On (m,n)-absolute clique number of planar graphs

Susobhan Bandopadhyay

Joint work with Sandip Das, Sagnik Sen & Soumen Nandi

NISER, Bhubaneswar

04/05/2021

This work

This work

Based on the paper:

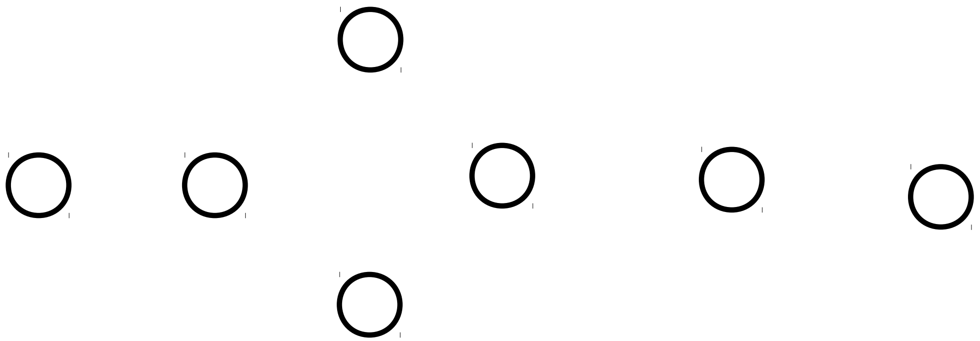
This work

Based on the paper:

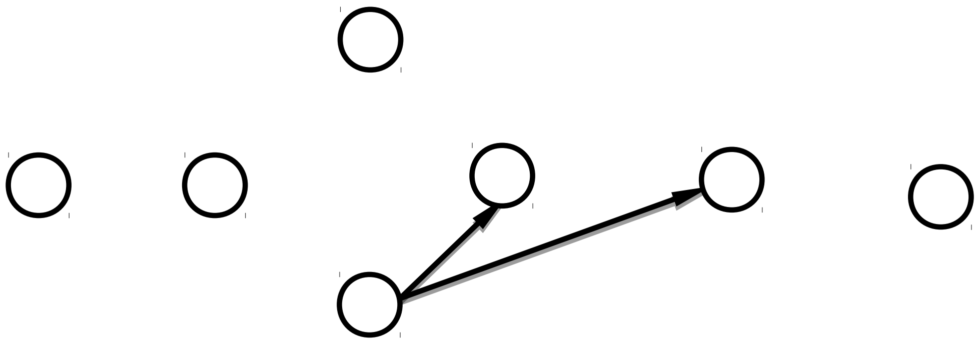
Analogues of Cliques for (m, n)-Colored Mixed Graphs.
Julien Bensmail, Christopher Duffy and Sagnik Sen.
Graphs and Combinatorics (2017) 33(4): 735-750.

Colored mixed graphs

Colored mixed graphs

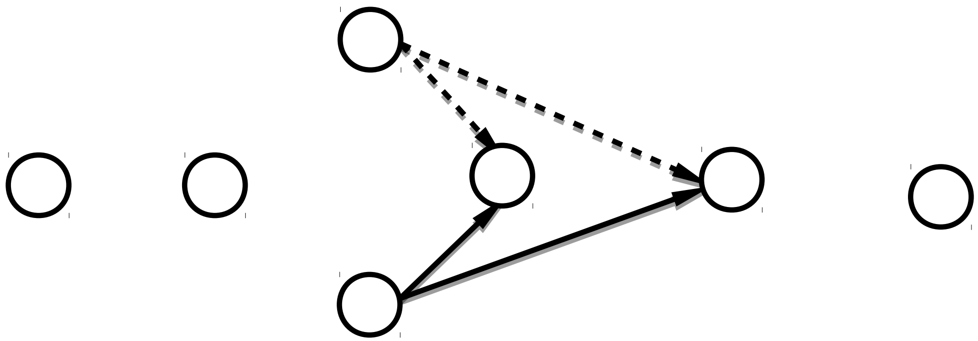


Colored mixed graphs



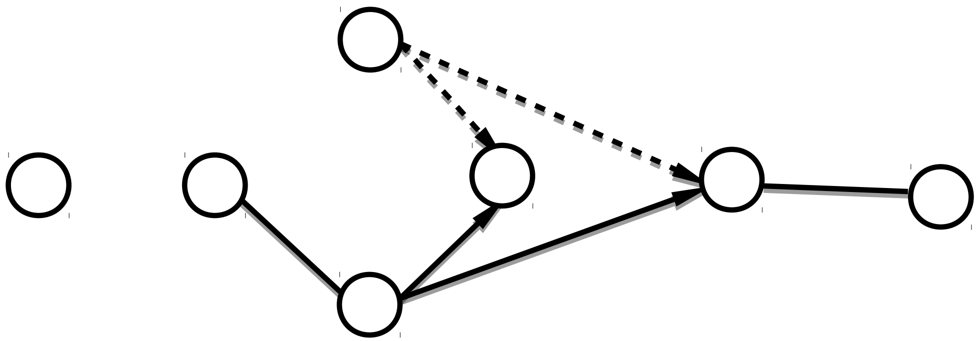
- Type1 ARC 

Colored mixed graphs



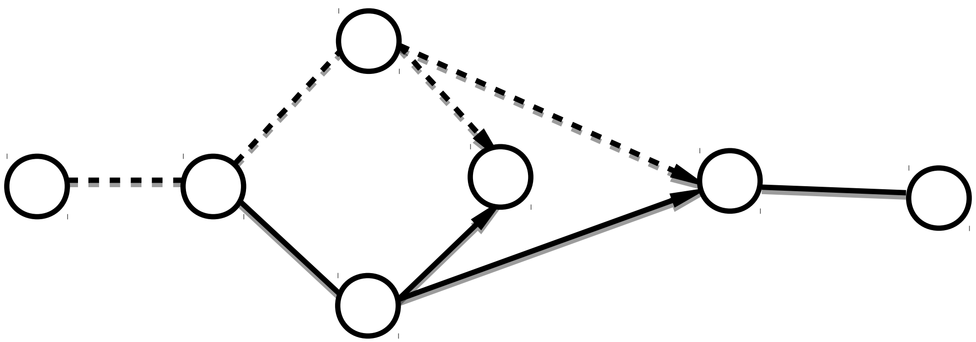
- Type1 ARC 
- Type2 ARC 

Colored mixed graphs



- Type1 ARC
- Type2 ARC
- Type1 EDGE

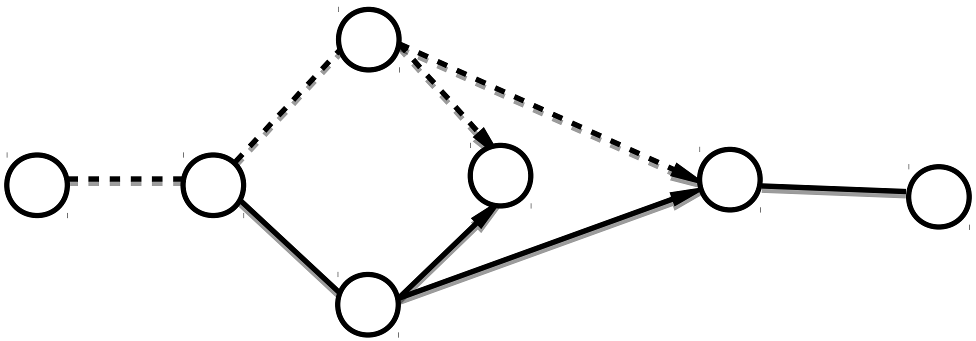
Colored mixed graphs



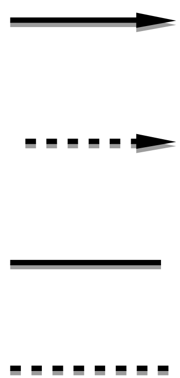
- Type1 ARC
- Type2 ARC
- Type1 EDGE
- Type2 EDGE

2 types of arcs &
2 types of edges...

Colored mixed graphs

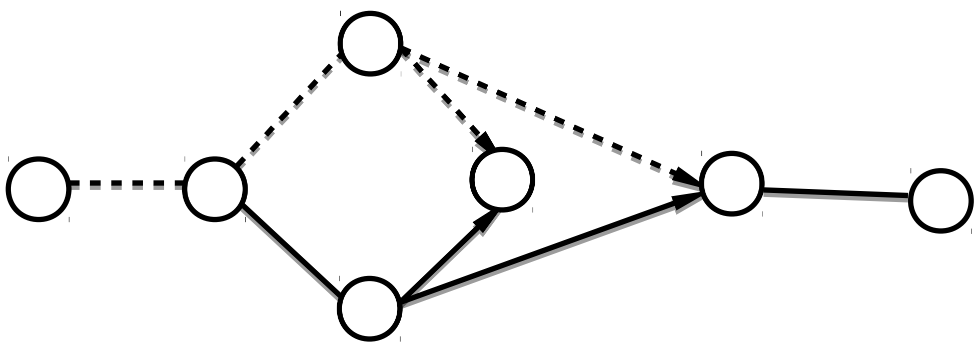


- Type1 ARC
- Type2 ARC
- Type1 EDGE
- Type2 EDGE



(2,2)-colored mixed graph

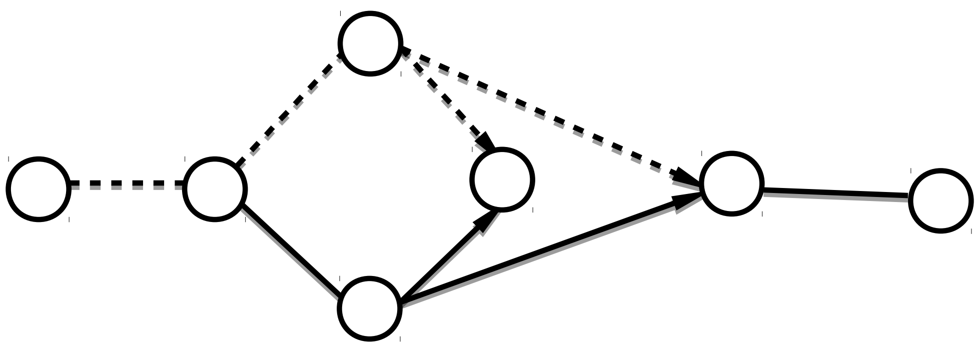
Colored mixed graphs



- Type1 ARC
- Type2 ARC
- Type1 EDGE
- Type2 EDGE

(2,2)-mixed graph

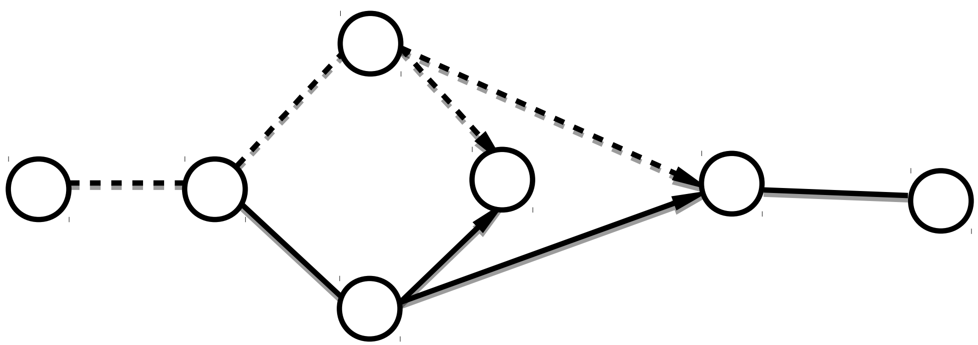
Colored mixed graphs



(m,n)-mixed graph

[generalization by Nešetřil and Raspaud, J. Combin. Theory Ser. B(2000)]

