The relative signed clique number of planar graphs is 8

Ritesh Seth

Project Linked Person
Indian Statistical Institute, Kolkata

04/05/2021

This work

This work

Has been published as:

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The relative signed clique number of planar graphs is 8.
Sandip Das, Soumen Nandi, Sagnik Sen and Ritesh Seth.
CALDAM 2019: Algorithms and Discrete Applied Mathematics.

An extension of this work

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As journal version (in preparation)

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Joint work of

Sandip Das, Soumen Nandi, Sagnik Sen,
Ritesh Seth, Dibyayan Chakraborty
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We restrict discussion to current work

And the work

And the work

Based on the paper:

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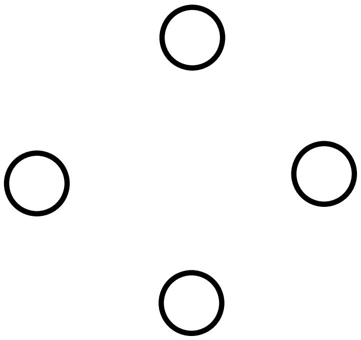
Based on the paper:

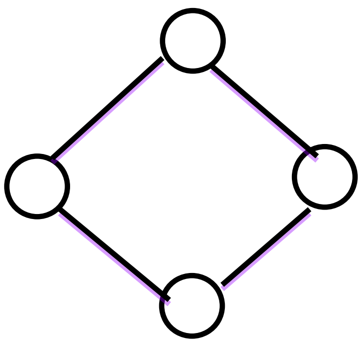
Relative clique number of planar signed graphs.
Sandip Das, Prantar Ghosh, Swathy Prabhu and Sagnik Sen.
Discrete Applied Mathematics (in press).

Signed graphs

Signed graphs

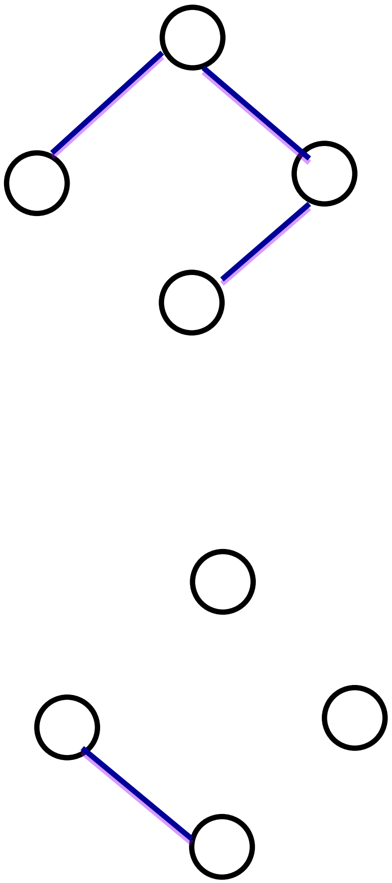
- simple graphs
-
-





Signed graphs

- simple graphs
- has positive edges
-



Signed graphs

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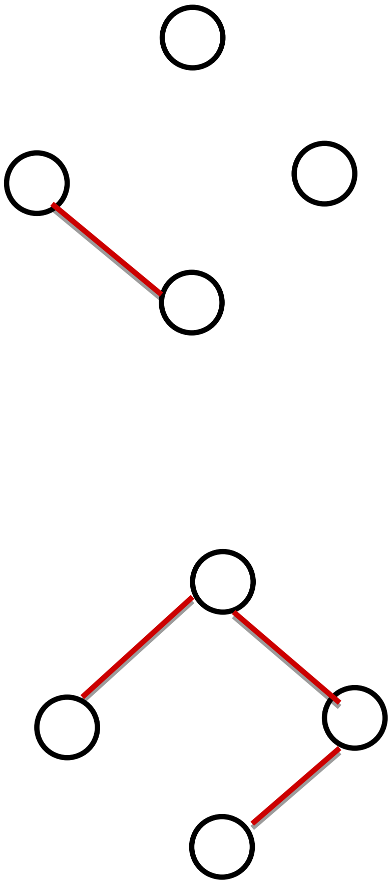

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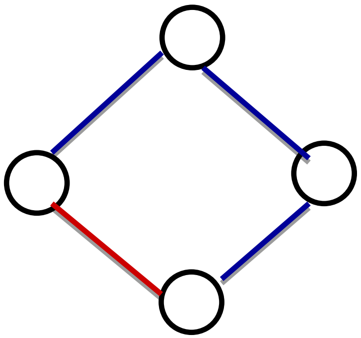

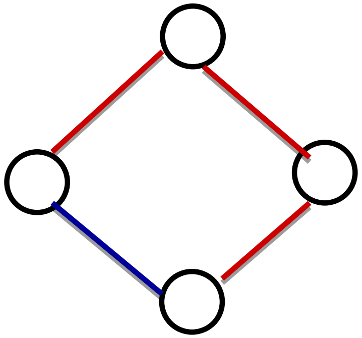

Signed graphs

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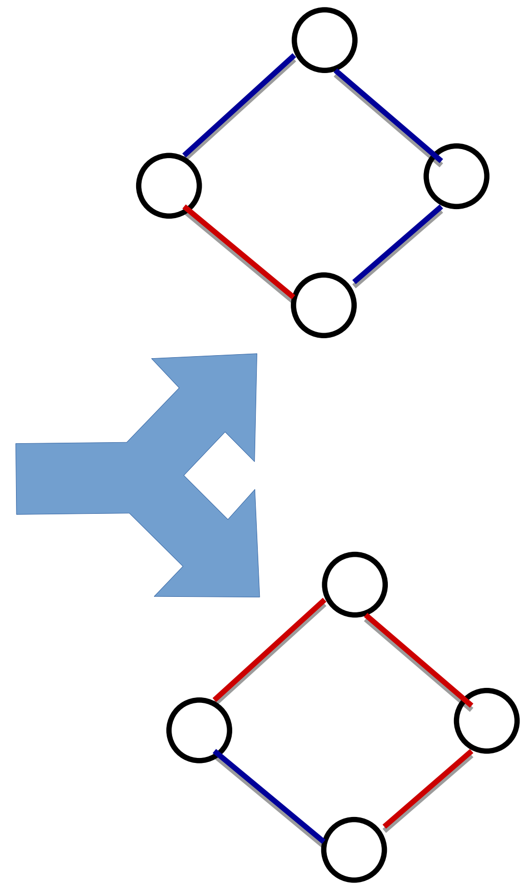
Unbalanced 4-cycle



signed relative clique

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vertex subset R of a signed graph whose non-adjacent vertices are part of an unbalanced 4-cycle

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vertex subset R of a signed graph whose non-adjacent vertices are part of an unbalanced 4-cycle

$$\omega_{rs}(G)=\max\{|R|: R \text{ is a relative clique of } G\}$$
$$\omega_{rs}(F)=\max\{\omega_{rs}(G): G \text{ is in } F\}$$

Open question

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Question (Naserasr during 2^{nd} Autumn meeting on signed graphs 2013, communicated to me by Sen):
For the family P of planar graphs, what is

$$\omega_{rs}(P) = ?$$

Our result (Das, Nandi, Sen and Seth)

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Theorem: For the family P of planar graphs, $\omega_{rs}(P) = 8$.

proof sketch...

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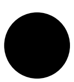
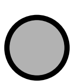
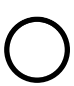
 = good vertices = vertices of the relative clique

 = helper vertices = not good vertices

 = maybe good or helper

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Let G be a planar signed graph with relative clique number at least 9 such that G is minimal. Then the following configurations are forbidden in G:

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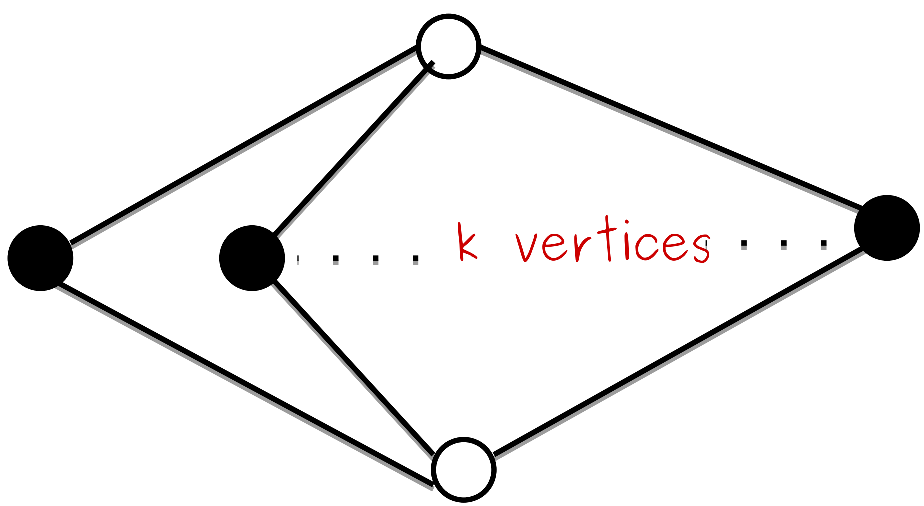
Proof (sketch)

