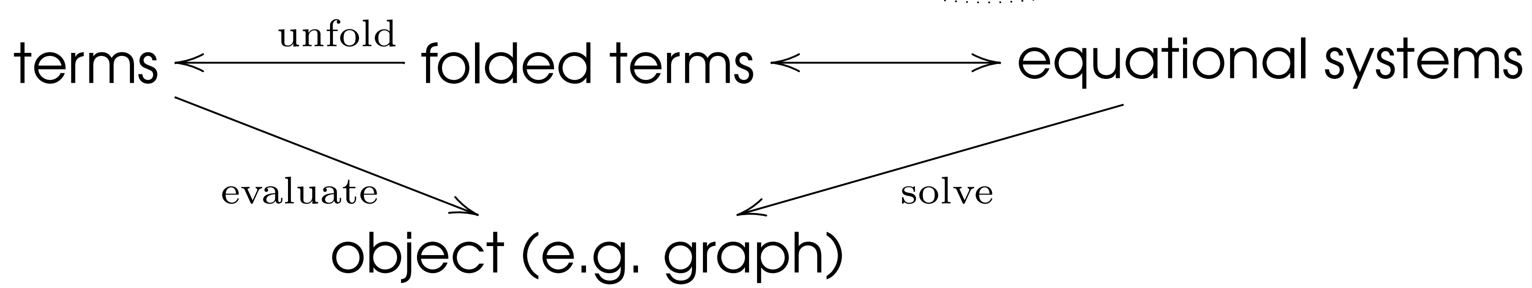

- develop tools to deal with equational systems

OVERVIEW

Objective: apply transformations to the represented objects

Approach: transform the representation

for more details see thesis of Thomas Colcombet

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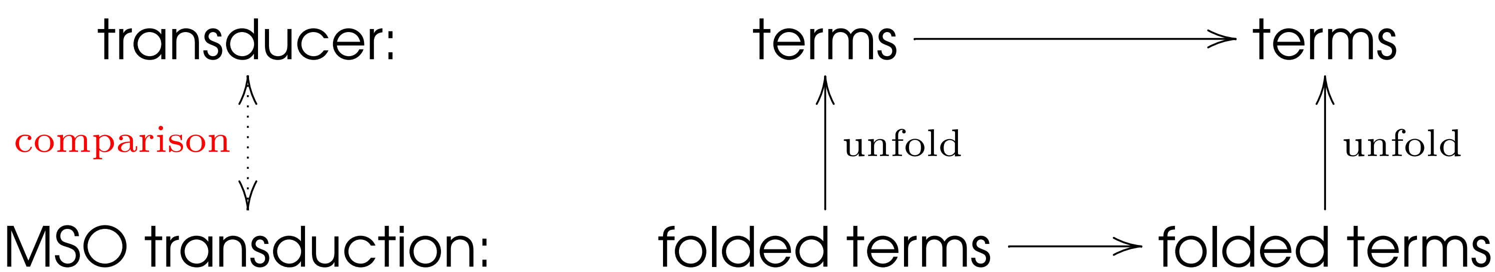

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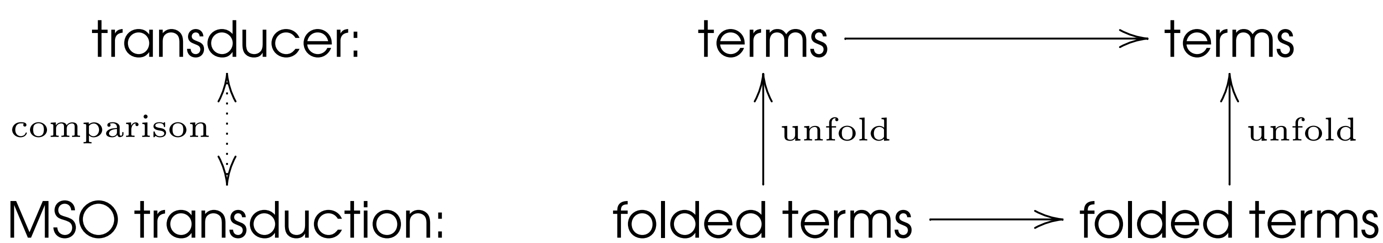

(3) Deterministic top-down tree transducers with rational lookahead

(4) MSO transductions

(5) Main result: comparison of deterministic transducers and MSO transductions

$T = (Q, \mathcal{F}, \mathcal{F}', q_0, \Delta)$ with:

- $\mathcal{F}$, $\mathcal{F}'$ ranked alphabets (input and output alphabet)
- Q a finite set of states
- $q_0 \in Q$ the initial state
- Δ a finite set of rules of one of the following forms:

(production rule): $q(x) \to g(q_1(x), ..., q_{|g|}(x))$
$g \in \mathcal{F}'$, x a variable, and $q_1, \ldots, q_{|g|} \in Q$

(consumption rule): $q(f(x_1, ..., x_{|f|})) \to q'(x_i)$
$f \in \mathcal{F}$, $q, q' \in Q$, and $x_1, \ldots, x_{|f|}$ variables

(lookahead rule): $q(x \in L) \to q'(x)$
L a rational set of $\mathcal{F}$-terms (called lookahead set), $q, q' \in Q$, and x a variable

TOP-DOWN TREE TRANSDUCERS WITH RATIONAL LOOKAHEAD

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Semantics: Start with $q_0(t)$ and ‘apply rewriting rules to infinity’

Determinism: for any q, t no two rules apply to $q(t)$

Example

$\mathcal{F} = \mathcal{F}' = \{\oplus, \eta_{i,j}, \rho_{i\to j}, \underline{1}, \ldots, \underline{k}, \bot\}$

Goal: Remove isolated vertices from $val(t)$

For a set of colors C let f_C be the mapping that removes all vertices from G that are isolated and not of color C. We are interested in $f_\emptyset$.

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Invariants:

$f_C(\perp) = \perp$. $f_C(\underline{i}) = \underline{i}$ if $i \in C$ and $f_C(\underline{i}) = \perp$, otherwise.

$f_C(G \oplus G') = f_C(G) \oplus f_C(G')$

$f_C(\eta_{i,j}(G)) = f_{C'}(G)$ with $C' = \begin{cases} C \cup \{i,j\} \text{ if } G \text{ contains } i\text{- and } j\text{-vertices} \\ C \text{ otherwise} \end{cases}$

$f_C(\rho_{i\to j}(G)) = f_{C'}(G)$ with $C' = \begin{cases} C \cup \{i\} \text{ if } j \in C \\ C \setminus \{i\} \text{ if } j \notin C \end{cases}$

Implementation: Transducer keeps track of the set C using the invariants.

EXAMPLE

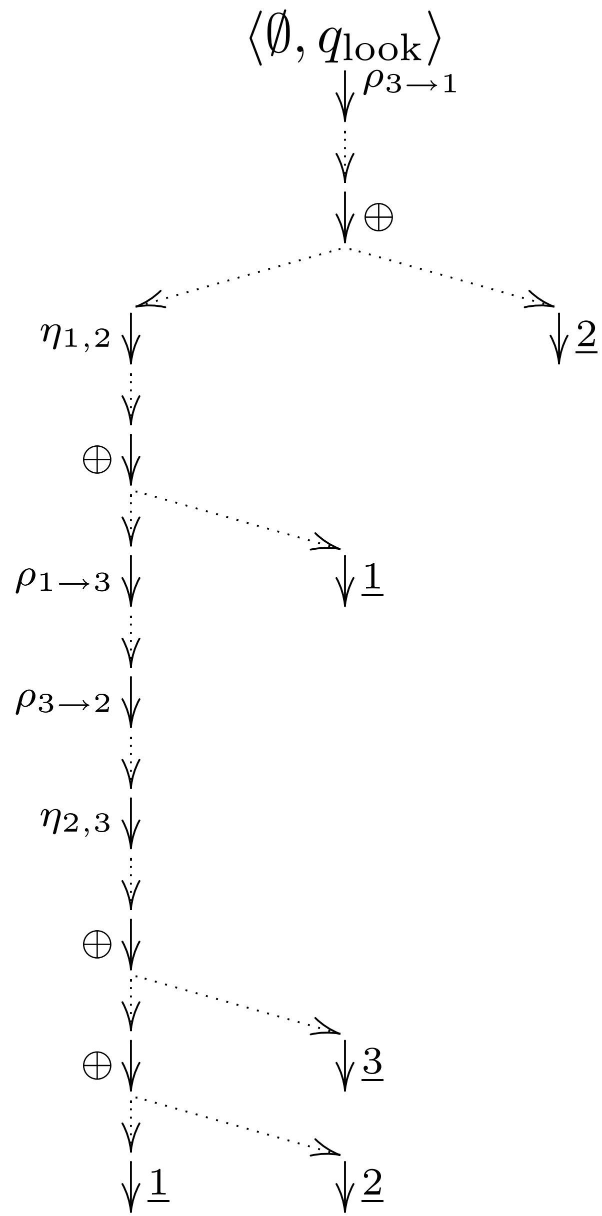
Lookahead sets:

$$
\begin{array}{lllllll}
L_{\underline{i}} & = & \{\underline{i}\} & L_{\perp} & = & \{\perp\} \\
L_{\oplus} & = & \{t \mid t = \oplus(t_1, t_2)\} & L_{\rho_{i\to j}} & = & \{t \mid t = \rho_{i\to j}(t_1)\} \\
L_{\eta_{i,j}} & = & \multicolumn{4}{l}{\{t \mid t = \eta_{i,j}(t_1) \text{ and } val(t_1) \text{ contains } i\text{- and } j\text{-vertices}\}} \\
\overline{L_{\eta_{i,j}}} & = & \multicolumn{4}{l}{\{t \mid t = \eta_{i,j}(t_1) \text{ and } val(t_1) \text{ does not contain } i\text{- and } j\text{-vertices}\}}
\end{array}
$$