Colored mixed graphs

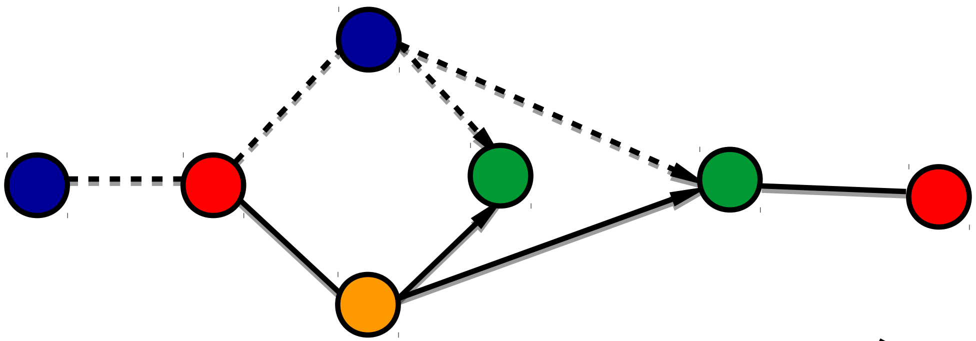


- m types of arcs
- n types of edges

(m,n)-mixed graph

[generalization by Nešetřil and Raspaud, J. Combin. Theory Ser. B(2000)]

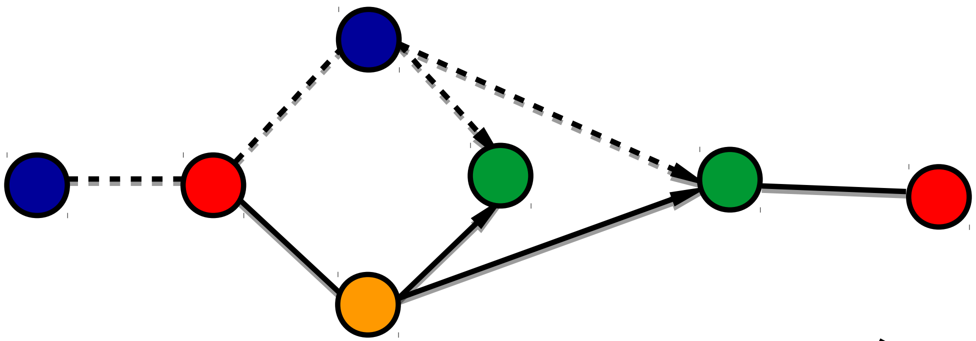
Colored mixed graphs



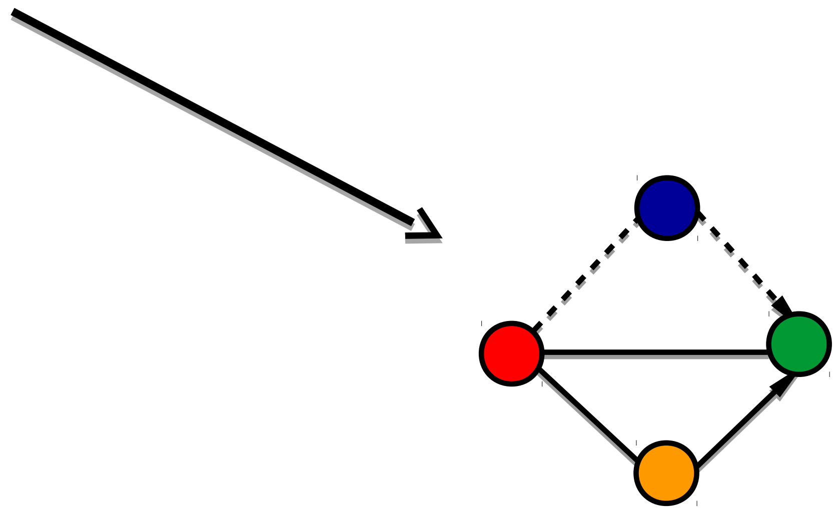
homomorphism?

- Type1 ARC
- Type2 ARC
- Type2 EDGE
- Type2 EDGE

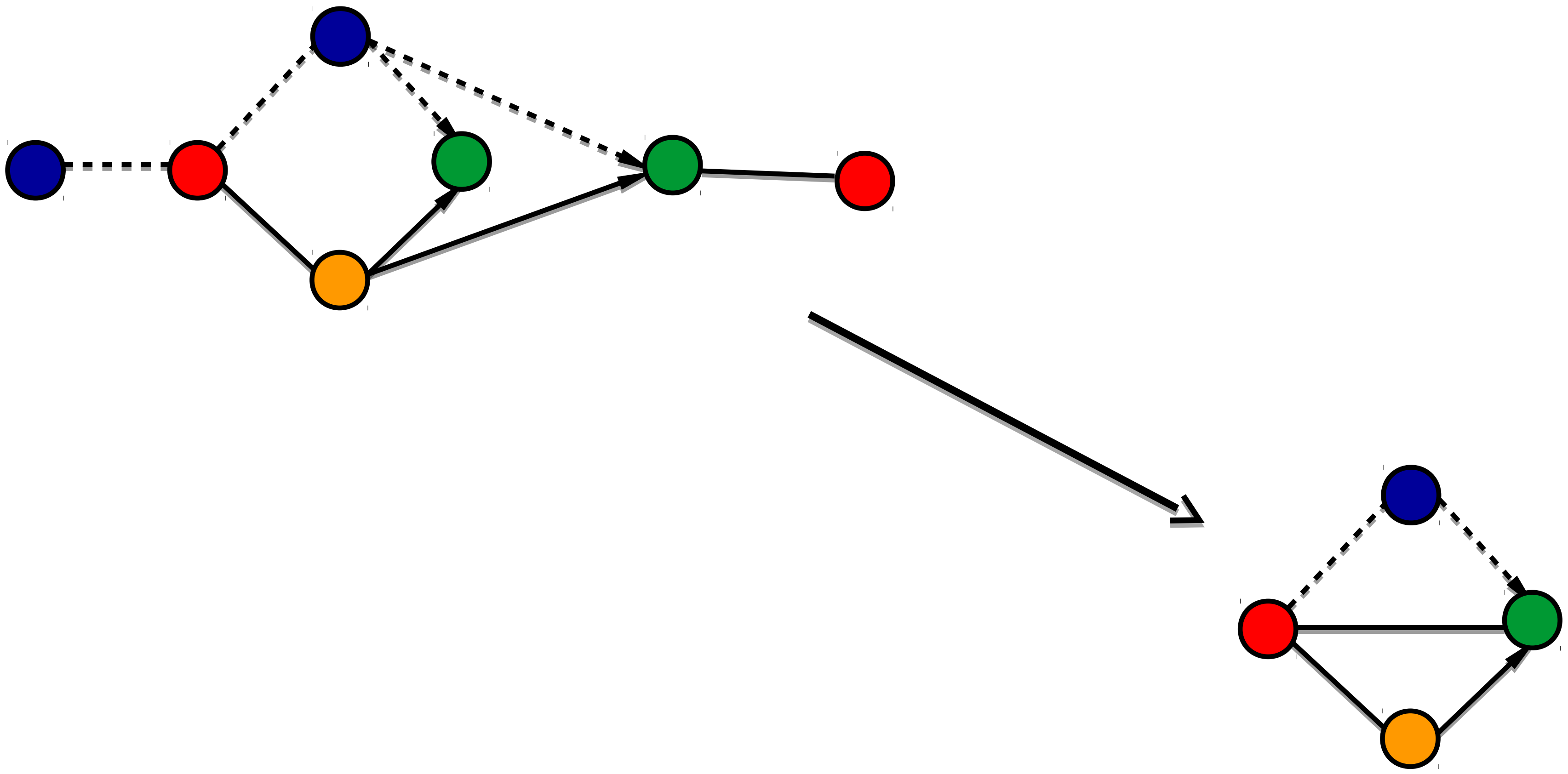
Colored mixed graphs



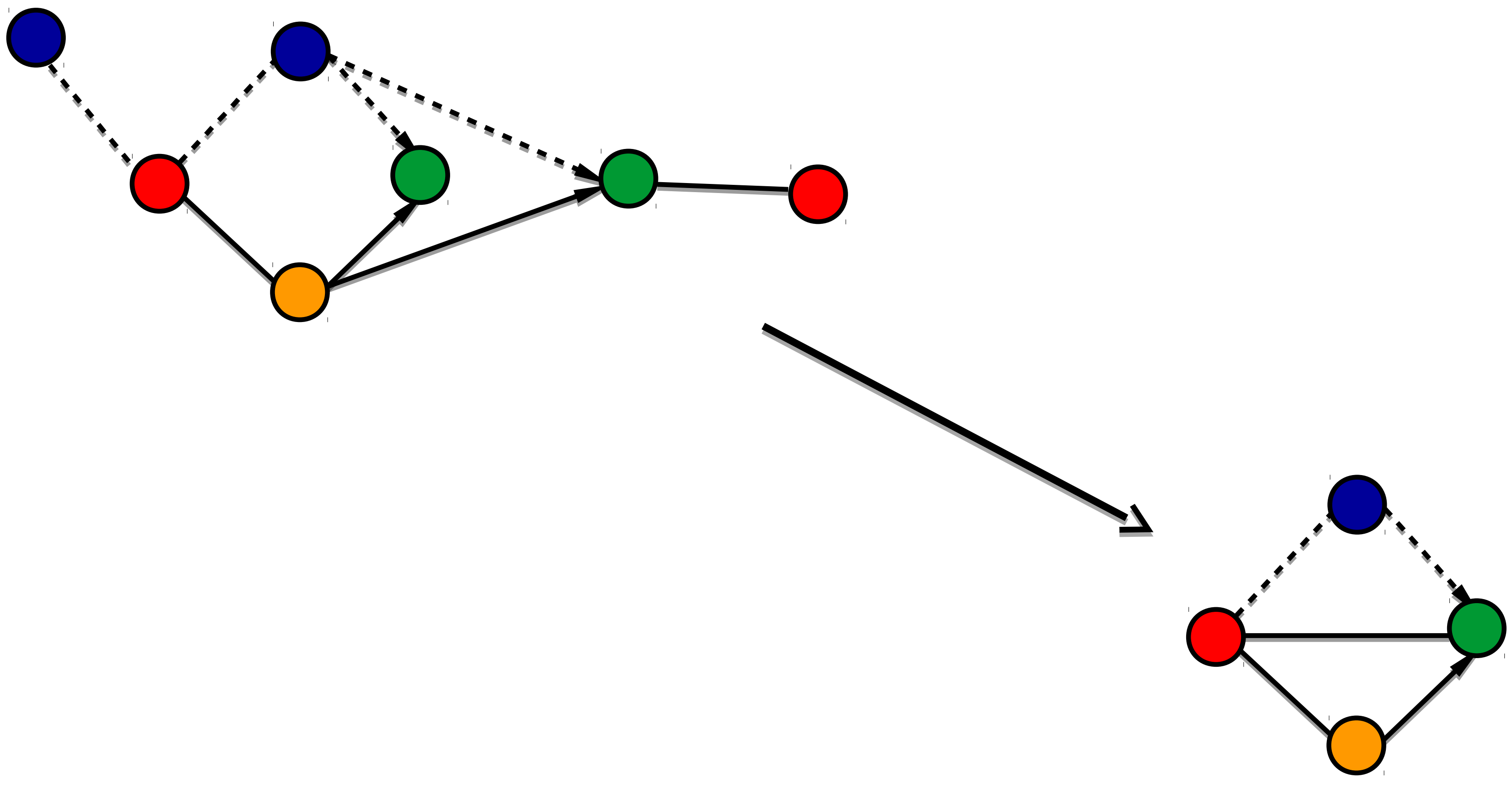
- Type1 ARC
- Type2 ARC
- Type1 EDGE
- Type2 EDGE