Let G be a planar signed graph with relative clique number at least 9. Then the following configurations (F_k) are forbidden in G:



$k = 3, 4, 5, 6, \ldots$

Switch operation

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- switch a vertex=switch signs of incident edges

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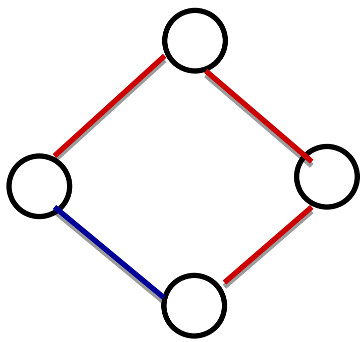

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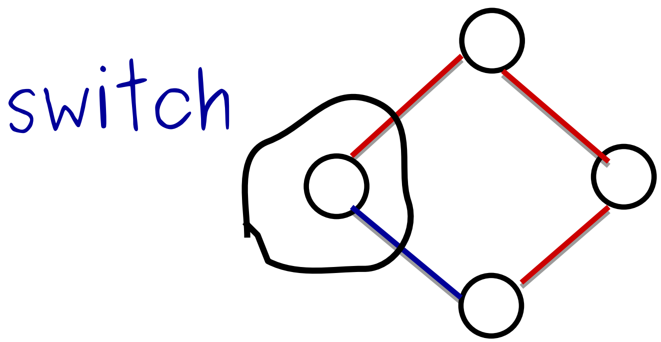

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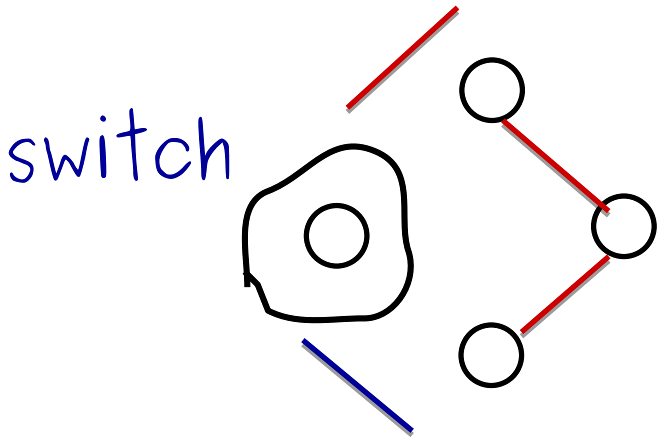

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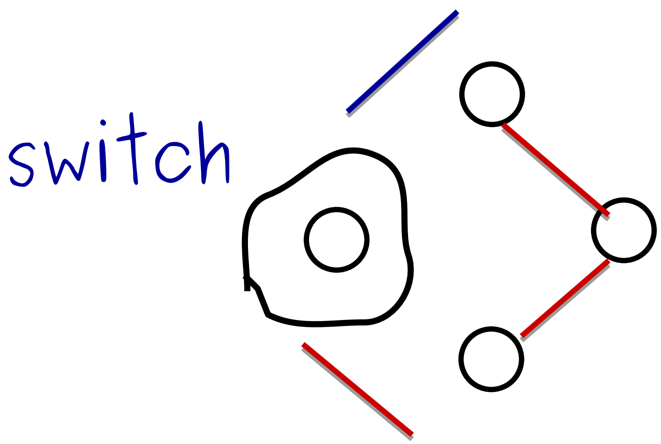

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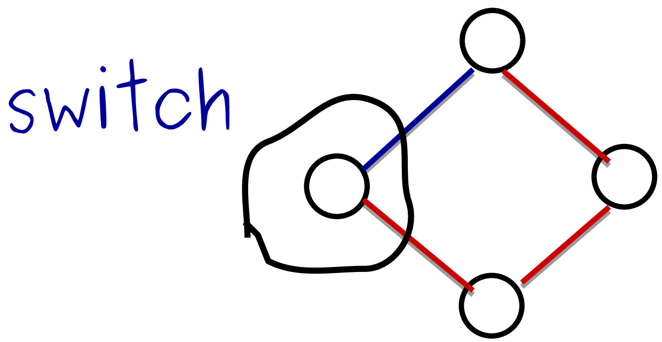

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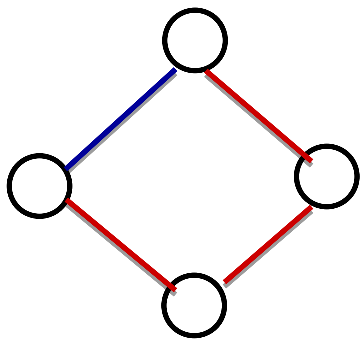

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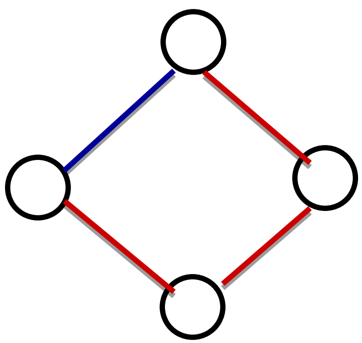

relative cliques
are switch
invariant

Forbidden configuration (F_k, $k \geq 7$)::

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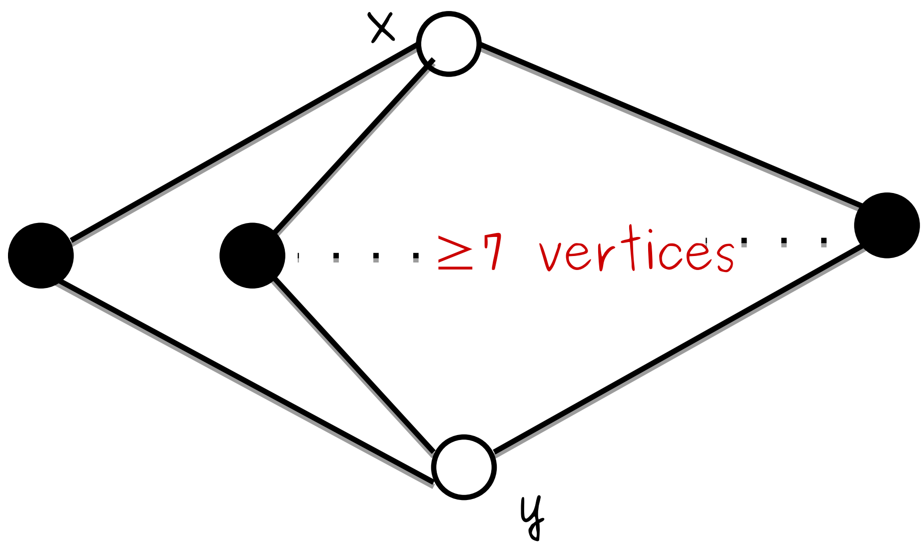