Some of the rewriting rules:

- $\langle C, q_{\text{look}}\rangle\,(x \in L_{\underline{i}}) \to \langle C, q_{\underline{i}}\rangle\,(x)$, $\quad \langle C, q_{\underline{i}}\rangle\,(x \in L_{\underline{i}}) \to \begin{cases} \underline{i} & \text{if } i \in C \\ \perp & \text{otherwise} \end{cases}$
- $\langle C, q_{\text{look}}\rangle\,(x \in \overline{L_{\eta_{i,j}}}) \to \langle C, q_{\text{cons}}\rangle\,(x)$
- $\langle C, q_{\text{look}}\rangle\,(x \in L_{\eta_{i,j}}) \to \langle C \cup \{i, j\}, q_{\eta_{i,j}}\rangle\,(x)$
- $\langle C, q_{\text{look}}\rangle\,(x \in L_{\oplus}) \to \langle C, q_{\oplus}\rangle\,(x)$, $\langle C, q_{\oplus}\rangle\,(x) \to \oplus(\langle C, q_{\oplus,1}\rangle\,(x), \langle C, q_{\oplus,2}\rangle\,(x))$
- $\langle C, q_{\text{look}}\rangle\,(x \in L_{\rho_{i\to j}}) \to \langle C' \cup \{i\}, q_{i\to j}\rangle\,(x)$ with $C' = \begin{cases} C \cup \{i\} \text{ if } j \in C \\ C \setminus \{i\} \text{ if } j \notin C \end{cases}$

Sample Application



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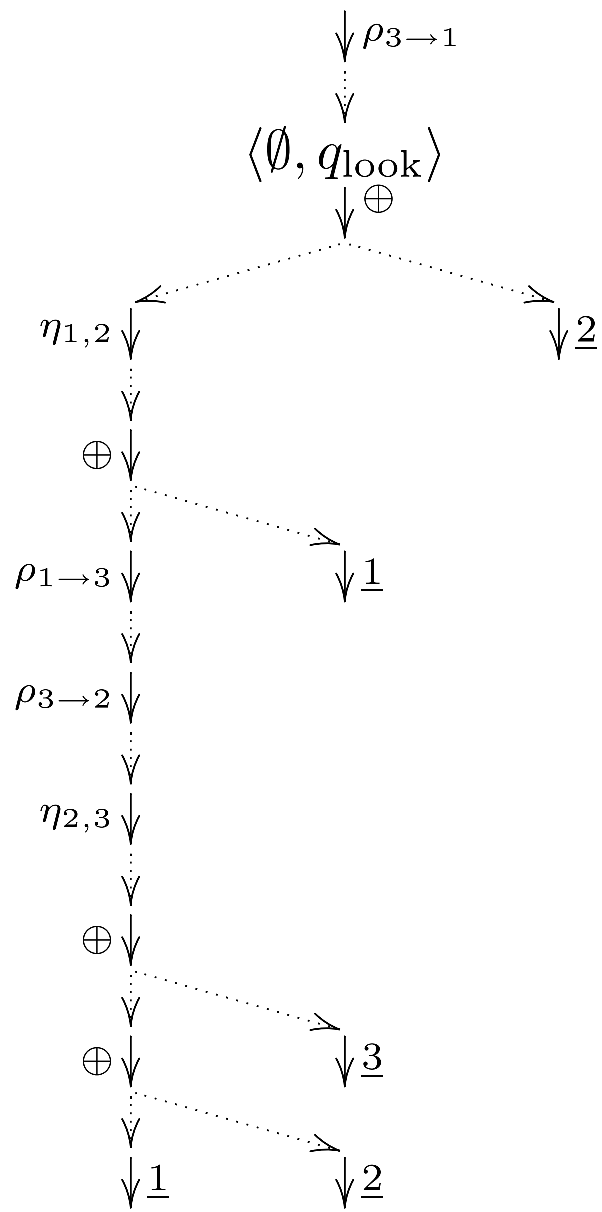

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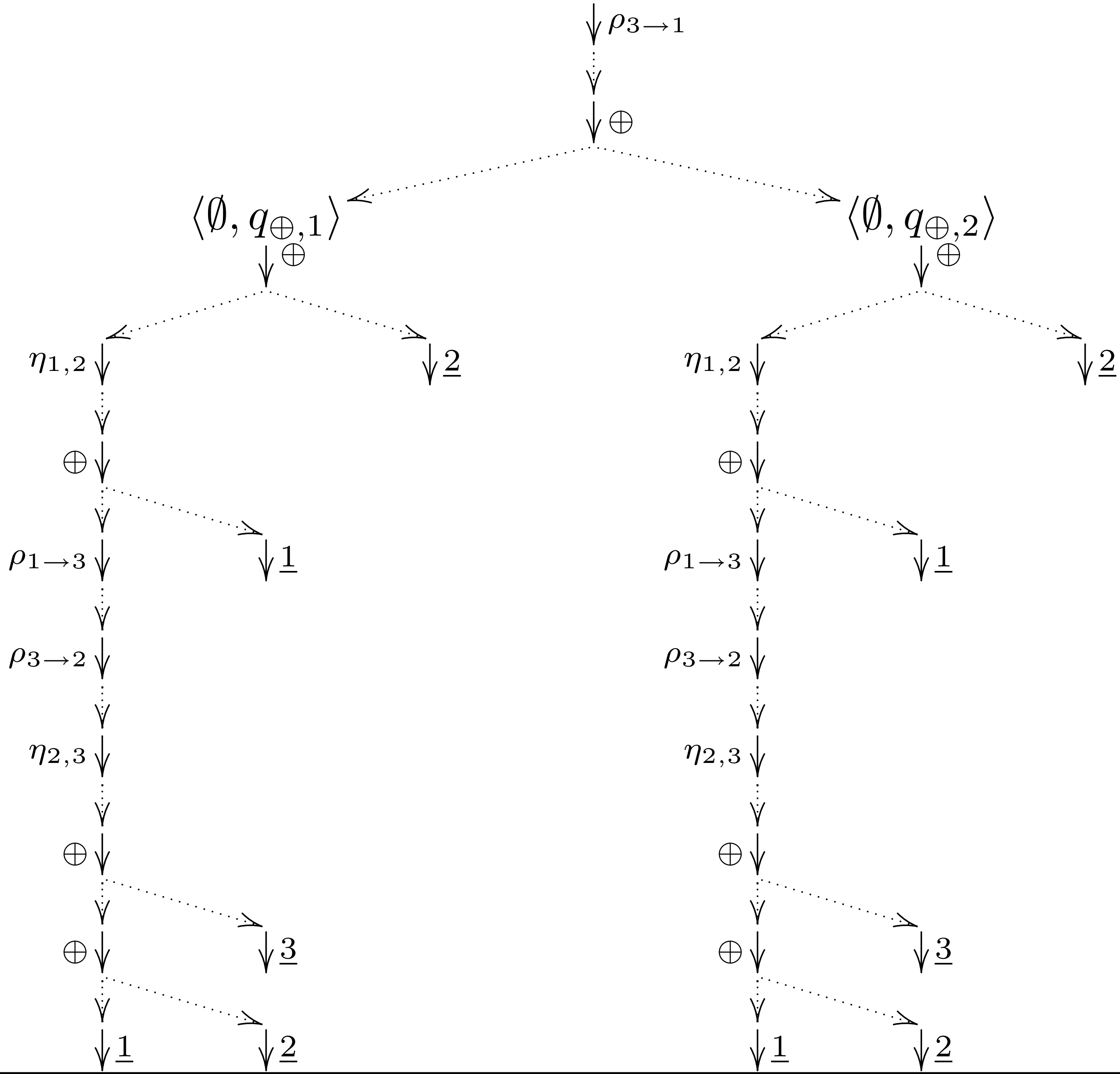

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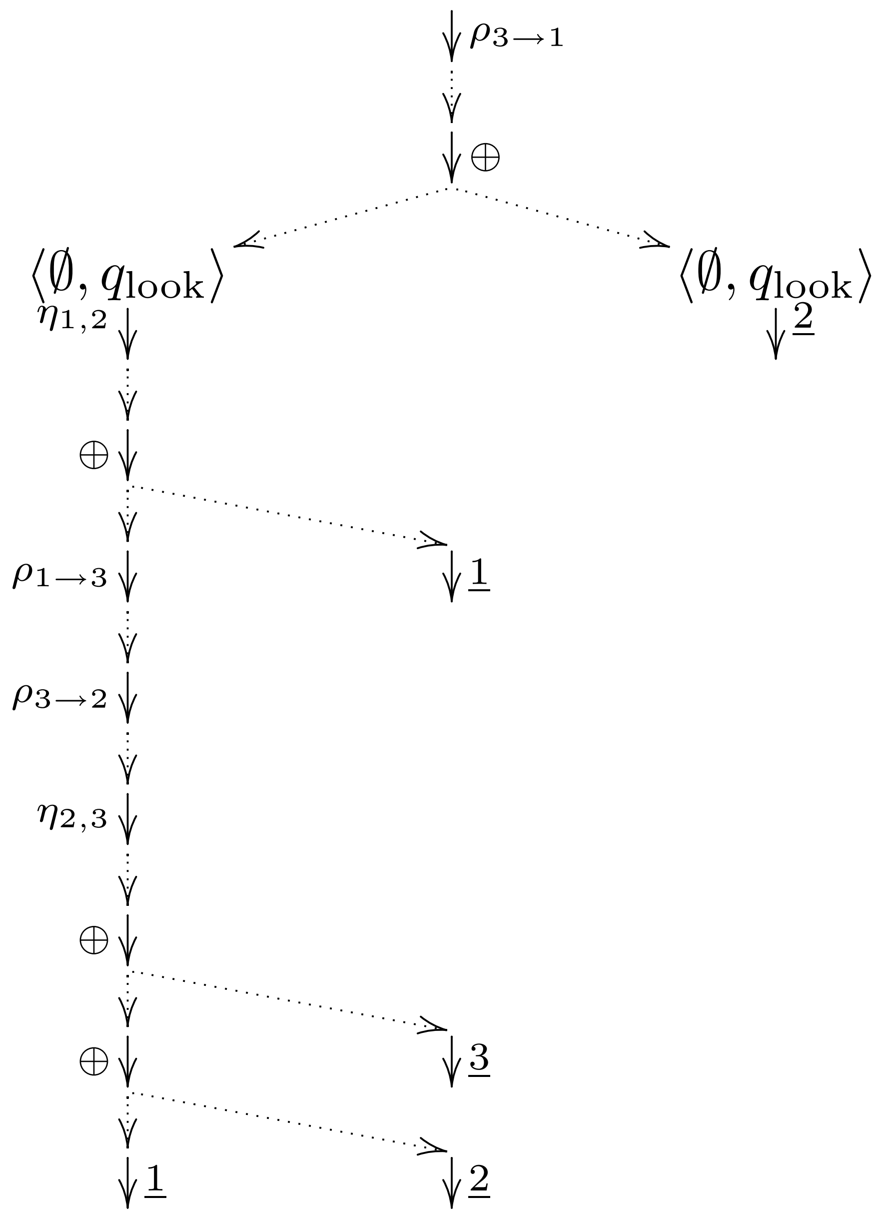