Colored mixed graphs



Colored mixed graphs



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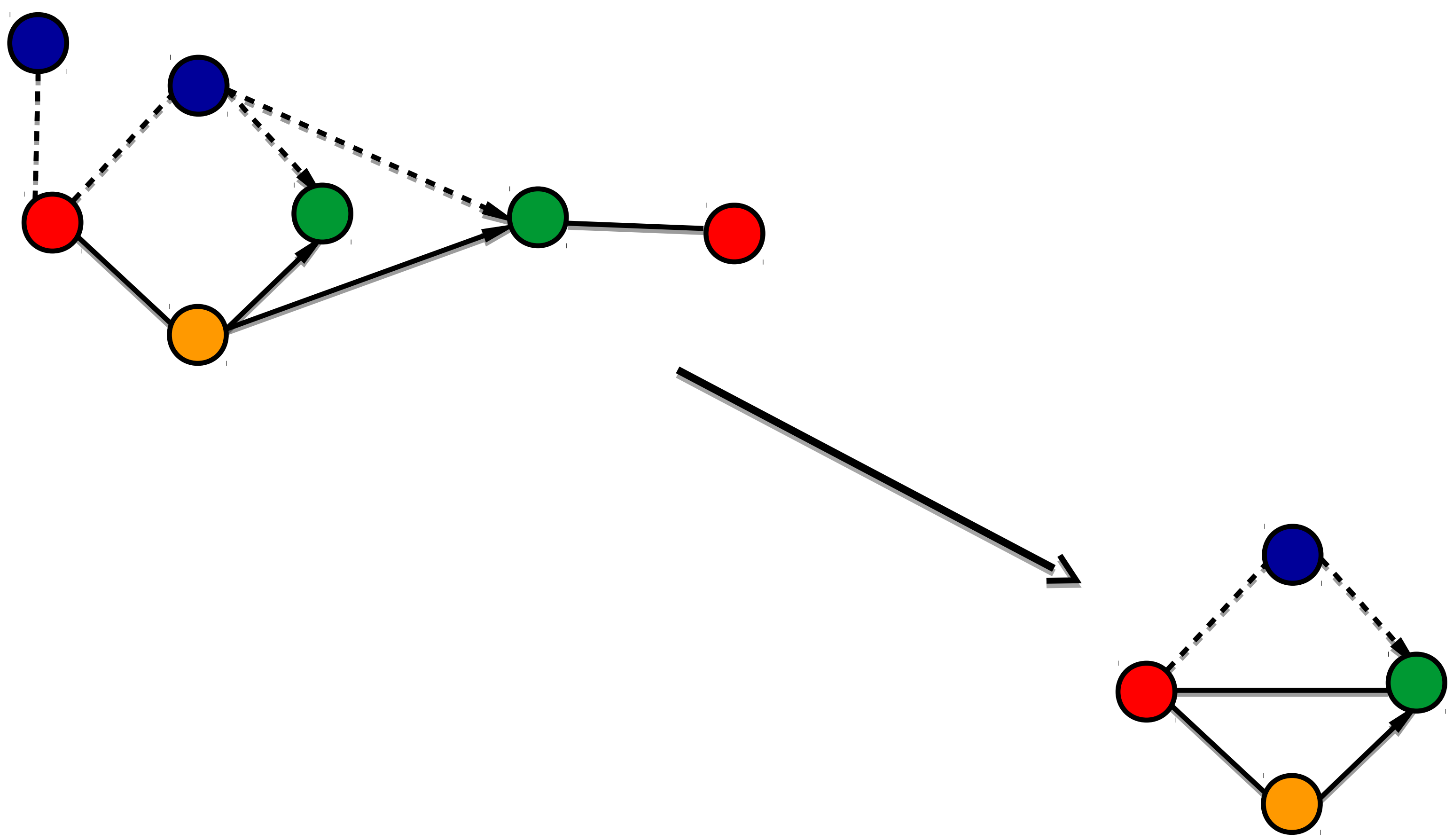

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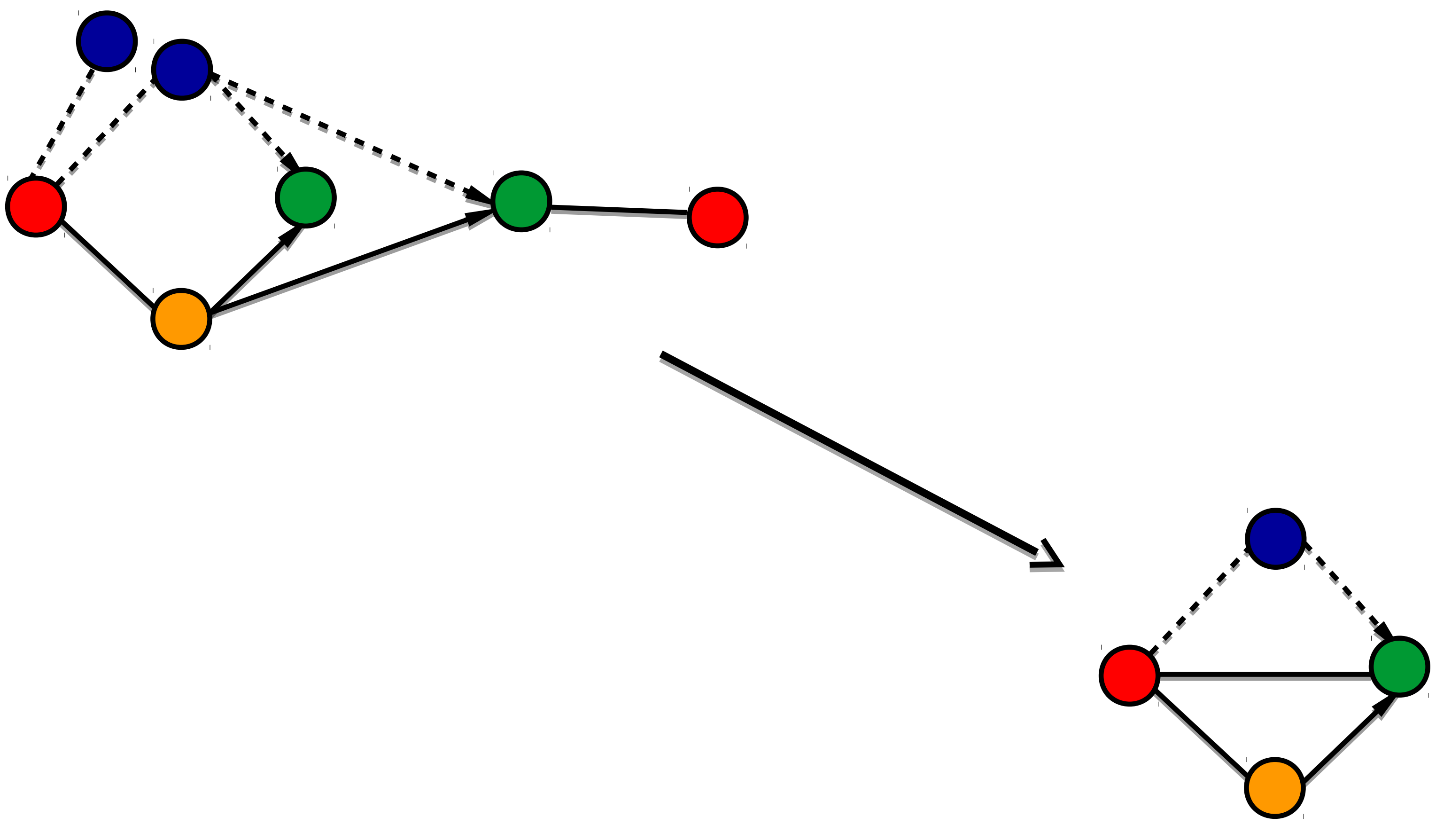