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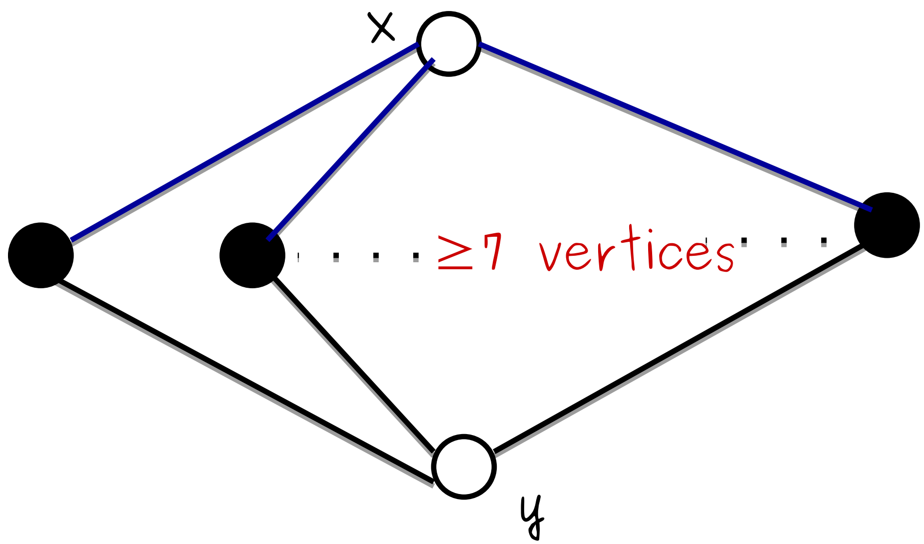
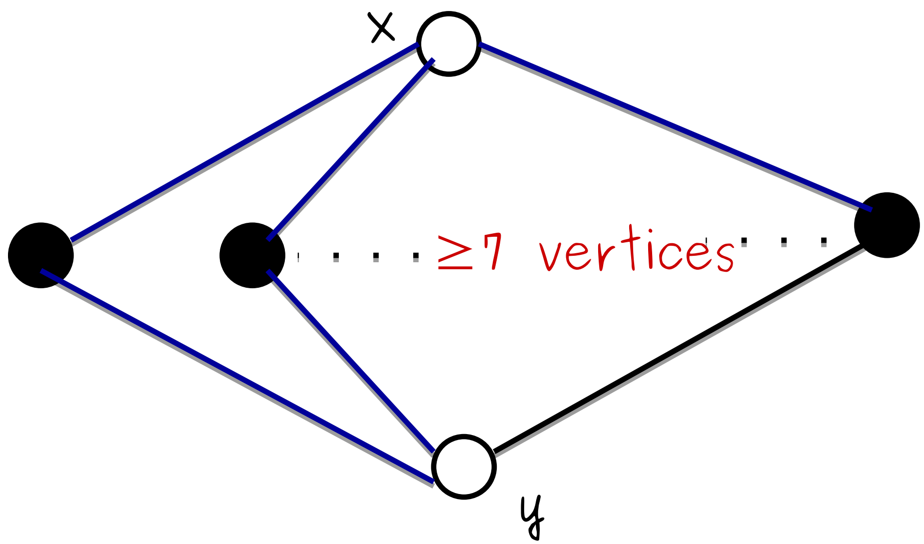


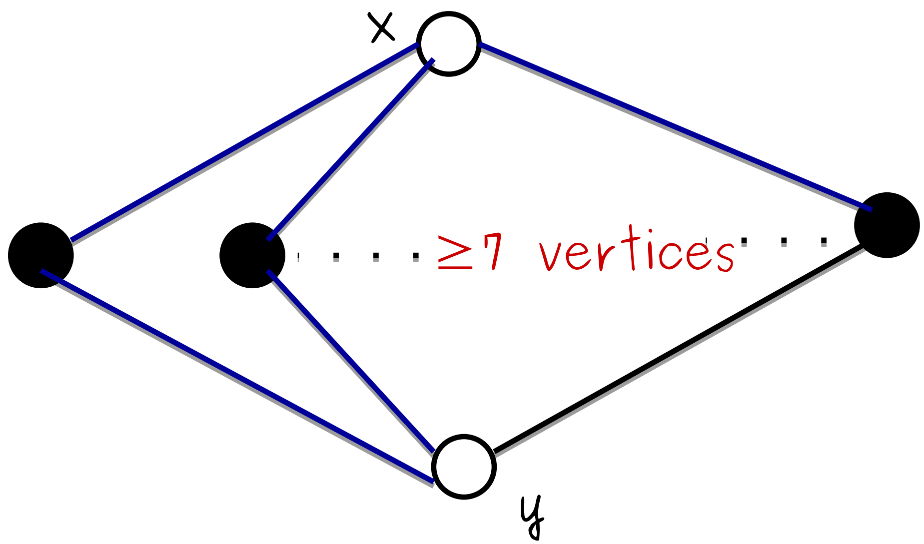
switch and assume all positive with x

Forbidden configuration (F_k, $k \geq 7$):



switch and by PHP least 4 are also positive with y

Forbidden configuration (F_k, $k \geq 7$):



these 4 can not reach each other

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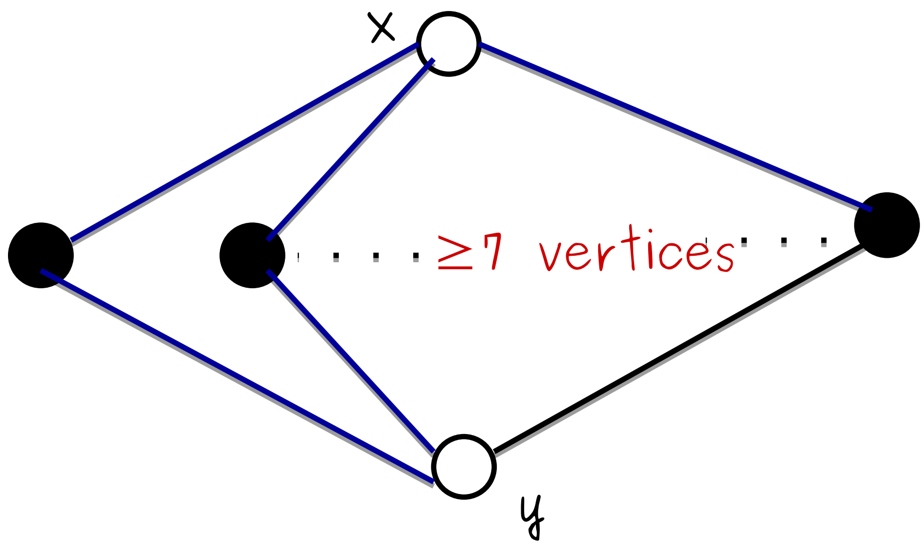

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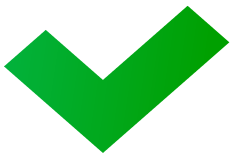

Forbidden configuration (F_6):

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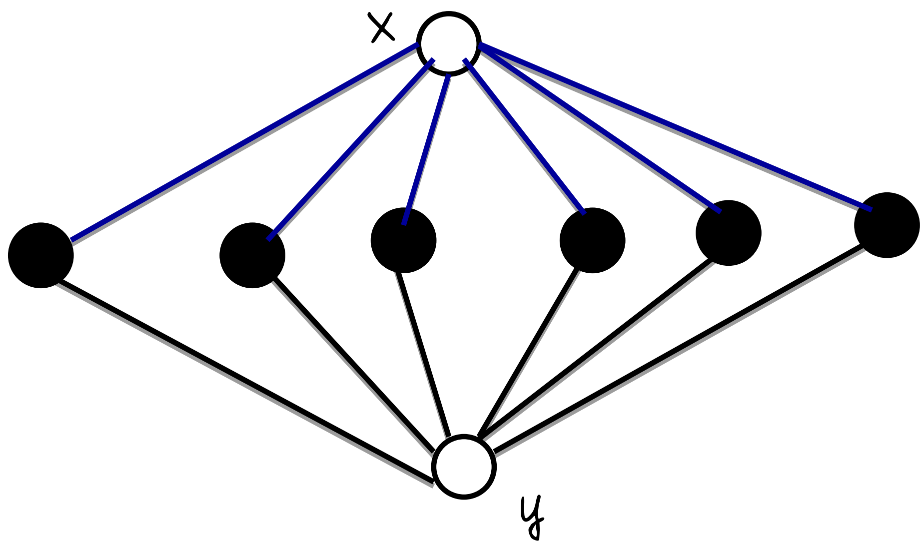