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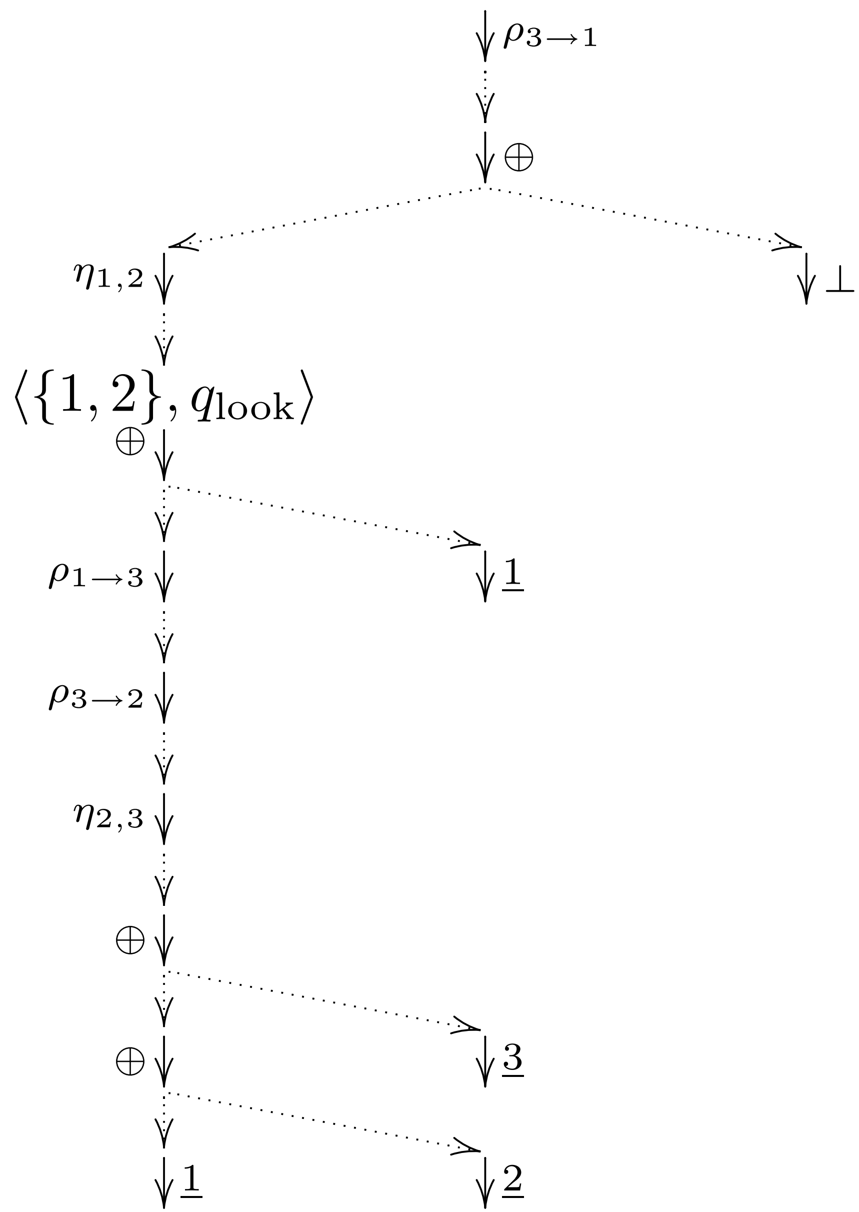

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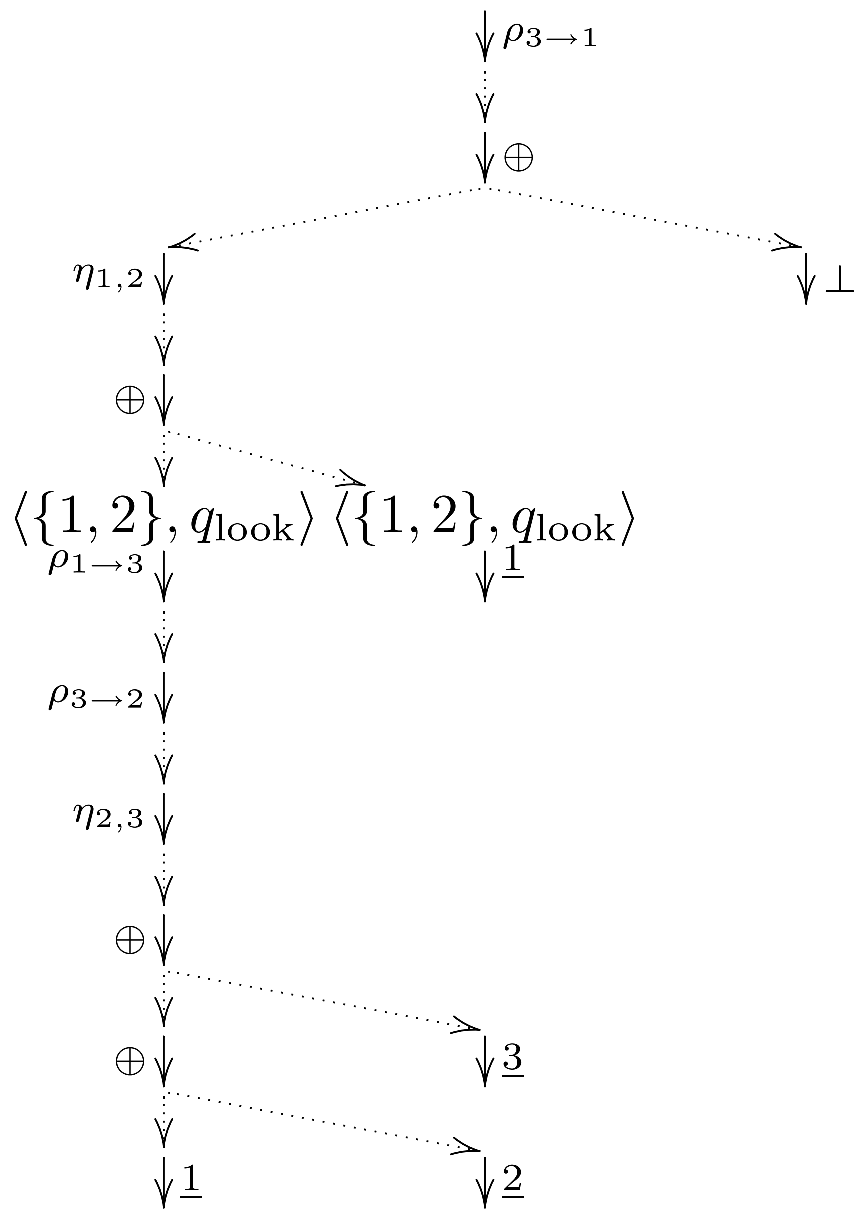