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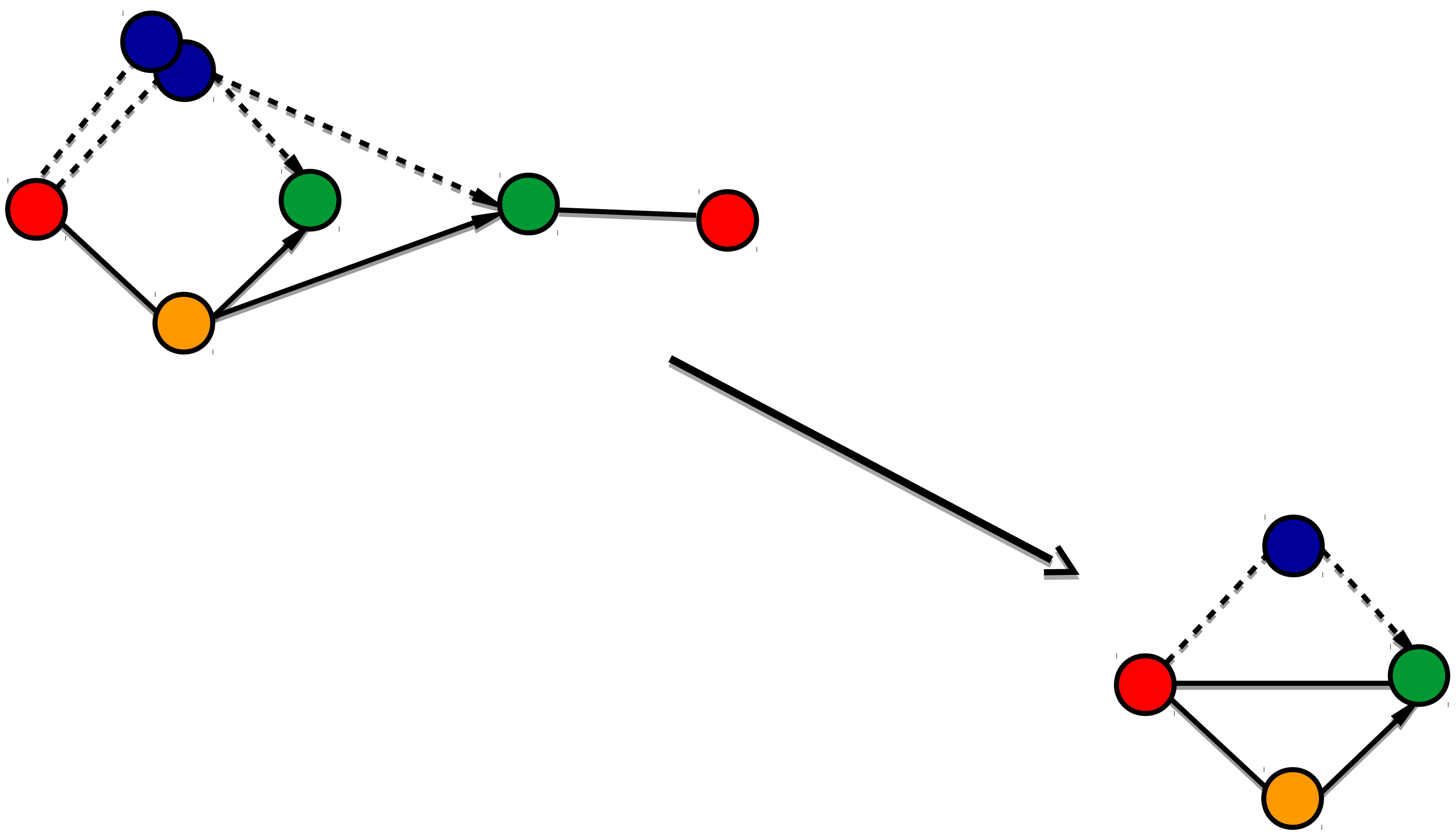
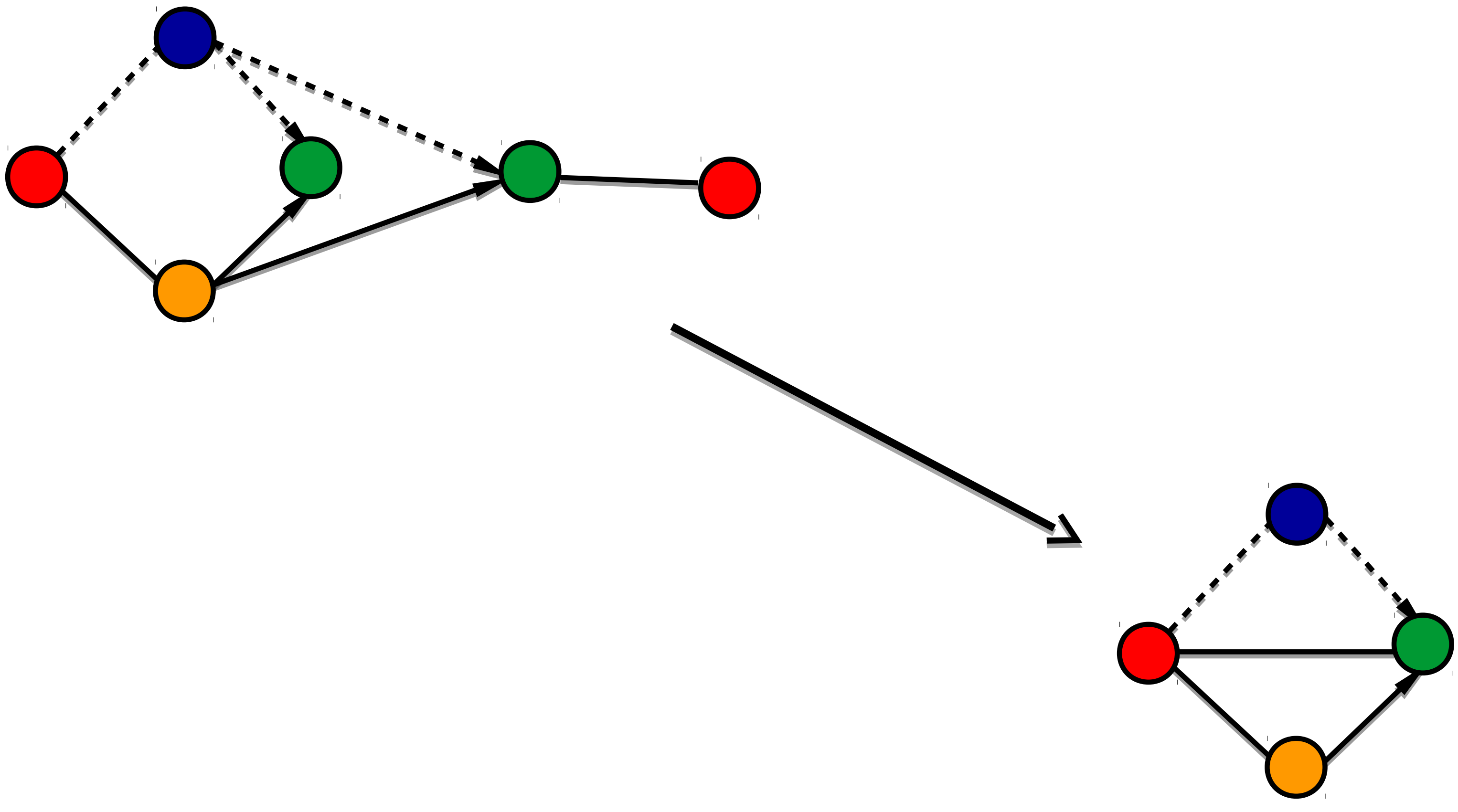


Colored mixed graphs



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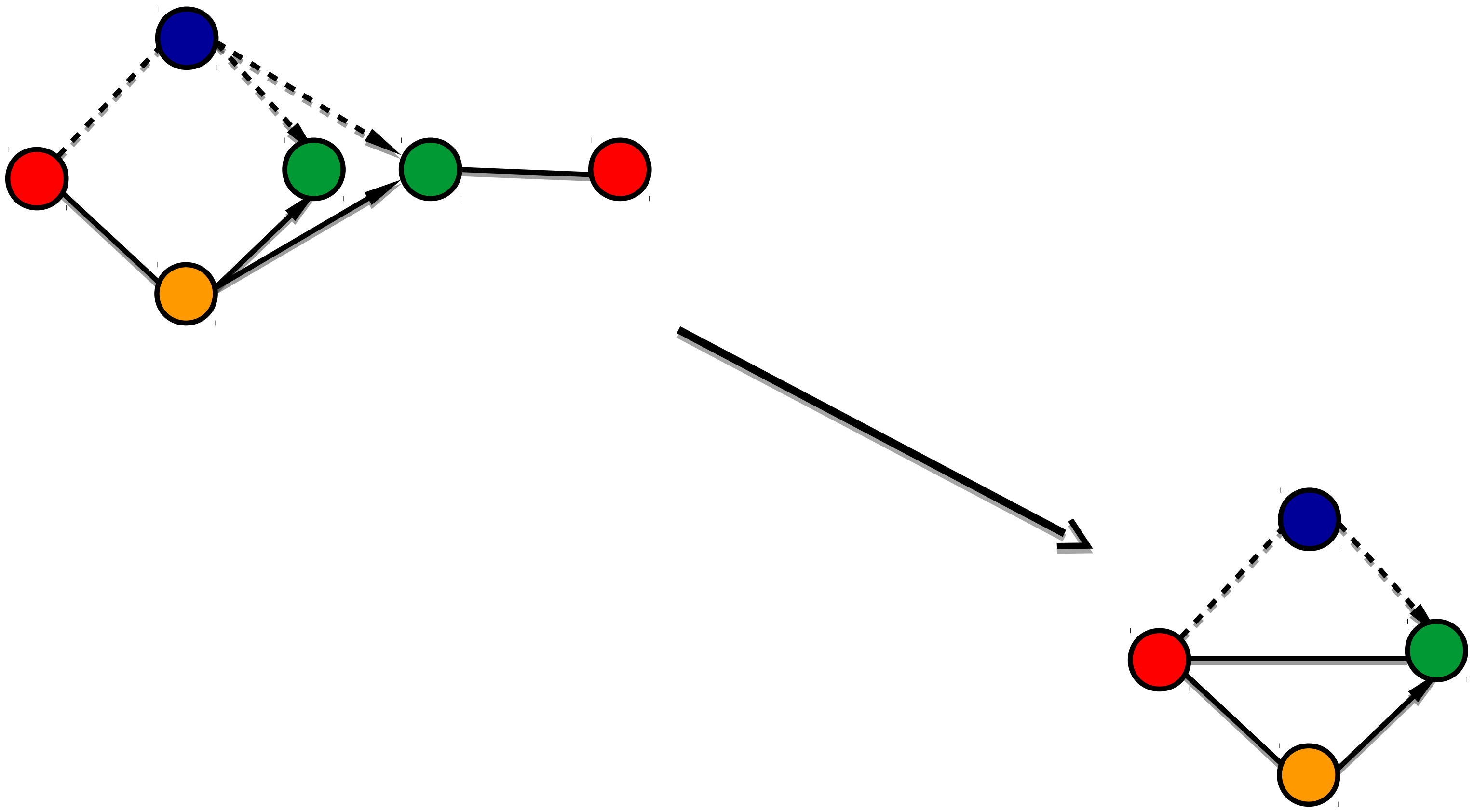