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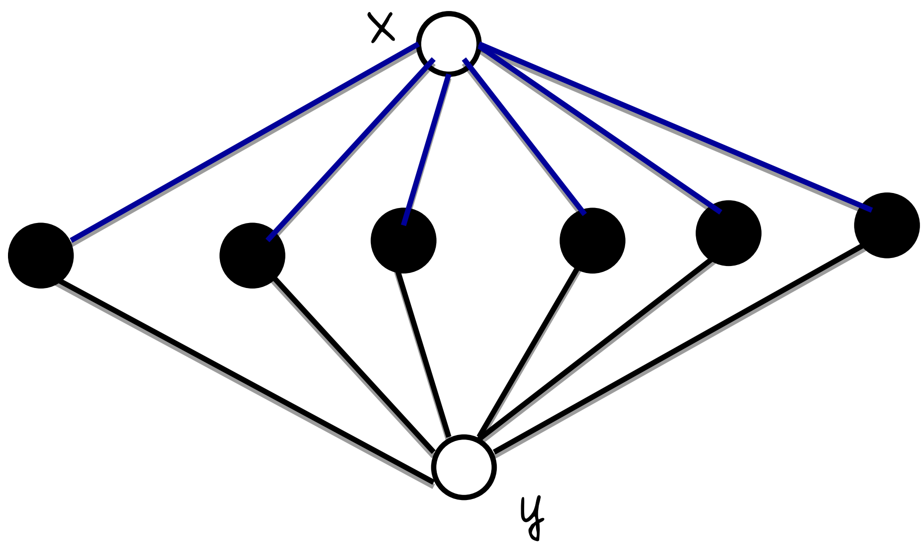
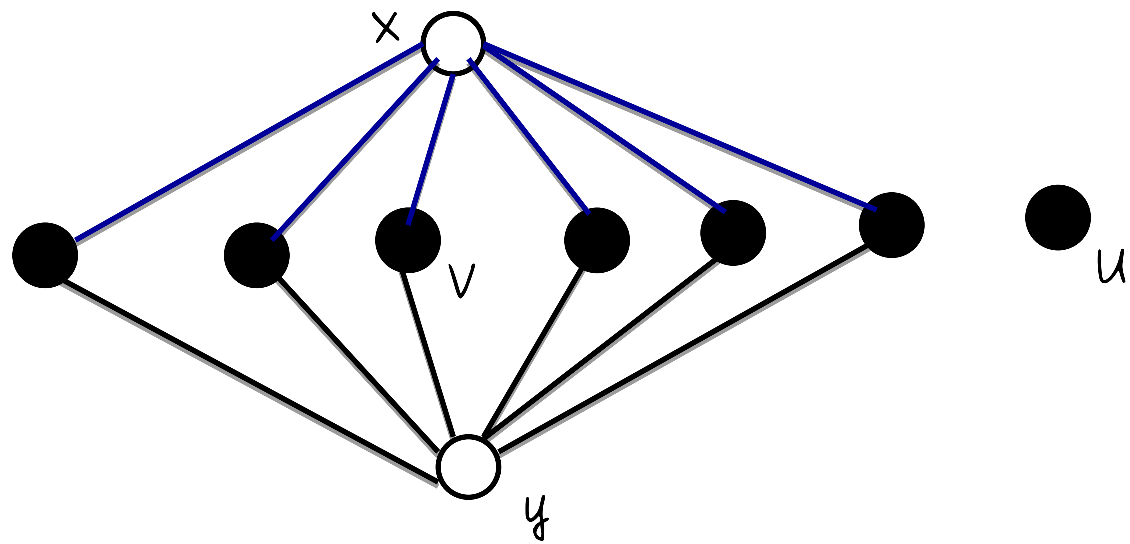


at least 9 good

Forbidden configuration (F_6):



u on outerface

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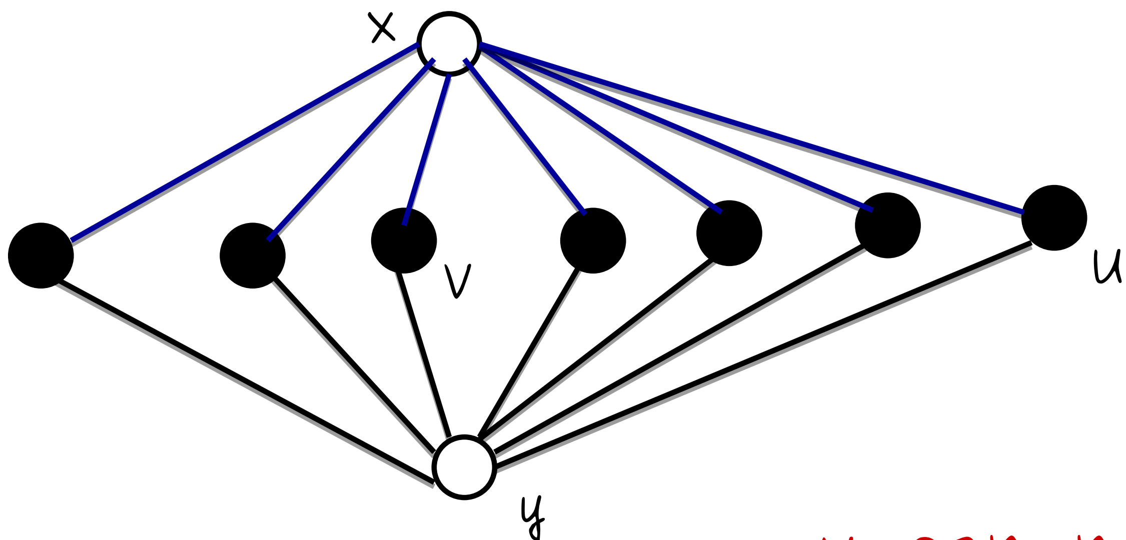

u can not reach v unless forcing F_7

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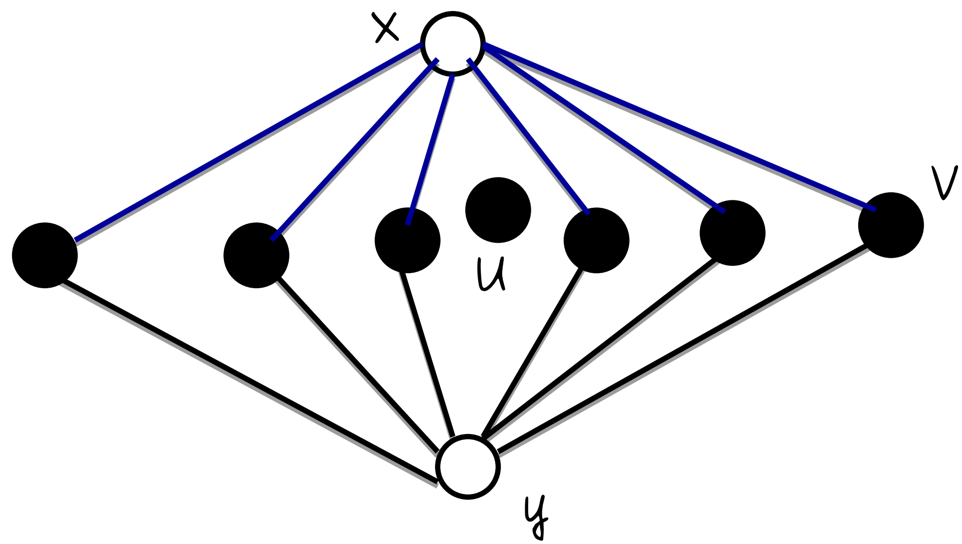

WLOG

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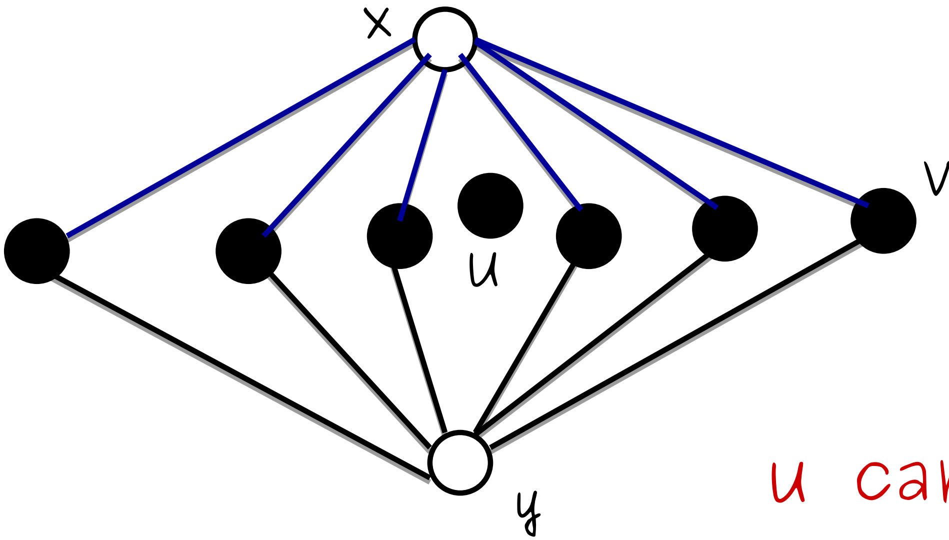