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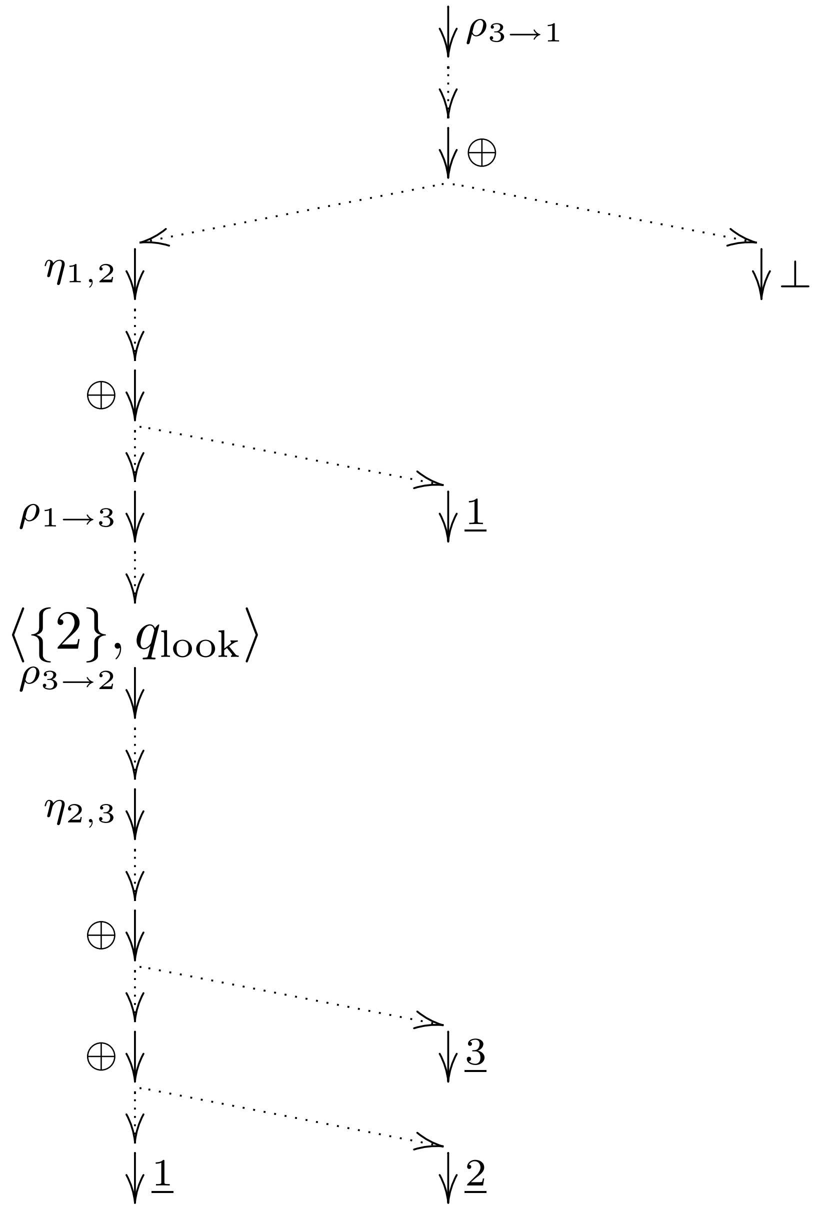

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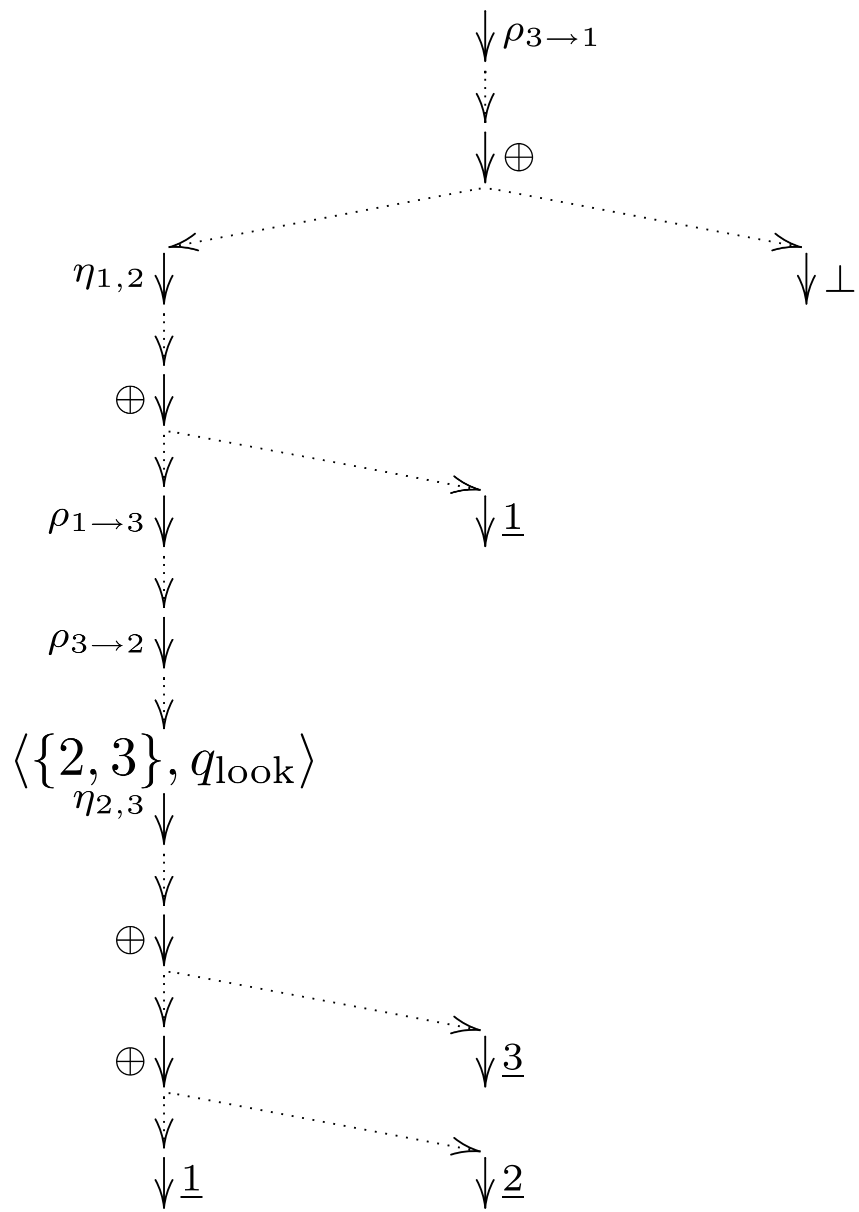

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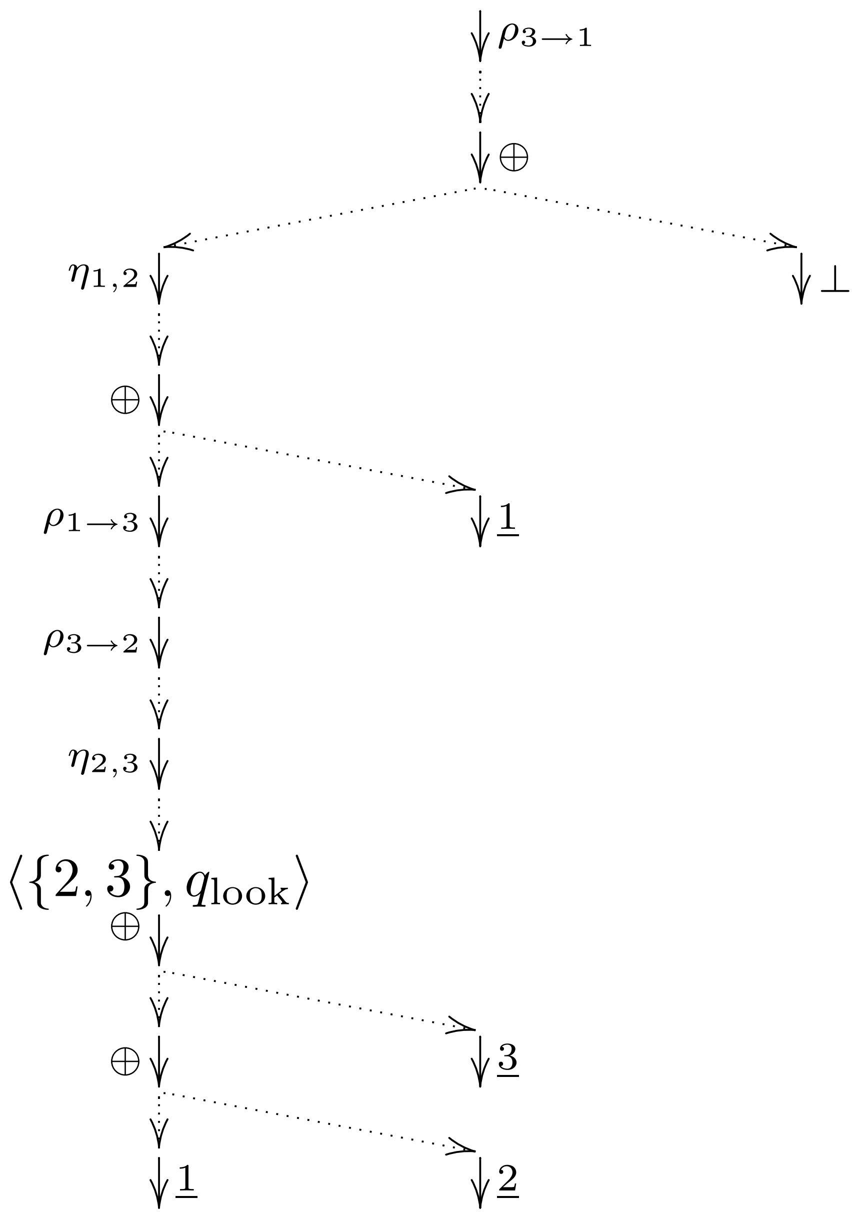