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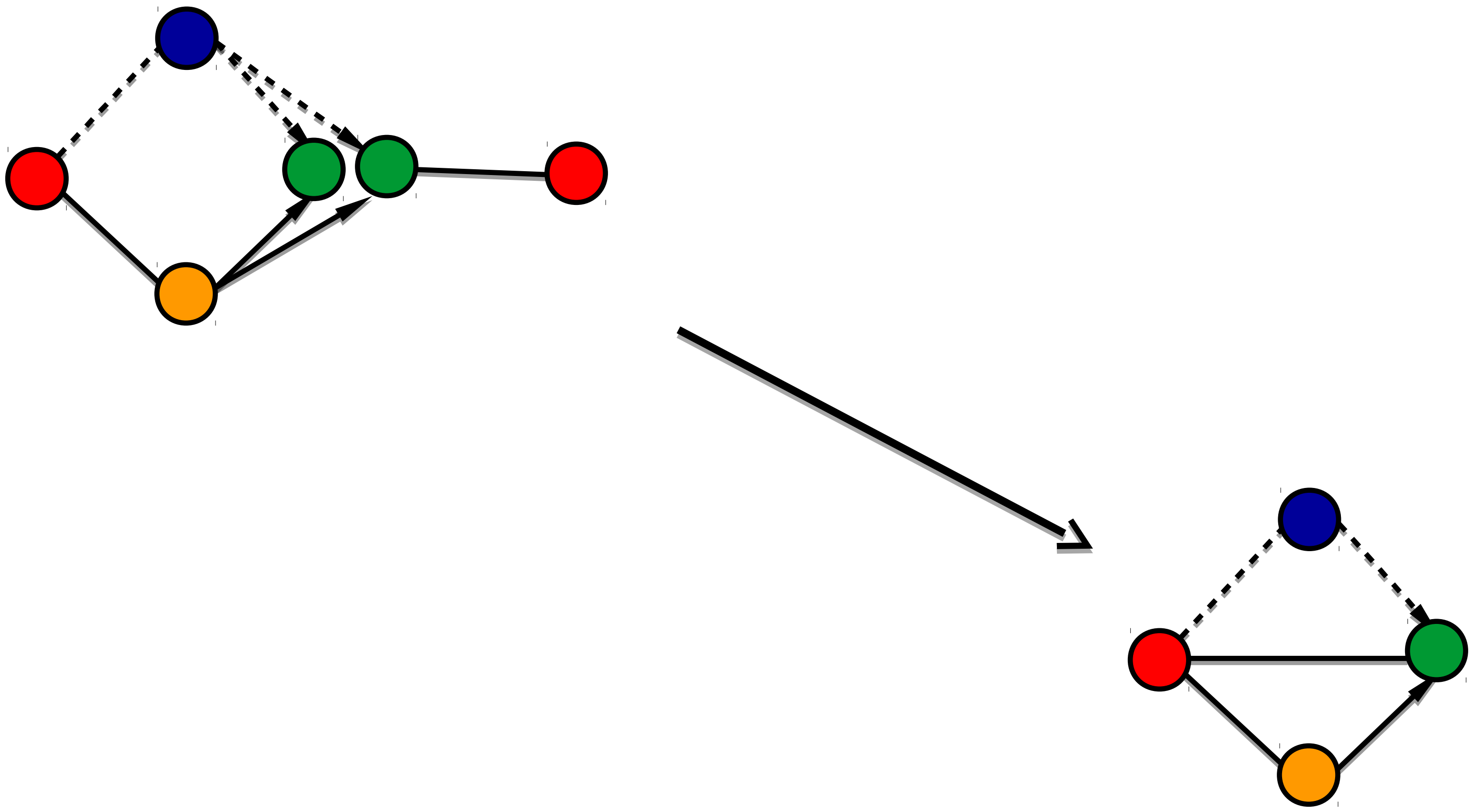

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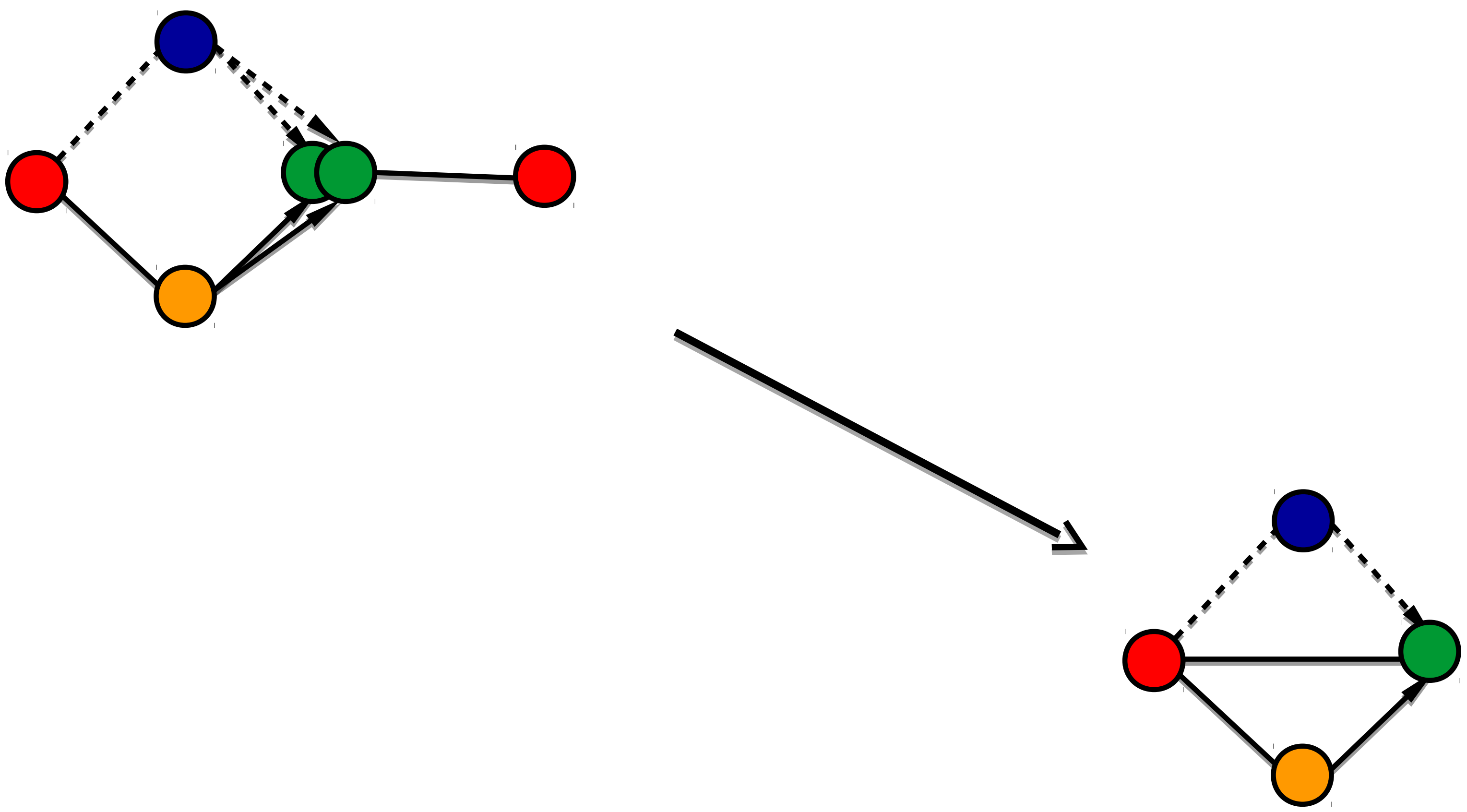
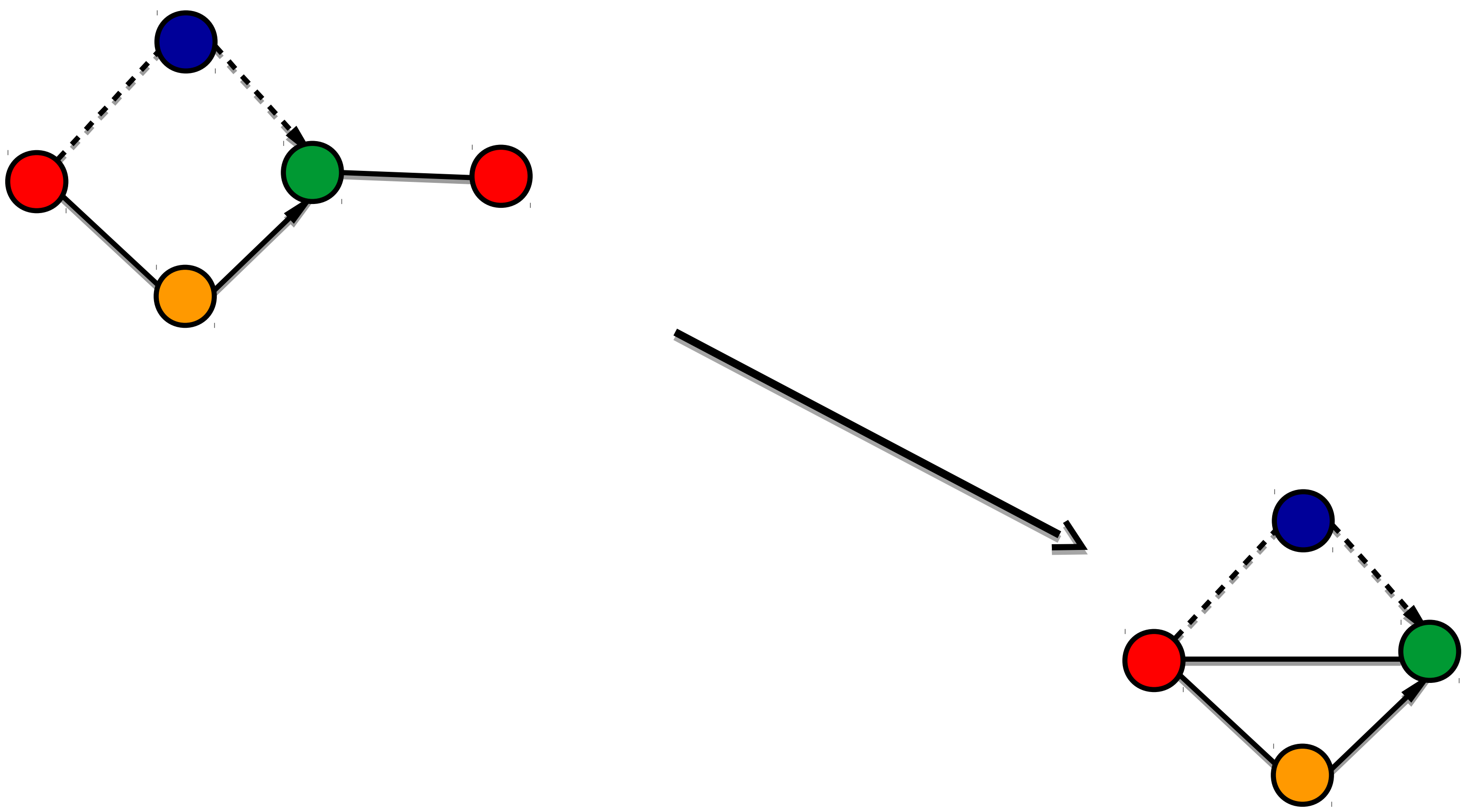