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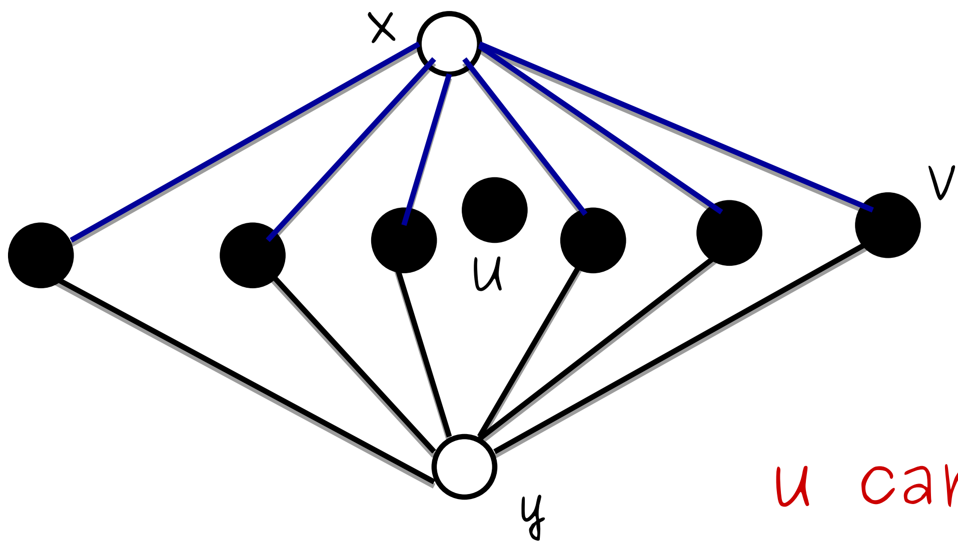

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Forbidden configuration (F_5):

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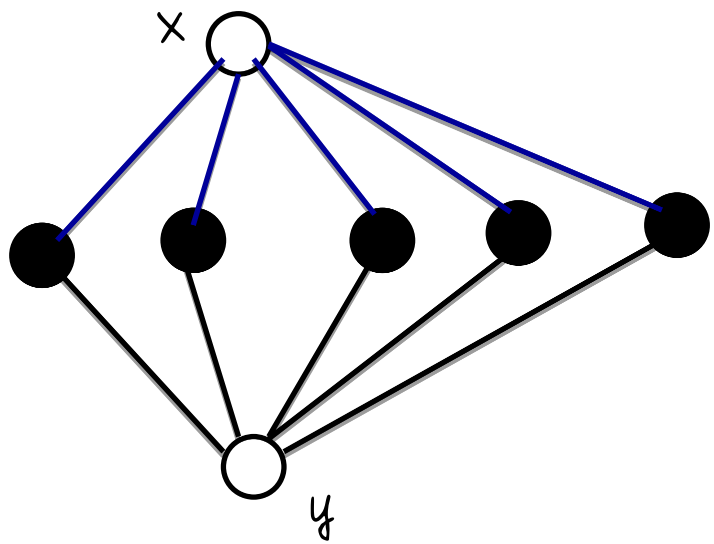

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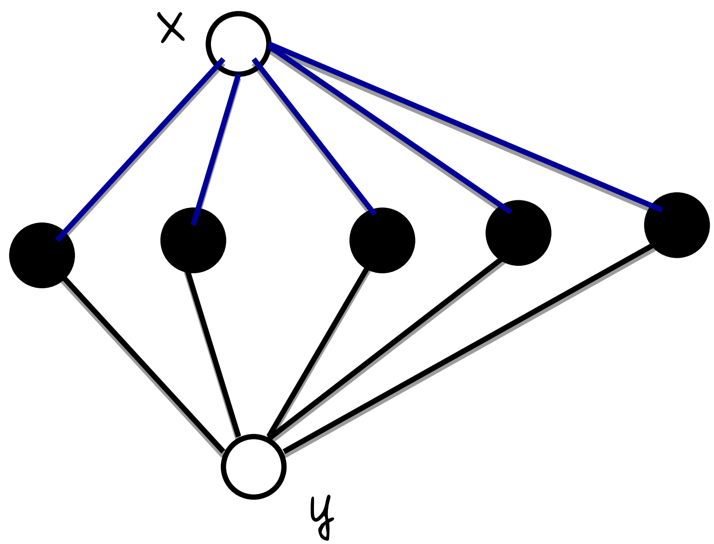

similar to F_6

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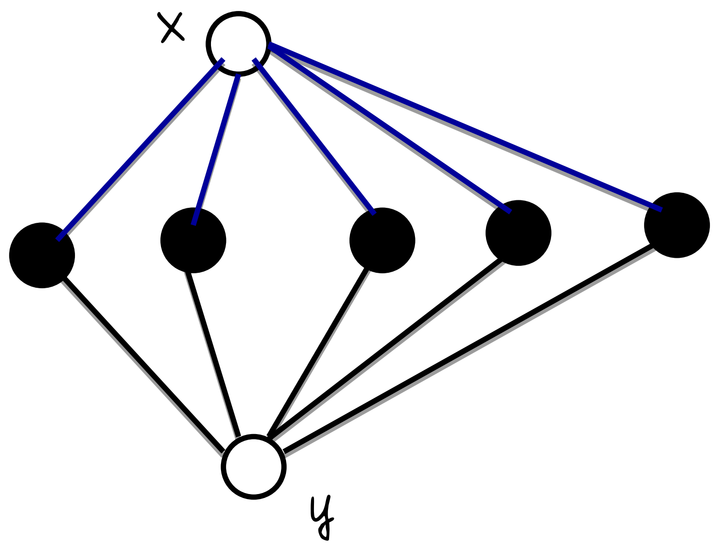

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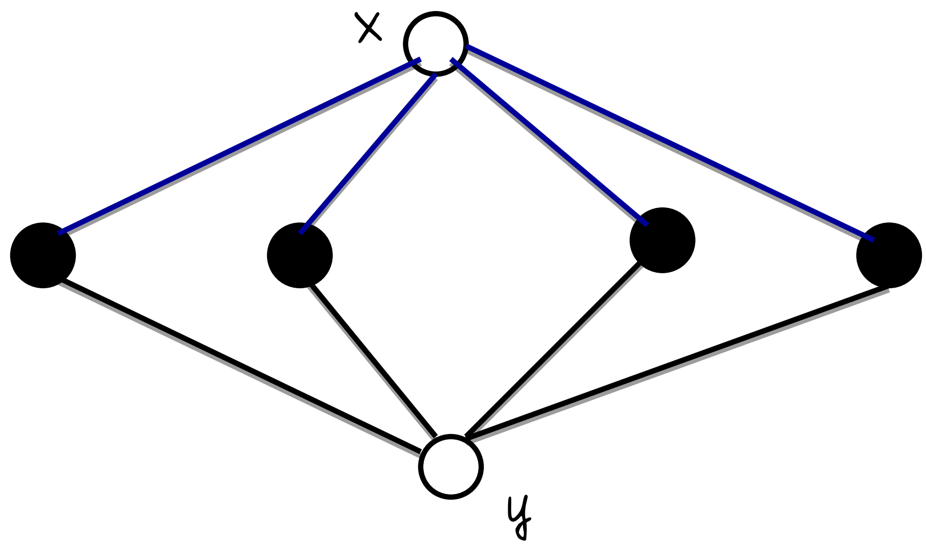

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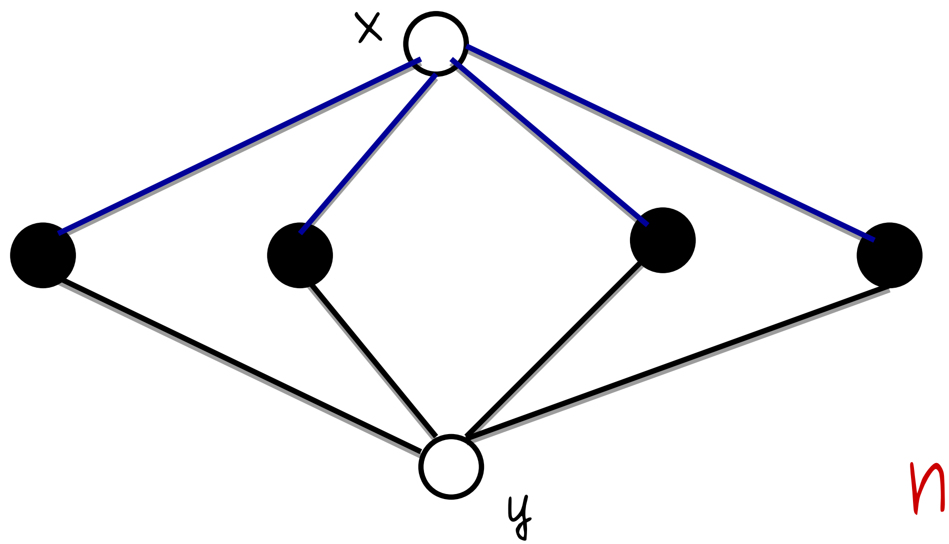
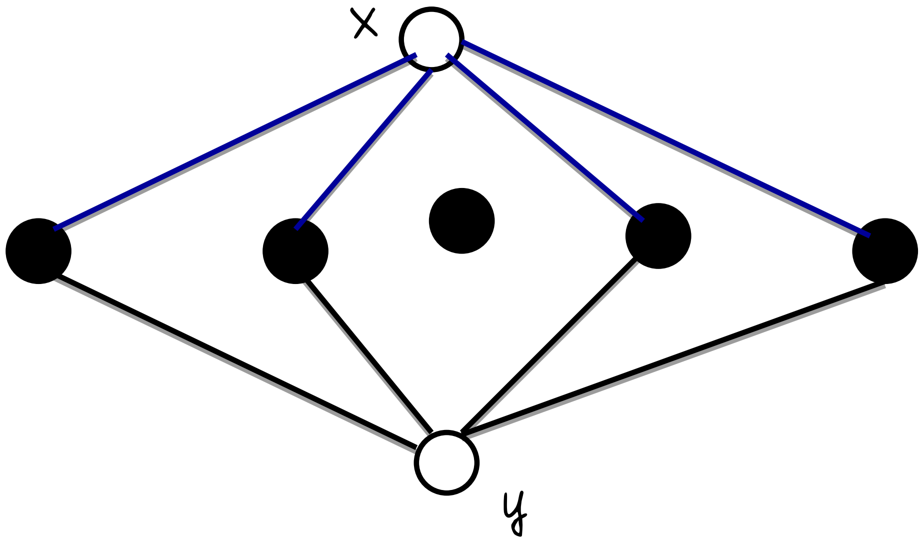