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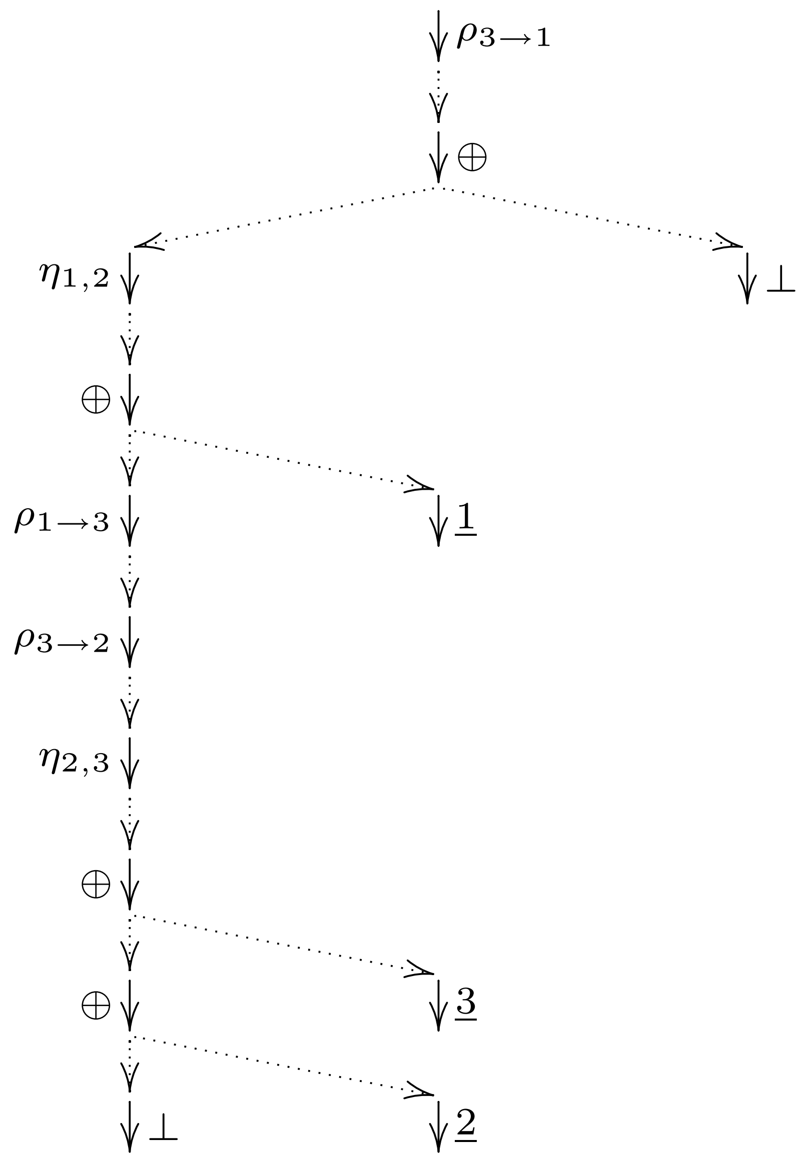

PROPERTIES OF DETERMINISTIC TRANSDUCERS

- The inverse image of a rational set of terms by a deterministic transducer is rational.
- The image of a rational set of terms by a deterministic transducer needs not to be rational.
- The image of a regular term (unfolding of a finite folded term) by a deterministic transducer is a regular term.
- Deterministic transducers are closed under composition.

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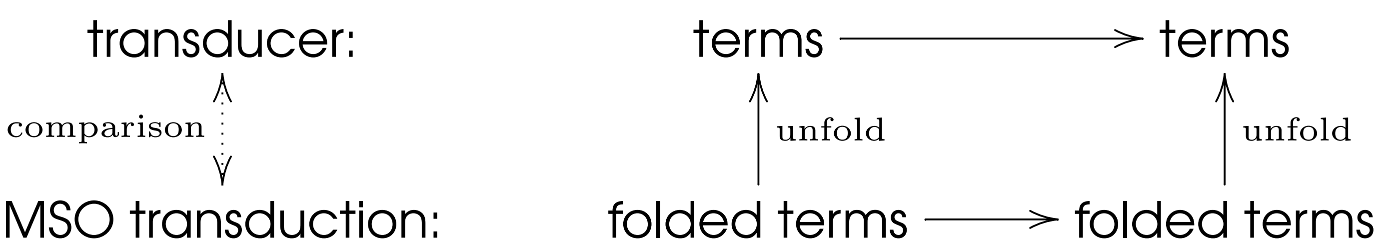

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MSO TRANSDUCTIONS

$n \in \mathbb{N}$

$$M = (\Sigma_{\mathcal{F}}, \Sigma_{\mathcal{F}'}, (\phi_{a,i,j}(x,y)), (\rho_i(x,y)), n)$$

$a \in \Sigma_{\mathcal{F}'}, i, j \in \{1, \ldots, n\}$ $i \in \{1, \ldots, n\}$

MSO-formulas $\phi_{a,i,j}(x,y)$ and $\rho_i(x,y)$ over the signature $(E_a)_{a \in \Sigma_{\mathcal{F}}}$

For a folded term $G = (V_G, E_G)$ with root r_G, M defines a folded term $M(G) = (V_{M(G)}, E_{M(G)})$ with root $r_{M(G)}$:

- $V_{M(G)} = V \times [1, n]$
- $((v,i), a, (u,j)) \in E_{M(G)}$ iff $G \models \phi_{a,i,j}(v,u)$
- $r_{M(G)} = (u,i)$ for the unique u and i with $G \models \rho_i(r_G, u)$.

semantic conditions

EXAMPLE

$\mathcal{F} = \mathcal{F}' = \{f, g, c\}$ with f, g binary and c constant.
Swap subterms of f if the right subterm contains c

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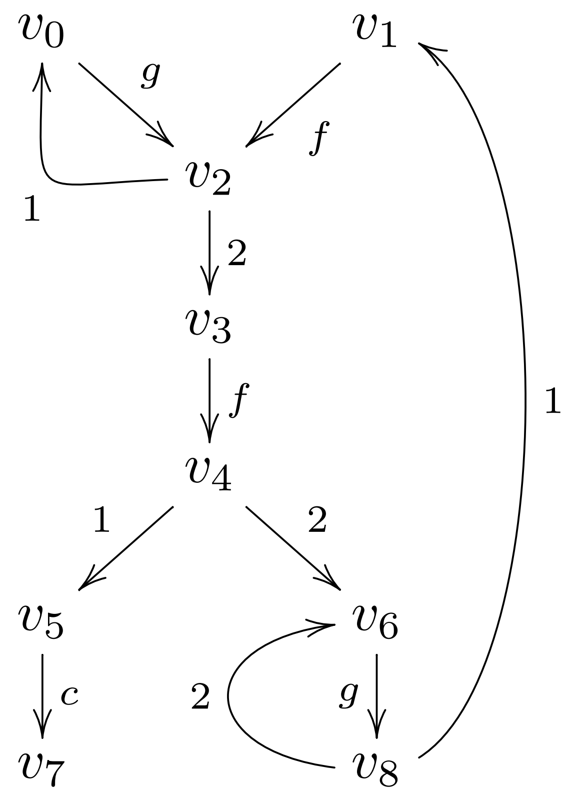

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Root:

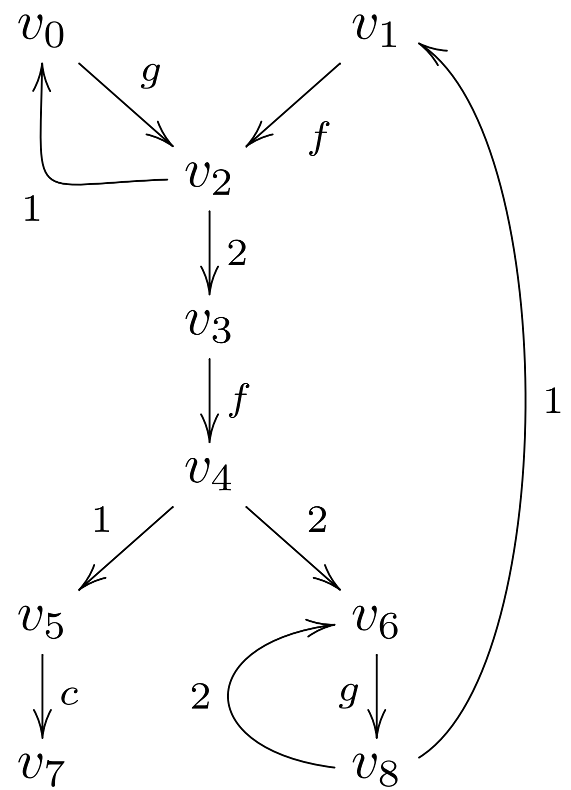
$\rho_1(x, y) = (x = y)$

Edges:

$\phi_{a,1,1}(x, y) = E_a(x, y)$
for $a \in \{g, c, 1, 2\}$

$\phi_{1,2,1}(x, y) = E_2(x, y)$
$\phi_{2,2,1}(x, y) = E_1(x, y)$

$\phi_{f,1,1}(x, y) =$
$E_f(x, y) \wedge \neg \phi_{f,1,2}(x, y)$



$$\phi_{f,1,2}(x, y) = \quad E_f(x, y) \wedge \exists z[E_2(y, z) \wedge$$
$$\forall X(z \in X \wedge \forall z', z''(z' \in X \wedge E(z', z'') \rightarrow z'' \in X)$$
$$\rightarrow \exists z', z'' \in X(E_c(z', z'')))]$$