Colored mixed graphs



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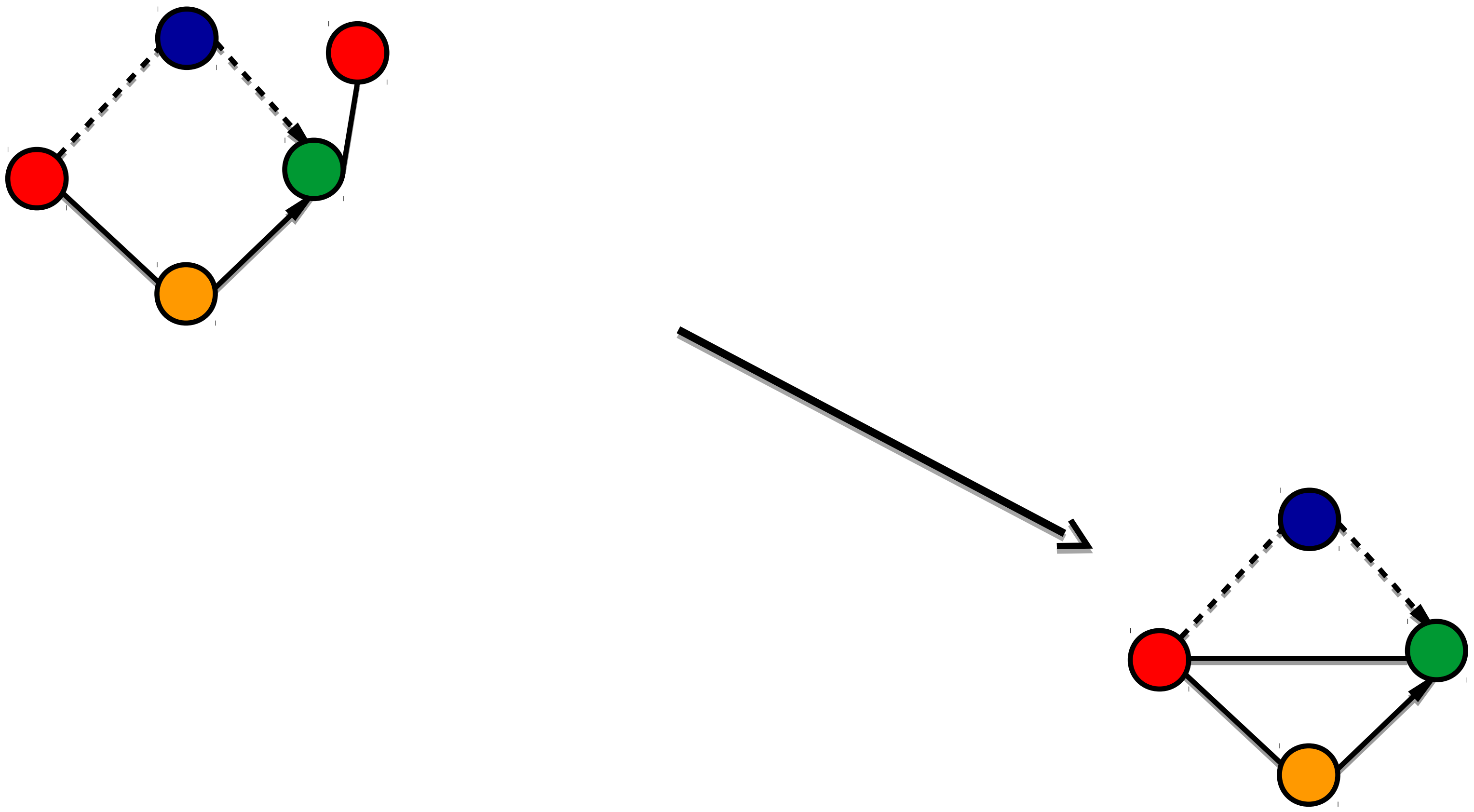

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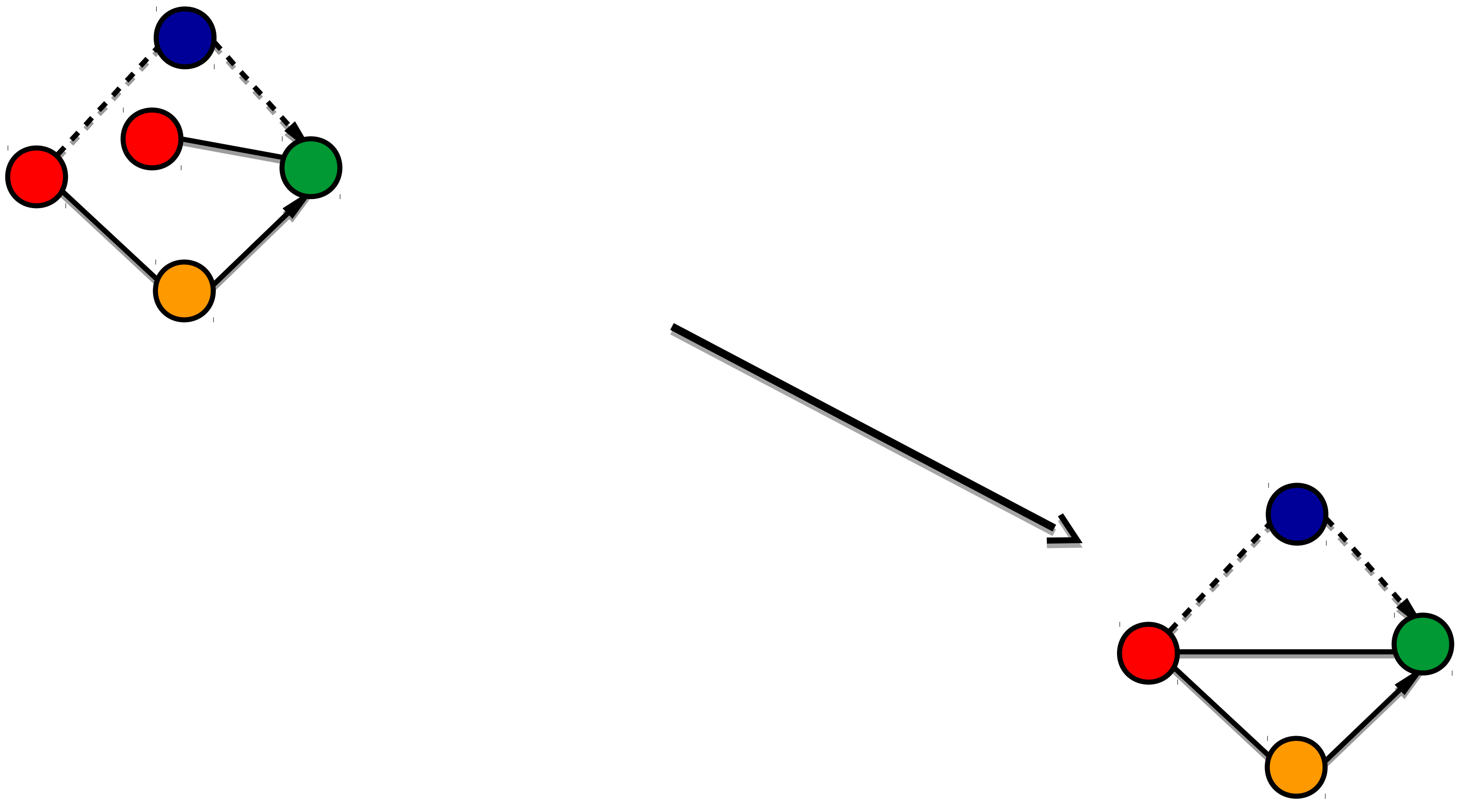
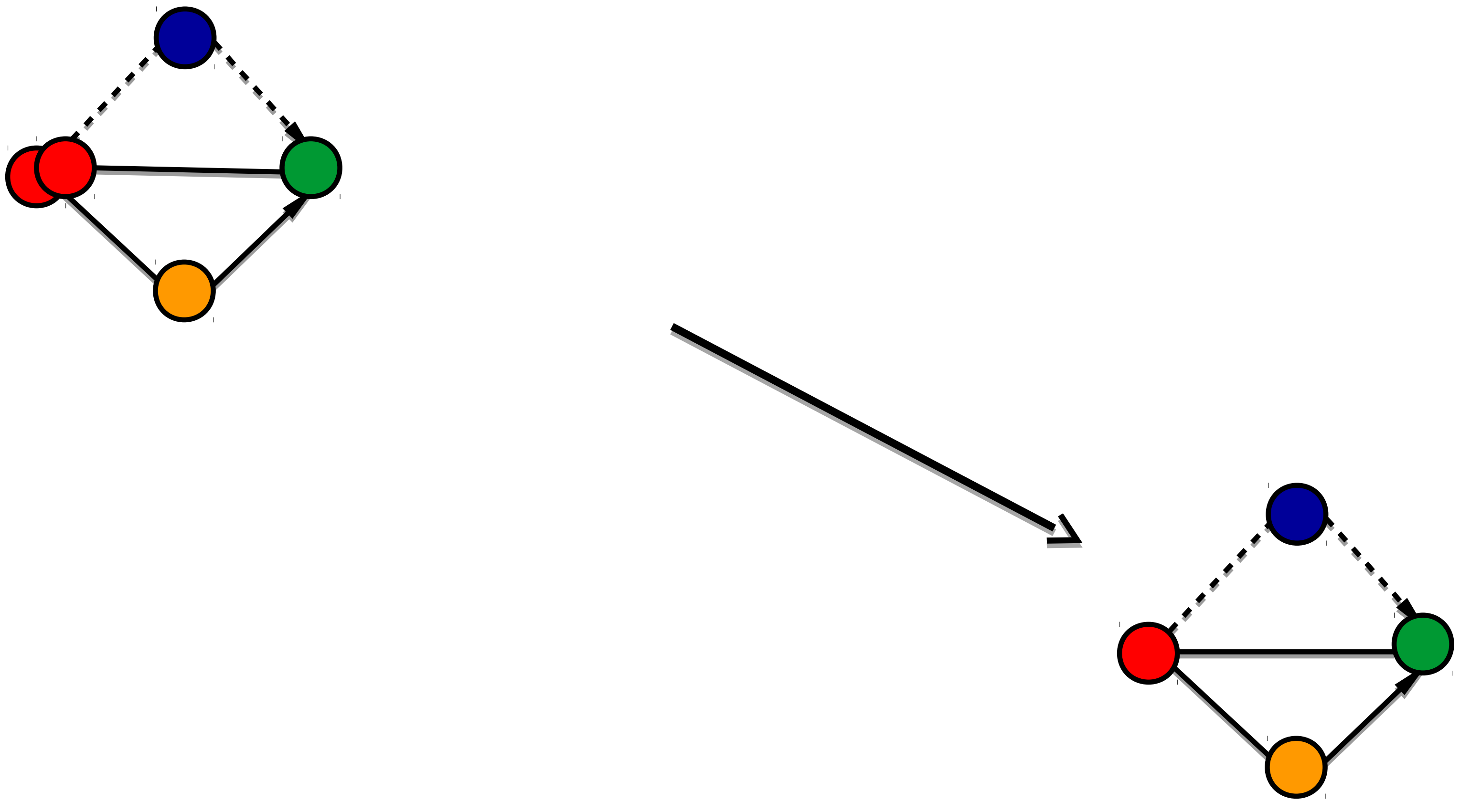
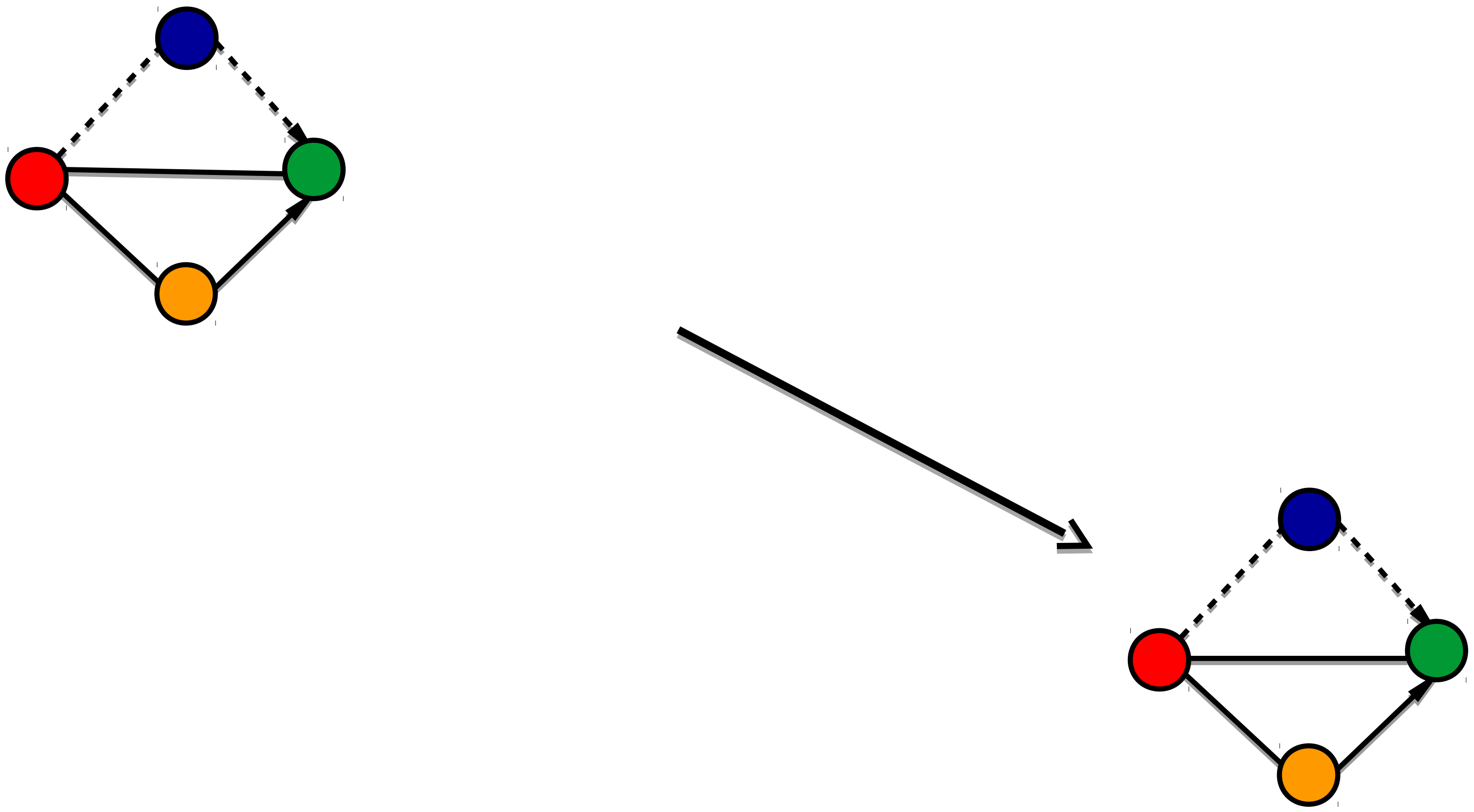