need to place 3 more good vertices

Forbidden configuration (F_4):



WLOG

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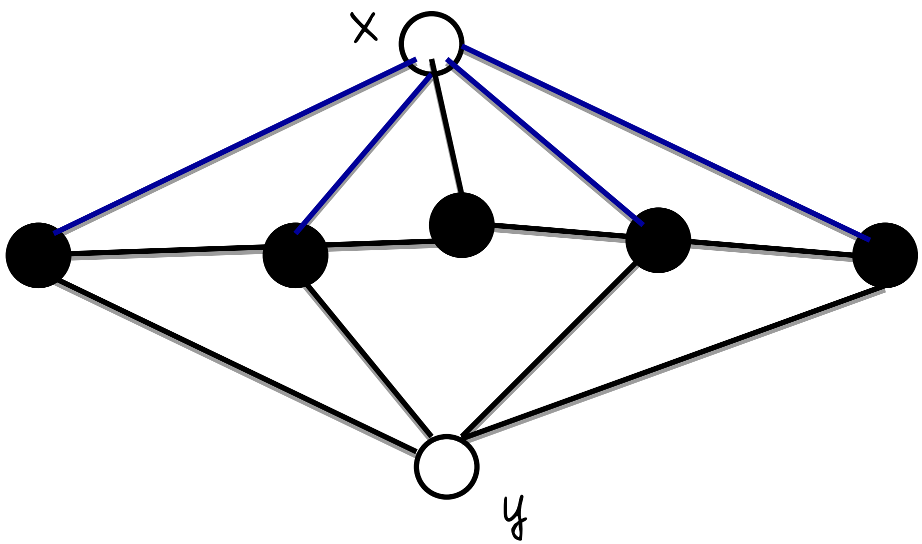
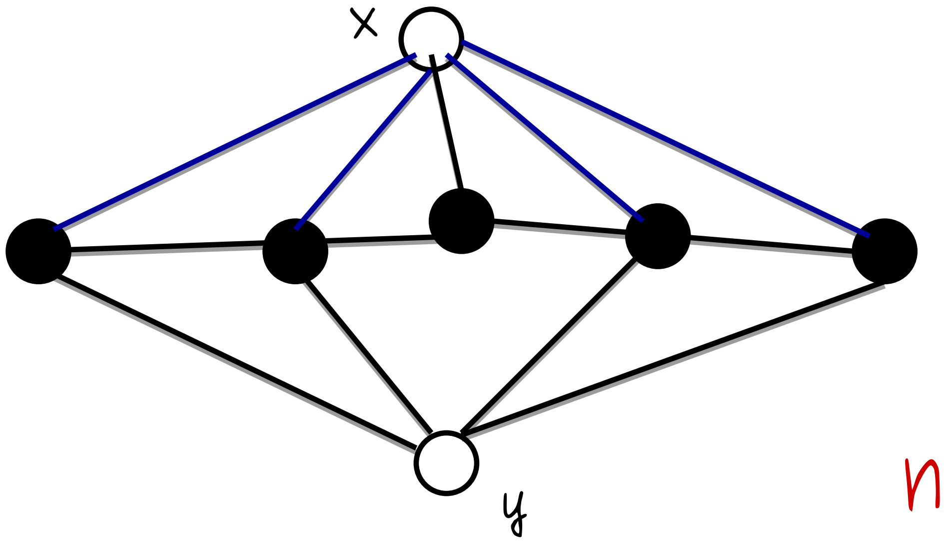


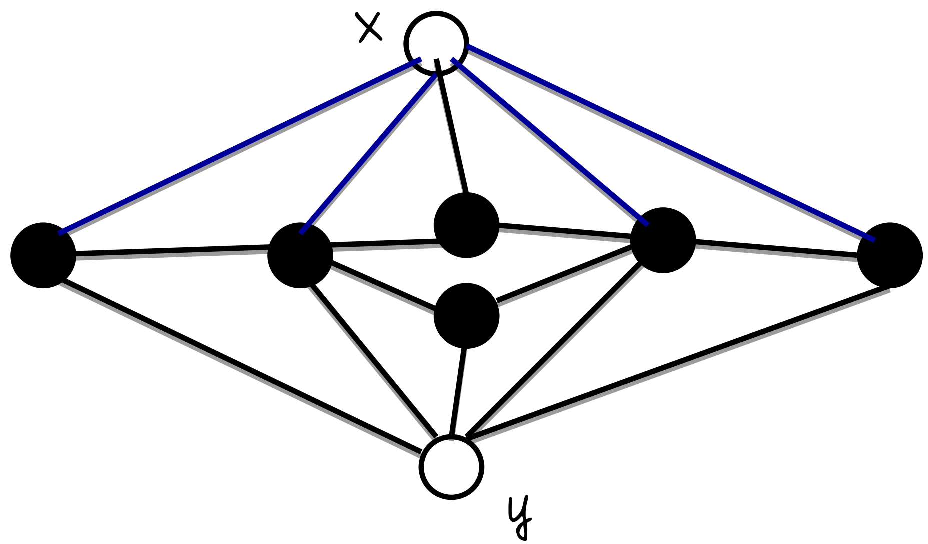
forced in order to avoid F_5

Forbidden configuration (F_4):



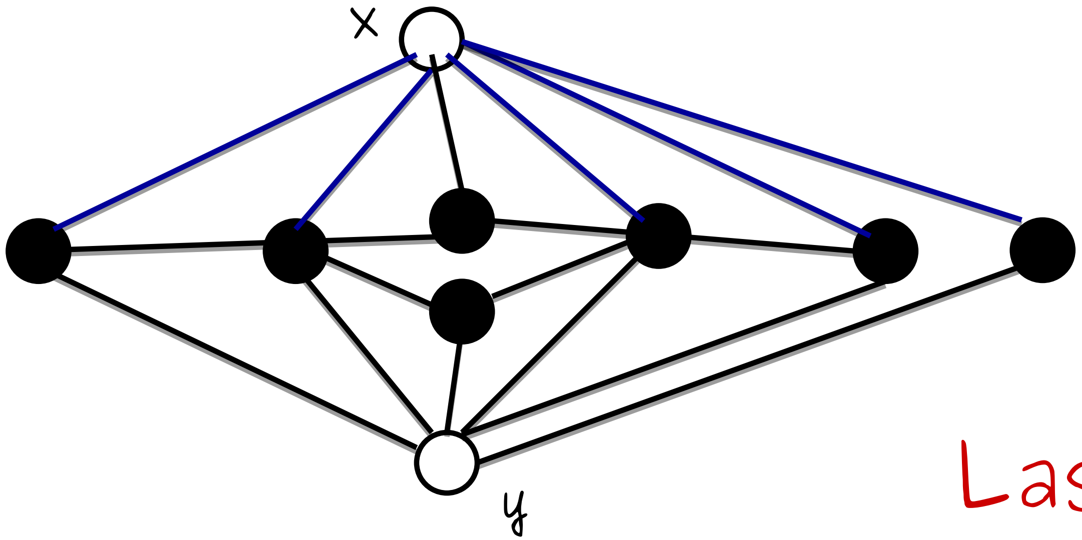
need to place 2 more good vertices

Forbidden configuration (F_4):