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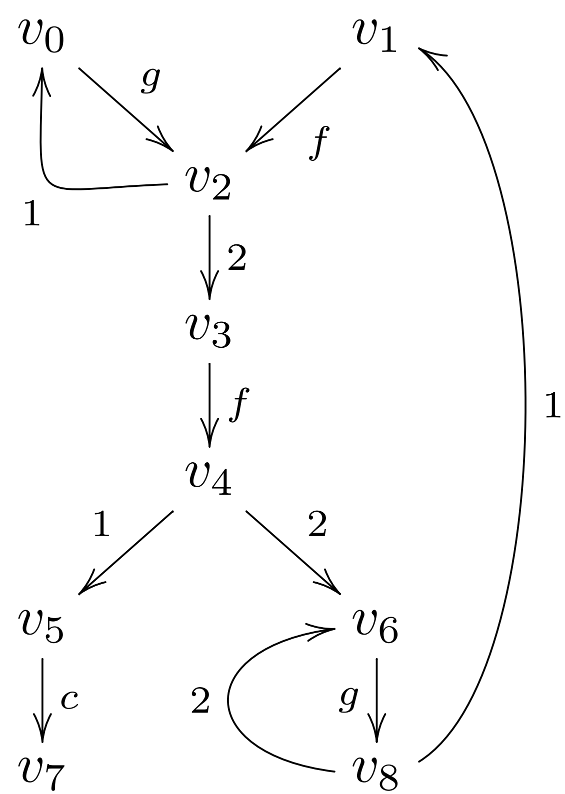
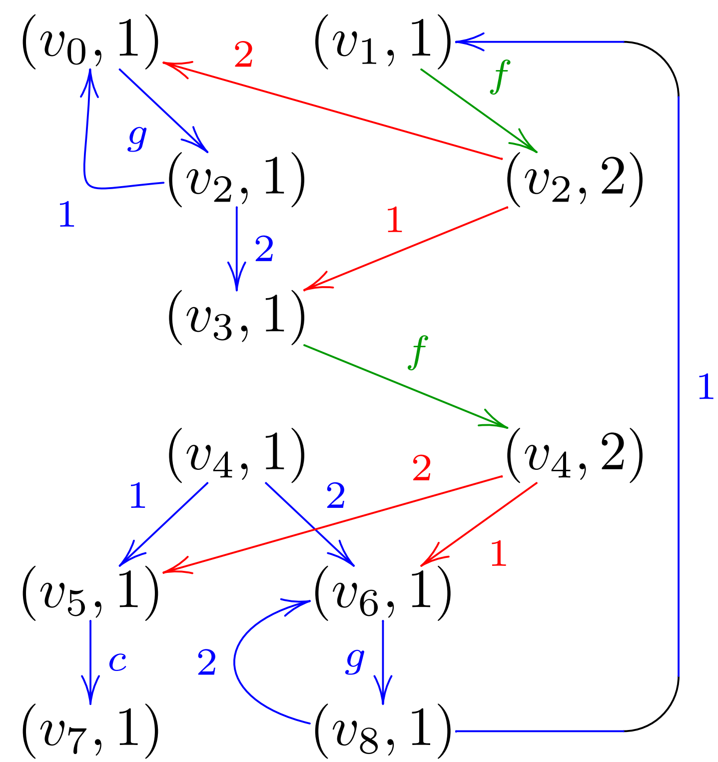
$\phi_{f,1,1}(x, y) =$
$E_f(x, y) \wedge \neg \phi_{f,1,2}(x, y)$





$$\phi_{f,1,2}(x, y) = \quad E_f(x, y) \wedge \exists z[E_2(y, z) \wedge$$
$$\forall X(z \in X \wedge \forall z', z''(z' \in X \wedge E(z', z'') \to z'' \in X)$$
$$\to \exists z', z'' \in X(E_c(z', z'')))]$$

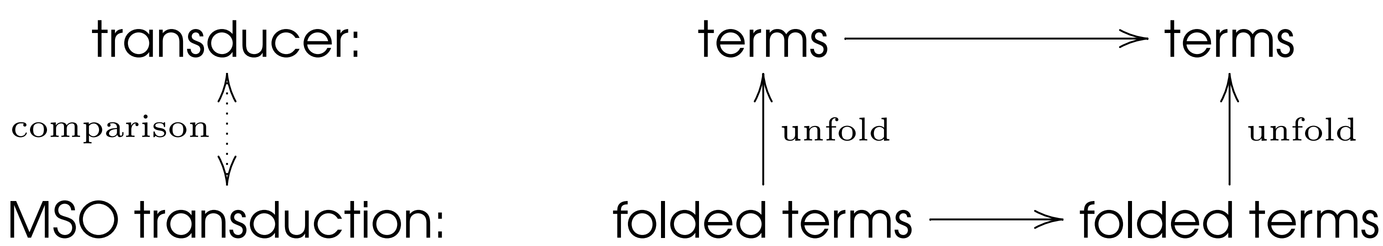
OUTLINE

(1) Basic definitions and terminology

terms, folded terms, MSO logic, rational sets of terms

(2) Overview and background

transformation of objects by transformation of representation



(3) Deterministic top-down tree transducers with rational lookahead

(4) MSO transductions

(5) Main result: comparison of deterministic transducers and MSO transductions

Bisimilarity Preserving Transductions

An MSO Transduction M is bisimilarity preserving if for any two rooted folded terms G, G':

$$\mathrm{unfold}(G) = \mathrm{unfold}(G') \Rightarrow \mathrm{unfold}(M(G)) = \mathrm{unfold}(M(G'))$$

MAIN RESULT

Bisimilarity preserving MSO Transductions and deterministic transducers have the same expressive power.

MAIN RESULT

Bisimilarity preserving MSO Transductions and deterministic transducers have the same expressive power.

More precisely:

(i) For each deterministic transducer T there exists a bisimilarity preserving MSO transduction M_T such that for all folded terms G:

$$\mathrm{unfold}(M_T(G)) = T(\mathrm{unfold}(G))$$

(ii) For each bisimilarity preserving MSO transduction M there exists a deterministic transducer T_M such that for all folded terms G:

$$\mathrm{unfold}(M(G)) = T_M(\mathrm{unfold}(G))$$

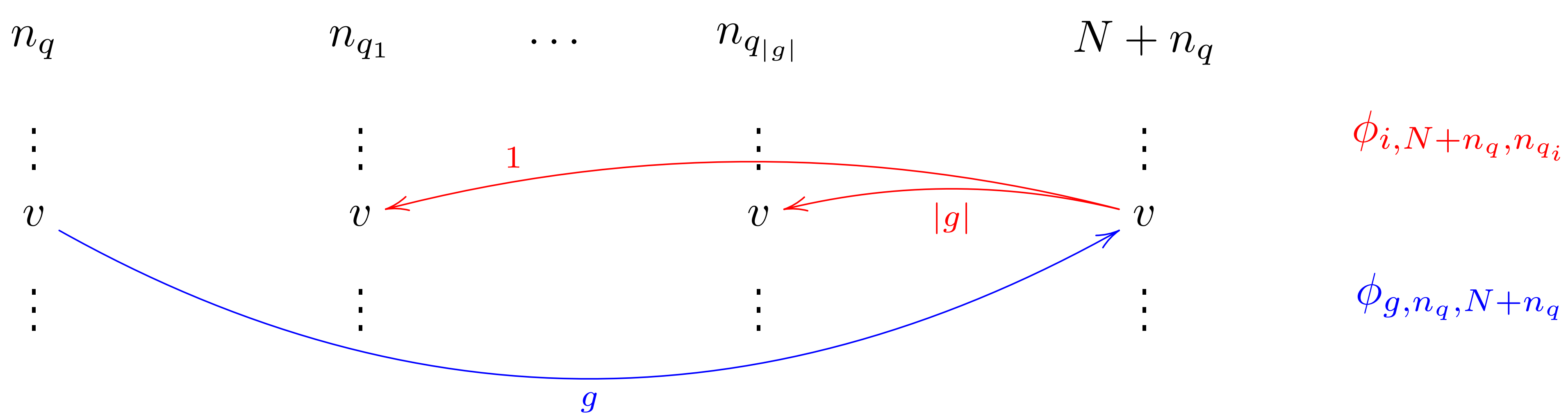
TRANSDUCER $\longrightarrow$ MSO TRANSDUCTION

- If T has N states, then M_T uses $2 \cdot N$ copies of G.
- State q identified uniquely with a number n_q.
- To deal with consumption and lookahead rules a new symbol ε of arity 1 is introduced. This can be removed by a second MSO transduction.

Transducer $\longrightarrow$ MSO Transduction

- If T has N states, then M_T uses $2 \cdot N$ copies of G.
- State q identified uniquely with a number n_q.
- To deal with consumption and lookahead rules a new symbol ε of arity 1 is introduced. This can be removed by a second MSO transduction.

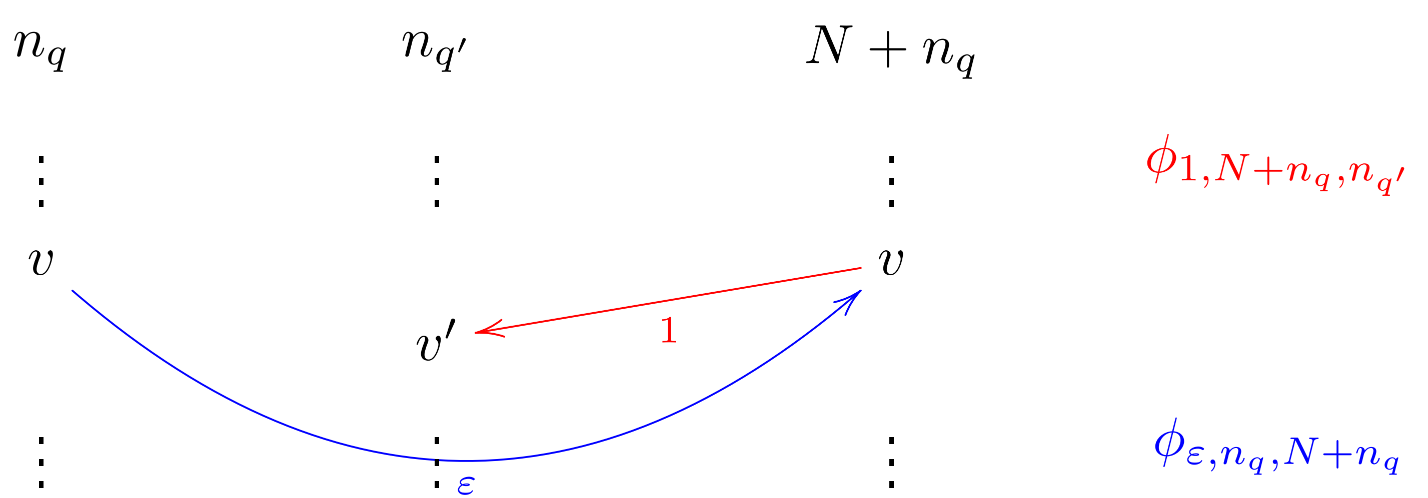
Production rule $q(x) \to g(q_1(x), \ldots, q_{|g|}(x))$



Transducer $\longrightarrow$ MSO Transduction

- If T has N states, then M_T uses $2 \cdot N$ copies of G.
- State q identified uniquely with a number n_q.
- To deal with consumption and lookahead rules a new symbol ε of arity 1 is introduced. This can be removed by a second MSO transduction.