Colored mixed graphs

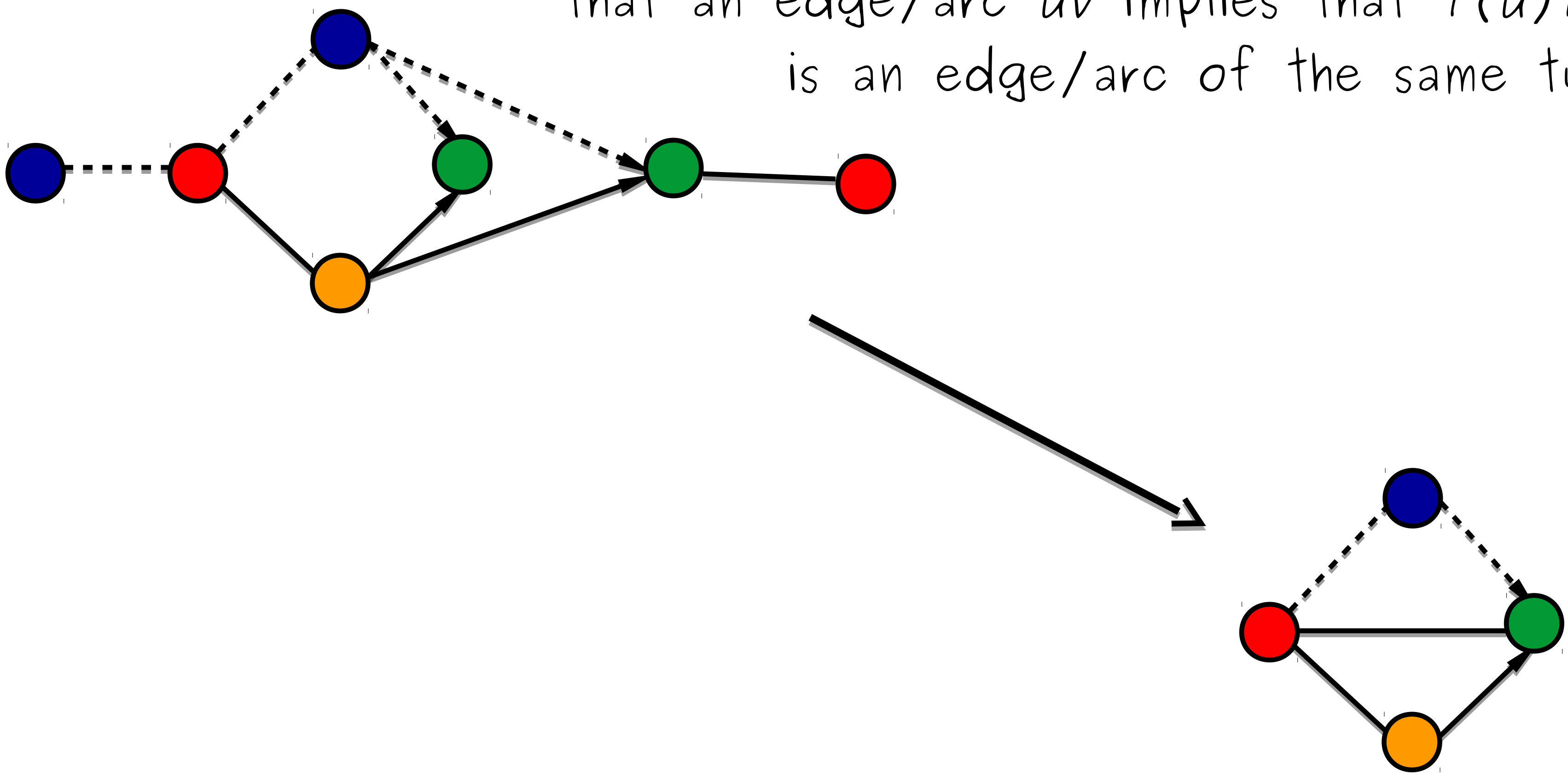


Colored mixed graphs



Colored mixed graphs

homomorphism: a vertex mapping f such that an edge/arc uv implies that $f(u)f(v)$ is an edge/arc of the same type.



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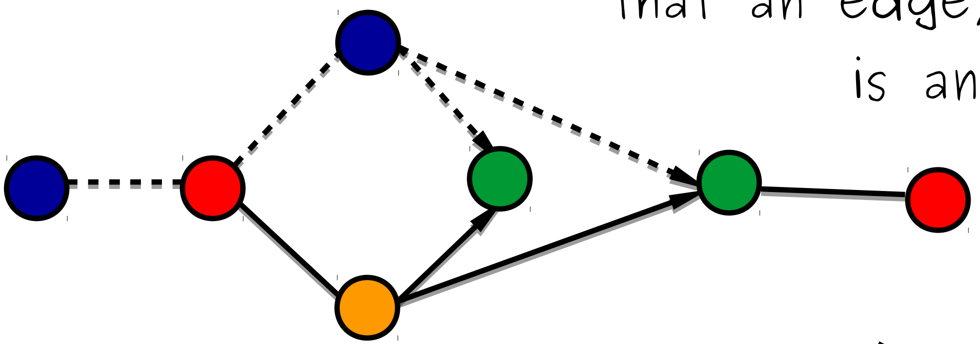

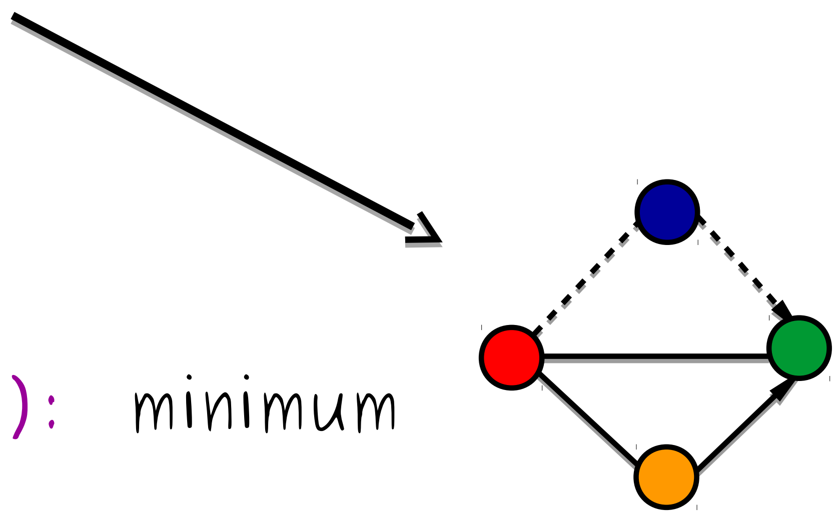

chromatic number ($\chi_{(m,n)}$): minimum $|H|$ such that $G \to H$.