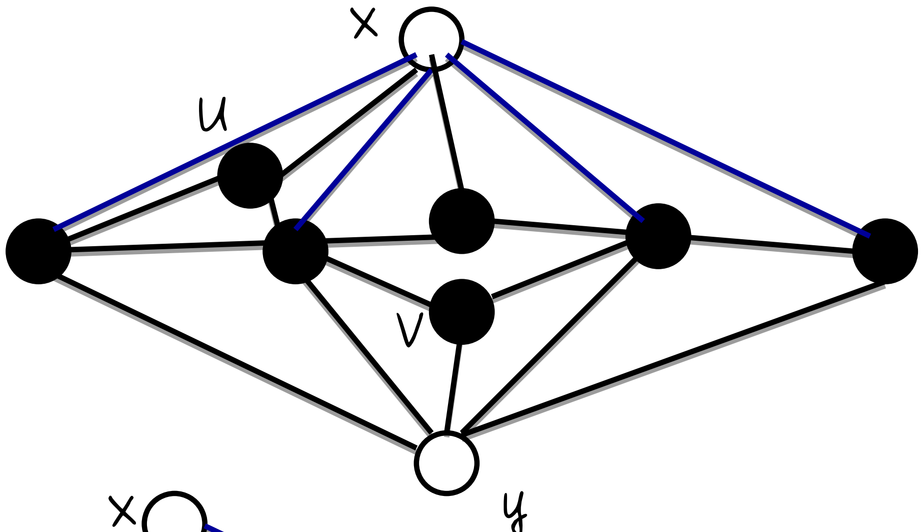
Adding another vertex on same face

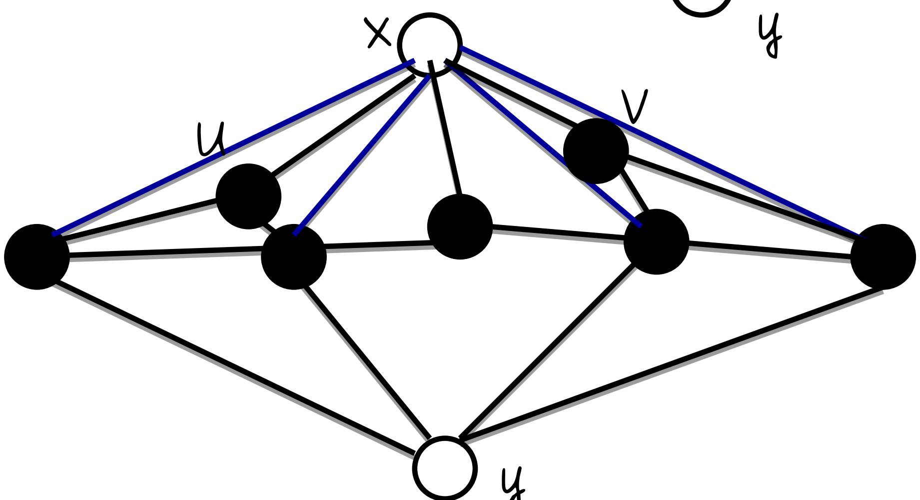
Forbidden configuration (F_4):



Last vertex
on outer face
enforces F_5

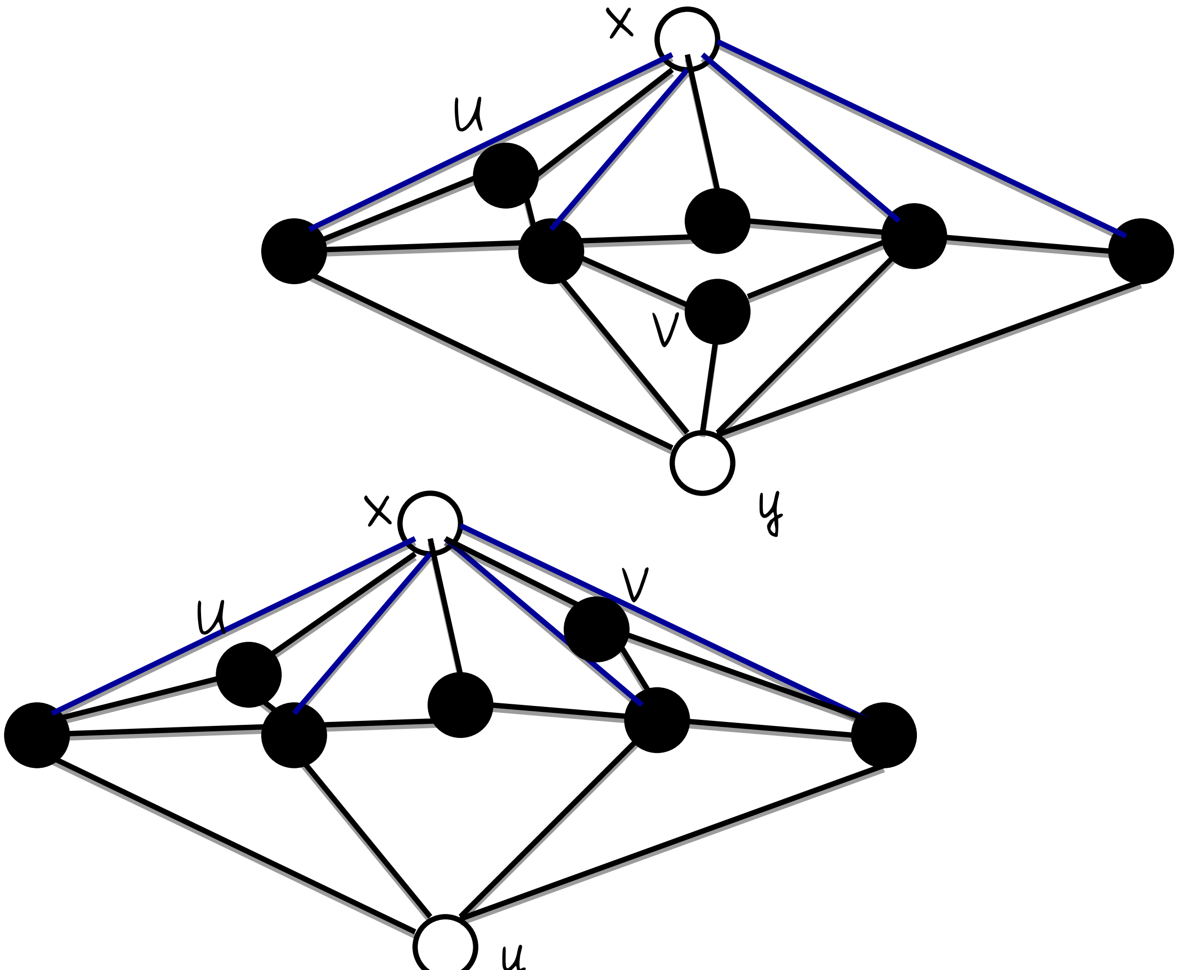
Forbidden configuration (F_4):





the only two possible options

Forbidden configuration (F_4):



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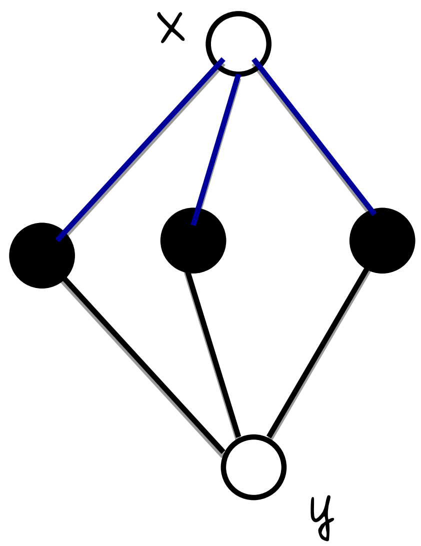
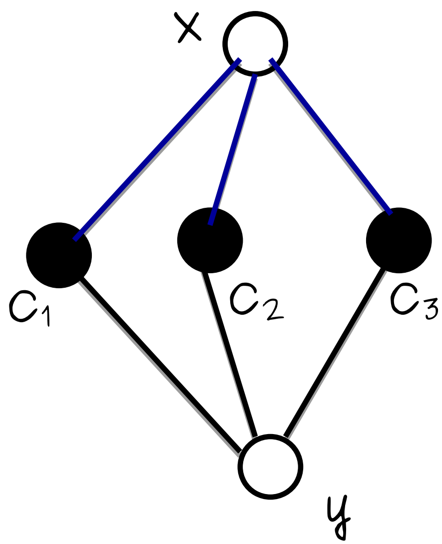