Consumption rule $q(f(x_1, \ldots, x_{|f|})) \rightarrow q'(x_i)$

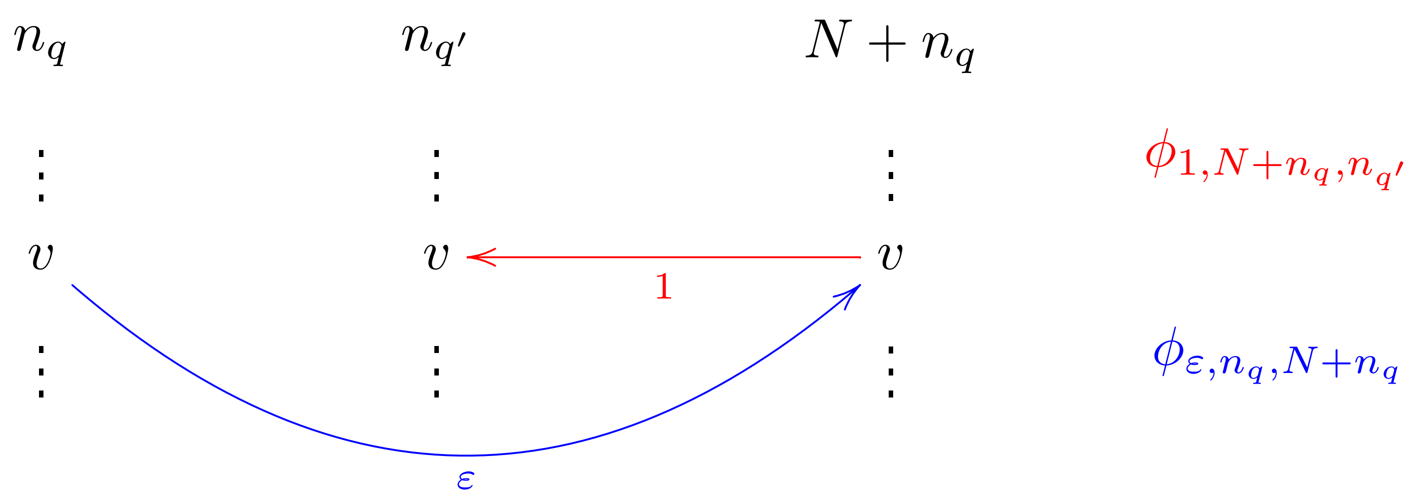


if exists u with $v \xrightarrow{f} u \xrightarrow{i} v'$ in G

Transducer $\longrightarrow$ MSO Transduction

- If T has N states, then M_T uses $2 \cdot N$ copies of G.
- State q identified uniquely with a number n_q.
- To deal with consumption and lookahead rules a new symbol ε of arity 1 is introduced. This can be removed by a second MSO transduction.

Lookahead rule $q(x \in L) \rightarrow q'(x)$



if $\mathrm{unfold}(G, v)$ is in L

MSO TRANSDUCTION $\longrightarrow$ TRANSDUCER

For each bisimilarity preserving MSO transduction M there exists a deterministic transducer T_M such that for all folded terms G:

$$\mathrm{unfold}(M(G)) = T_M(\mathrm{unfold}(G))$$

MSO TRANSDUCTION $\longrightarrow$ TRANSDUCER

For each bisimilarity preserving MSO transduction M there exists a deterministic transducer T_M such that for all folded terms G:

$$\mathrm{unfold}(M(G)) = T_M(\mathrm{unfold}(G))$$

It suffices to consider M on terms:

M bisimilarity preserving $\Rightarrow \mathrm{unfold}(M(G)) = \mathrm{unfold}(M(\mathrm{unfold}(G)))$

MSO TRANSDUCTION $\longrightarrow$ TRANSDUCER

For each bisimilarity preserving MSO transduction M there exists a deterministic transducer T_M such that for all terms t:

$$\mathrm{unfold}(M(t)) = T_M(t)$$

MSO Transduction $\longrightarrow$ Transducer

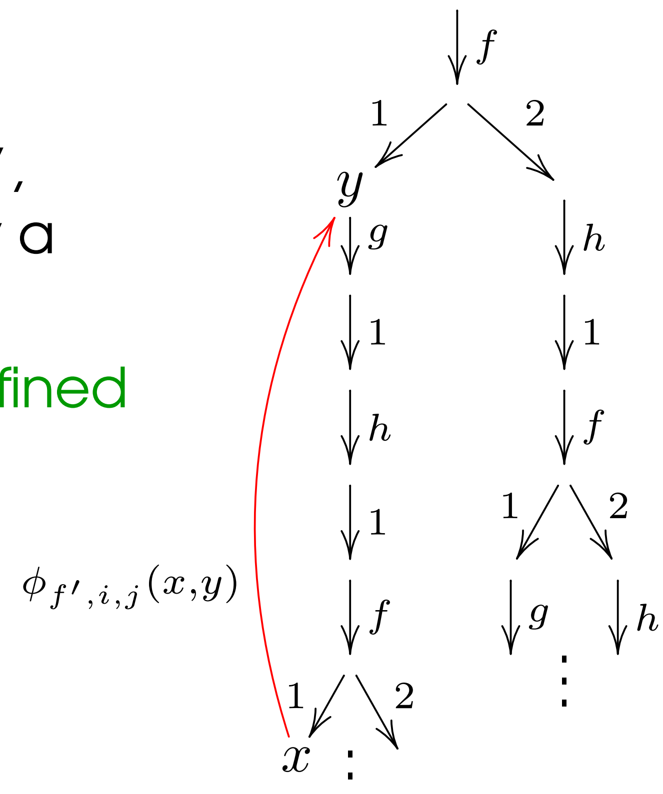
For each bisimilarity preserving MSO transduction M there exists a deterministic transducer T_M such that for all terms t:

$$\mathrm{unfold}(M(t)) = T_M(t)$$

Main difficulty:

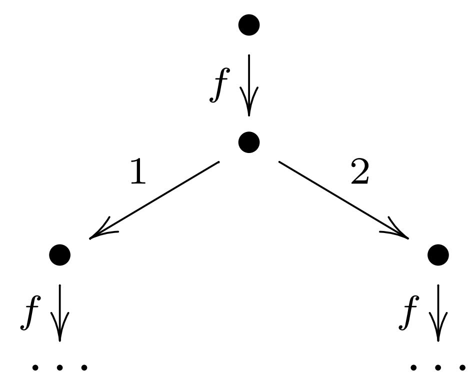
- Transducers work top-down.
- If M defines new edges 'going upward', these edges cannot be constructed by a finite state transducer.

$\Rightarrow$ In a first step normalize M such that defined edges are 'going downward'.



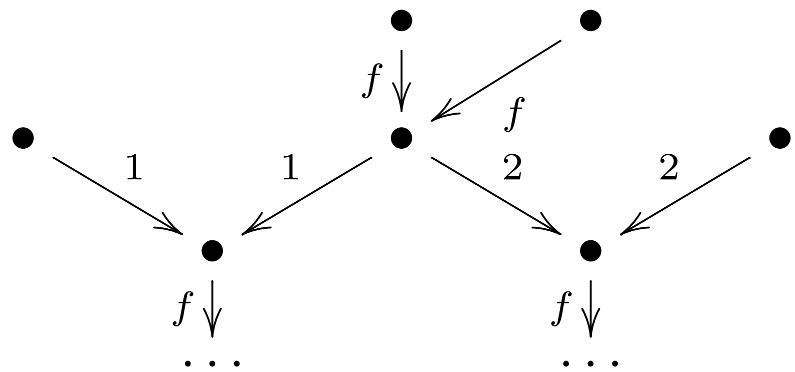
TOP-DOWN NORMALIZATION

t :