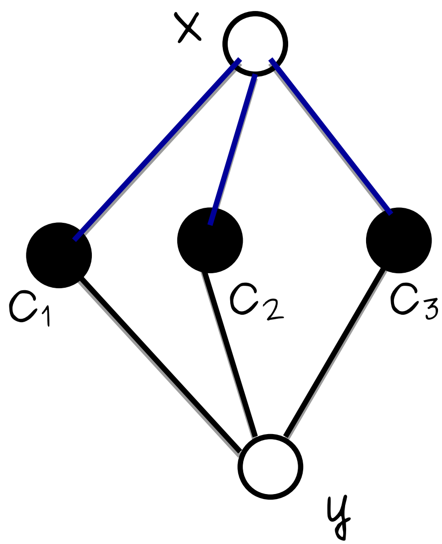
Needs to add 4 good vertices

Forbidden configuration (F_3):



It can be showed c_1, c_2, c_3 must be independent to avoid F_4

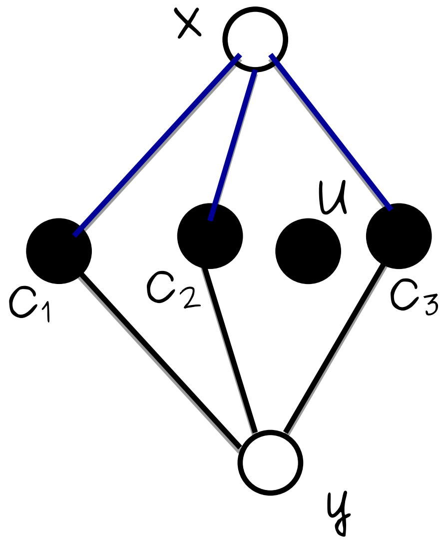
Forbidden configuration (F_3):



Proof requirs complicated case analysis.

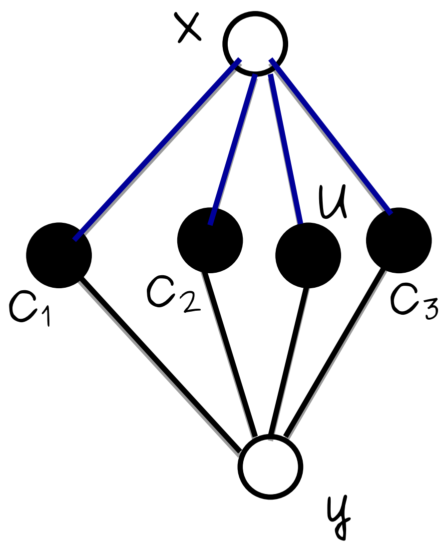
We skip it.

Forbidden configuration (F_3):



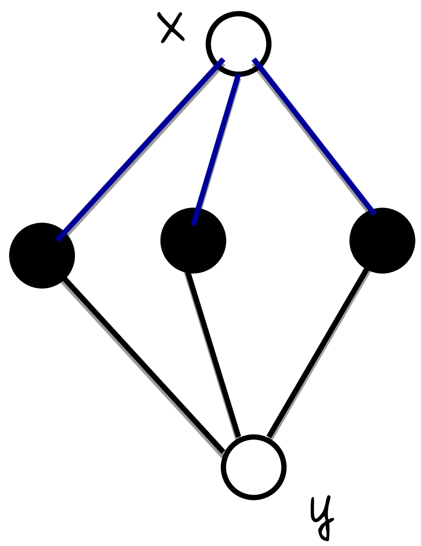
Add u.

Forbidden configuration (F_3):



Enforces to create F_4 to reach c_1

Forbidden configuration (F_3):



F_3 Forbidden

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Proof (sketch)

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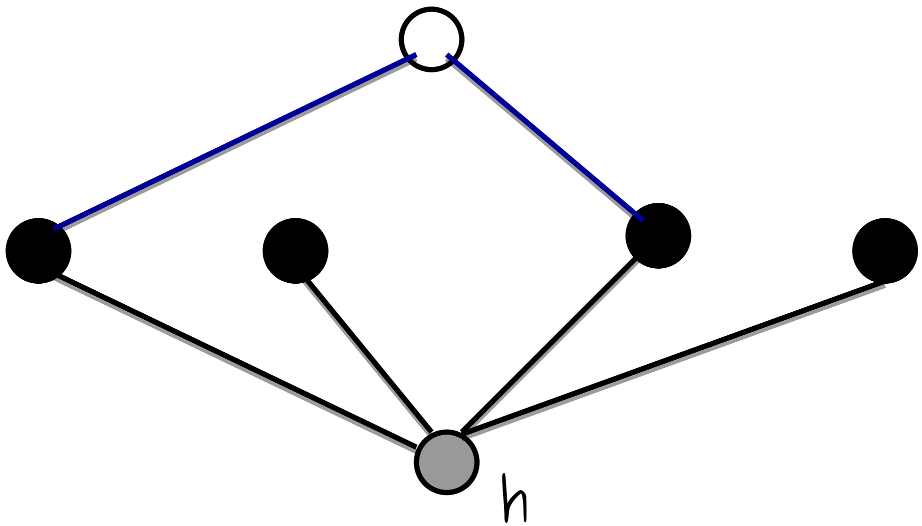
- a helper has $\deg \geq 4$ due to minimality.
- helpers are independent due to minimality.
- no helper implies done (Naserasr, Rollova, Sopena 2015)

Forced:



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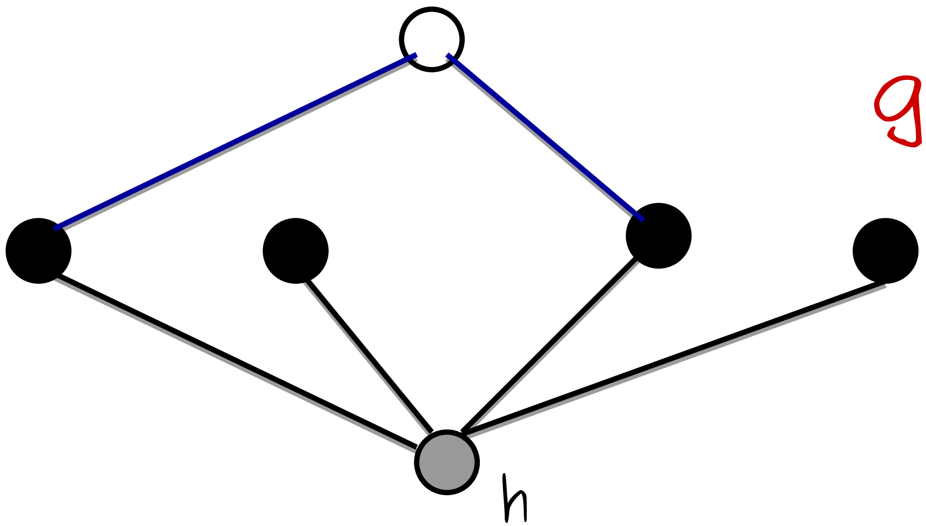

implies F_3 or an unique graph which is not a counter-example

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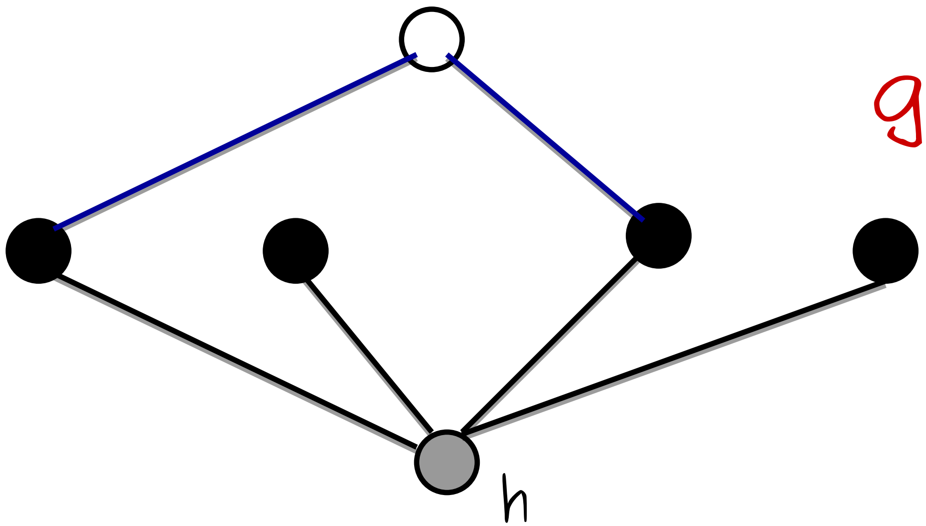

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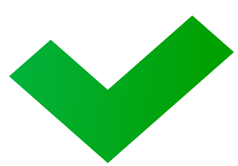

Thank You