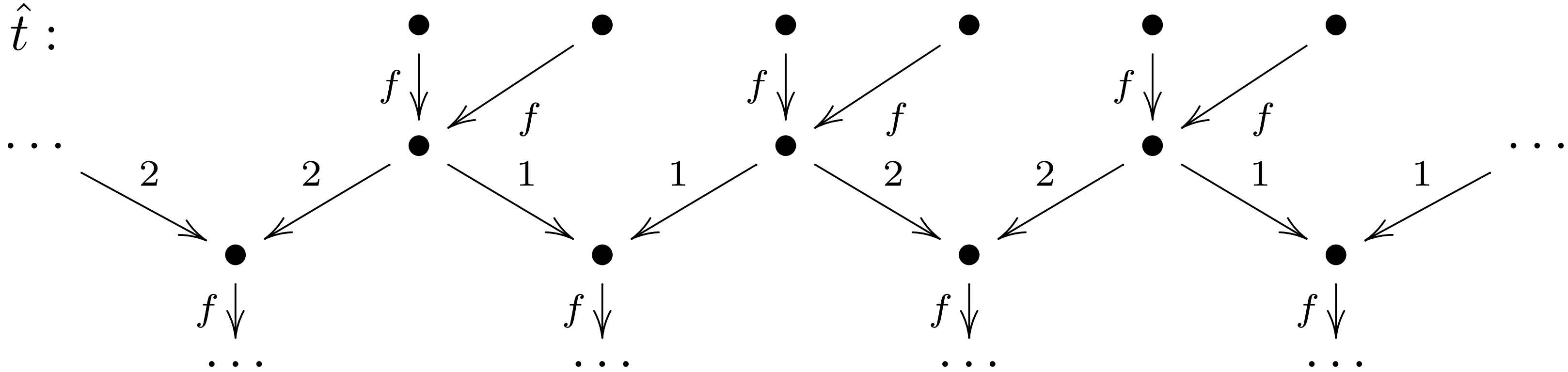


TOP-DOWN NORMALIZATION

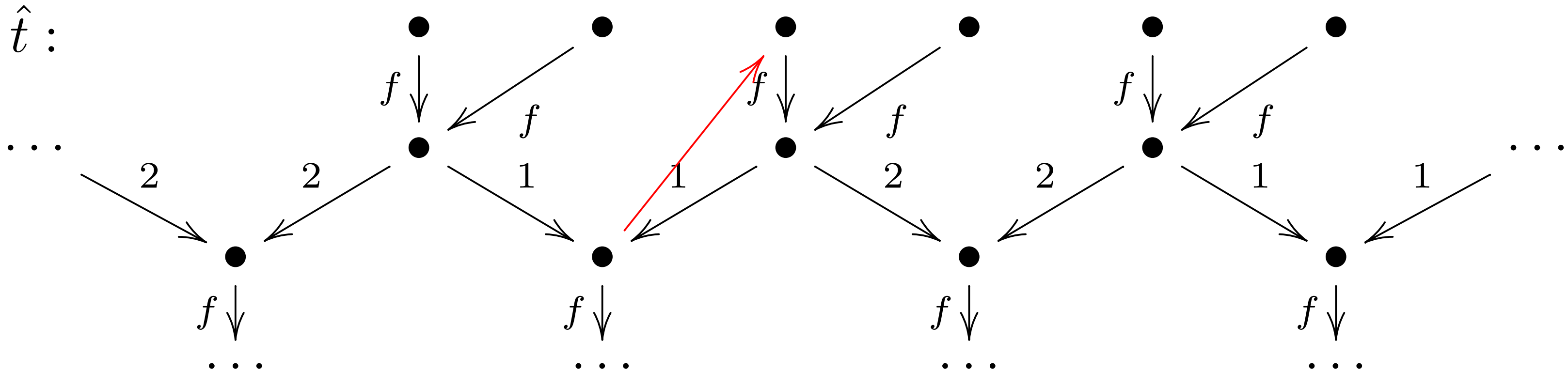
t :



TOP-DOWN NORMALIZATION

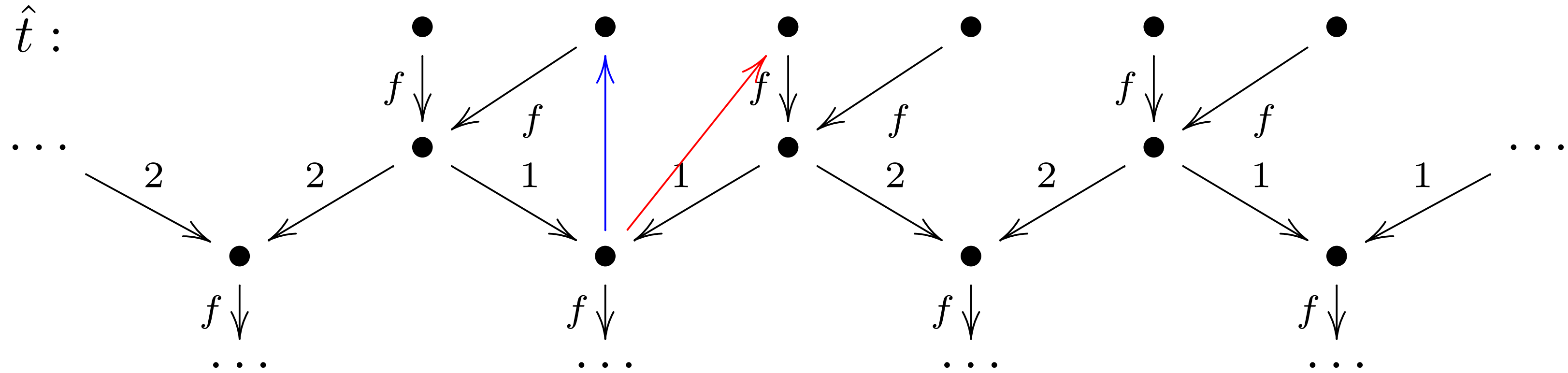


TOP-DOWN NORMALIZATION



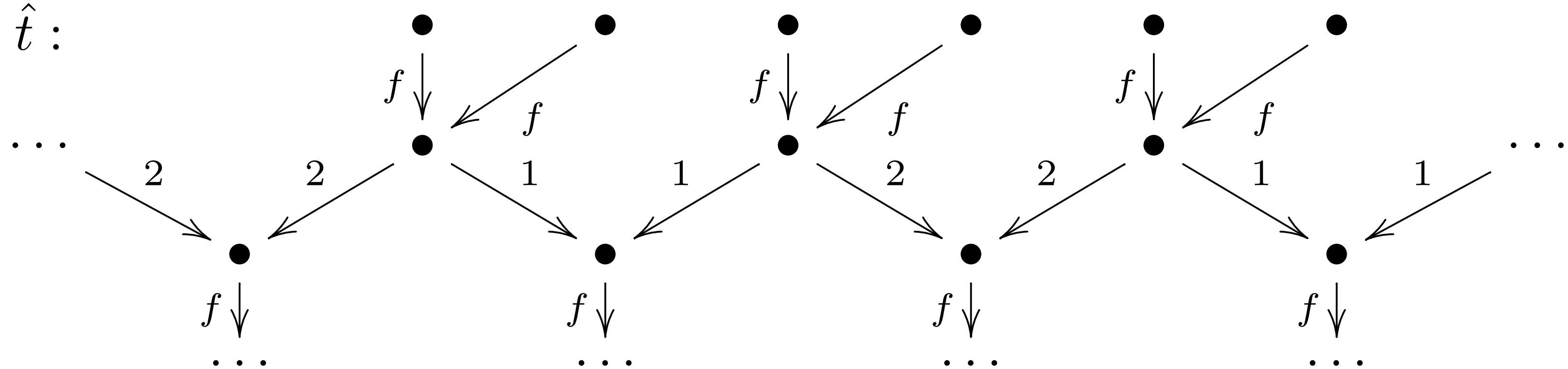
Consider M on $\hat{t}$ (with root inherited from t) and assume a new edge goes upward.

Top-Down Normalization



Consider M on $\hat{t}$ (with root inherited from t) and assume a new edge goes upward.
Then the same formula defines another edge with the same origin.
Hence $M(\hat{t})$ is not a folded term.

Top-Down Normalization

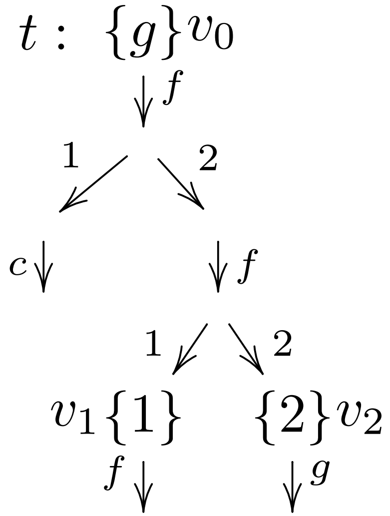


- In $\hat{t}$ the edges defined by M are going downward.
- The formulas $\phi_{a,i,j}$ on $\hat{t}$ can be transformed into formulas $\hat{\phi}_{a,i,j}$ on t ($\hat{t}$ can be obtained from t by the Muchnik/Walukiewicz construction).
- The new MSO transduction $\hat{M}$ using the formulas $\hat{\phi}_{a,i,j}$ has the following properties:
  - $\mathrm{unfold}(M(t)) = \mathrm{unfold}(\hat{M}(t))$
  - The edges defined by $\hat{M}$ are going downward.

NORMALIZED TRANSDUCTION → TRANSDUCER

Rough sketch:

- Normalized Transduction $M = (\Sigma_{\mathcal{F}}, \Sigma_{\mathcal{F}'}, (\phi_{a,i,j}(x,y)), (\rho_i(x,y)), n)$
- Transform formulas $\phi_{a,i,j}(x,y)$ into (Rabin) tree automata accepting 'marked terms':



$\mathcal{A}_{g,i,j_1,j_2}$ accepts t if for some ℓ and v

$$\begin{aligned} t &\models \phi_{g,i,\ell}(v_0, v) \\ t &\models \phi_{1,\ell,j_1}(v, v_1) \\ t &\models \phi_{1,\ell,j_2}(v, v_2) \end{aligned}$$

- Transducer T_M keeps track of the states of the automata $\mathcal{A}_{a,i,j_1,\ldots,j_k}$ while going through the term.
- The lookahead is used to check for which automaton there exists a marking that is accepted. This information is used to construct the next edge.

CONCLUSION

- For every deterministic transducer there is an equivalent MSO transduction.
  $\leadsto$ decidability of the MSO theory of terms is preserved
- For every bisimilarity preserving MSO transduction there is an equivalent deterministic transducer.
  $\leadsto$ deterministic transducers are expressively complete for MSO logic
- Transducers are more handy than MSO transductions concerning their construction and the proofs of correctness (cf. thesis of T. Colcombet)

Open:

- We assume that M is bisimilarity preserving for finite and infinite folded terms. Can one transfer the result if M has this property only for finite folded terms?
- Transfer (and analyze) other models of transducers that have been defined for finite terms to the infinite world.