Colored mixed graphs

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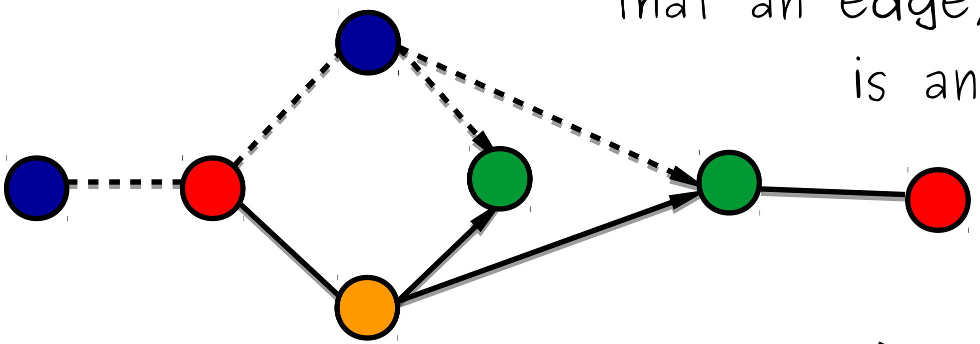
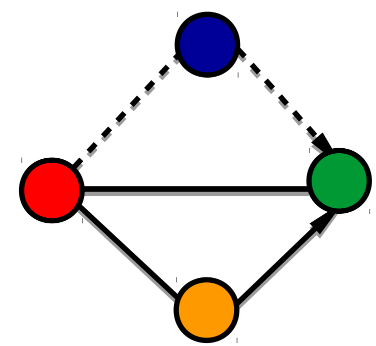


*vertex images=colors

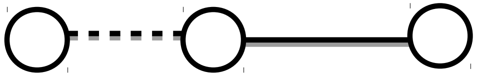


chromatic number ($\chi_{(m,n)}$): minimum $|H|$ such that $G \to H$.

Special 2-path uvw

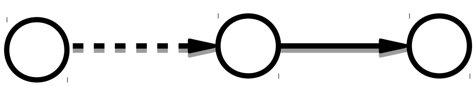
Special 2-path uvw

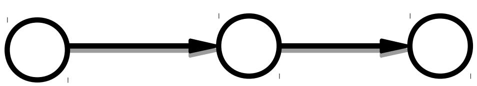
- uv, vw different types of edges
-
-
-
-



Special 2-path uvw

- uv, vw different types of edges
- uv, vw are arcs (maybe same type)
-
-
-





Special 2-path uvw

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- uv, wv are different types of arcs
-
-

Special 2-path uvw

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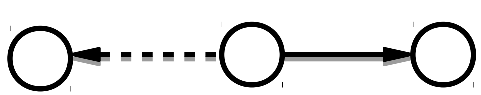
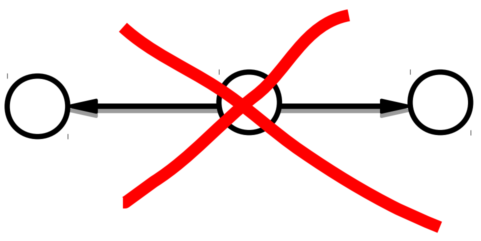
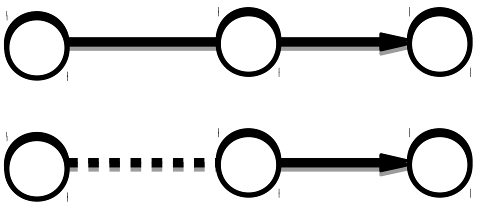
- uv, wv are different types of arcs
- vu, vw are different types of arcs
-

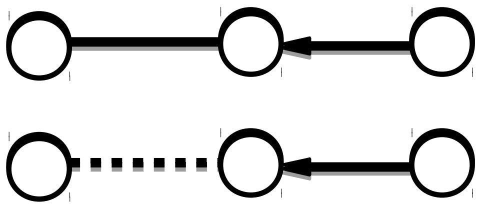




Special 2-path uvw

- uv, vw different types of edges
- uv, vw are arcs (maybe same type)
- uv, wv are different types of arcs
- vu, vw are different types of arcs
- in uv, vw exactly one is an edge





Absolute (m,n)-clique

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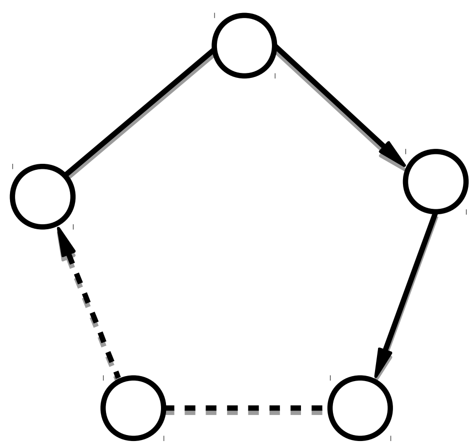
(m,n)-mixed graph whose non-adjacent vertices are connected by a special 2-path



Absolute (m,n)-clique

(m,n)-mixed graph whose non-adjacent vertices are connected by a special 2-path

$\omega_{a(m,n)}(F) = \max_{C \in F}\{\#\text{vertices in } C : C \text{ is } (m,n)\text{-clique}\}$

Known related results

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Theorem (Bensmail, Duffy and Sen, Graphs Combin., 2017): For the family $\mathcal{O}$ of outerplanar graphs

$$\omega_{a(m,n)}(\mathcal{O}) = 3(2m+n)+1, \text{ for all } (m,n) \neq (0,1).$$

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Theorem (Bensmail, Duffy and Sen, Graphs Combin., 2017): For the family $\mathcal{P}$ of planar graphs

$$3(2m+n)^2+(2m+n)+1 \leq \omega_{a(m,n)}(\mathcal{P}) \leq 9(2m+n)^2+2(2m+n)+2, \text{ for all } (m,n)\neq(0,1).$$

Known related results

Conjecture (Bensmail, Duffy and Sen, Graphs Combin., 2017):
For the family P of planar graphs

$$\omega_{a(m,n)}(P) = 3(2m+n)^2+(2m+n)+1, \text{ for all } (m,n)\neq(0,1).$$

Our result (Bandopadhyay, Das, Naandi and Sen)

Here we settle the conjecture.

Our result (Bandopadhyay, Das, Nandi and Sen)

Theorem: For the family P of planar graphs $\omega_{a(m,n)}(P) = 3(2m+n)^2+(2m+n)+1$, for all $(m,n)\neq(0,1)$.

time for proof...

Theorem: For the family P of planar graphs $\omega_{a(m,n)}(P) = 3(2m+n)^2+(2m+n)+1$, for all $(m,n)\neq(0,1)$.

Proof (sketch)

Theorem: For the family P of planar graphs
$$\omega_{a(m,n)}(P) = 3(2m+n)^2+(2m+n)+1 \text{ , for all } (m,n)\neq(0,1).$$

Proof (sketch)

- We will use the notion of Dominating Set(D) and *Domination Number* in this proof.

Theorem: For the family P of planar graphs $\omega_{a(m,n)}(P) = 3(2m+n)^2+(2m+n)+1$, for all $(m,n) \neq (0,1)$.

Proof (sketch)

- We will use the notion of Dominating Set(D) and Domination Number in this proof.
- Goddard and Henning(J. Graph. Th., 2002) showed that any planar graph with at least 10 vertices and diameter 2 has domination number at most 2.

Theorem: For the family P of planar graphs $\omega_{a(m,n)}(P) = 3(2m+n)^2+(2m+n)+1$, for all $(m,n)\neq(0,1)$.

Proof (sketch)

- We will use the notion of Dominating Set(D) and *Domination Number* in this proof.
- Goddard and Henning(J. Graph. Th., 2002) showed that any planar graph with at least 10 vertices and diameter 2 has domination number at most 2.
- We have to proof the theorem for domination number 1 and 2.

Theorem: For the family $\mathcal{P}$ of planar graphs $\omega_{a(m,n)}(\mathcal{P}) = 3(2m+n)^2+(2m+n)+1$, for all $(m,n)\neq(0,1)$.

Proof (sketch)

- We adapt the technique of Nandy, Sen and Sopena(J. Graph Th., 2016) for our proof.

Theorem: For the family $\mathcal{P}$ of planar graphs

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Proof (sketch)

- We adapt the technique of Nandy, Sen and Sopena(J. Graph Th., 2016) for our proof.

Lemma: For triangulated planar (m,n)-absolute clique G with domination number 1

$$\omega_{a(m,n)}(G) \leq 3(2m+n)^2+(2m+n)+1,$$

for all $(m,n)\neq(0,1)$.

$3p+1$

Theorem: For the family P of planar graphs
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Proof (sketch)

Theorem: For the family P of planar graphs $\omega_{a(m,n)}(P) = 3(2m+n)^2+(2m+n)+1$, for all $(m,n) \neq (0,1)$.

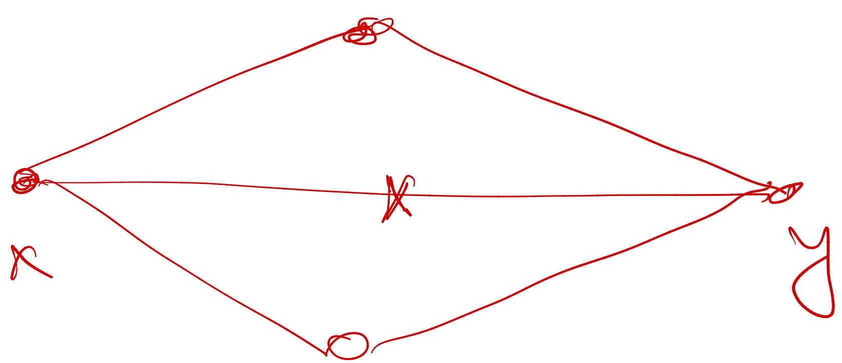
Proof (sketch)

For triangulated planar (m, n)-absolute clique G with domination number 2, Theorem: For the family P of planar graphs $\omega_{a(m,n)}(P) = 3(2m+n)^2+(2m+n)+1$, for all $(m,n)\neq(0,1)$.

Proof (sketch)

For triangulated planar (m, n)-absolute clique G with domination number 2,

Lemma: $2 \leq |C| \leq 3(2m+n)^2$, where C is the common neighbor of x, y and $D=\{x, y\}$.



Theorem: For the family P of planar graphs

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Proof (sketch)

For triangulated planar (m, n)-absolute clique G with domination number 2,

Lemma: $2 \leq |C| \leq 3(2m+n)^2$, where C is the common neighbor of x, y and $D=\{x, y\}$.

Lemma: If $3 \leq |C| \leq 3(2m+n)^2$ then,

$$\omega_{a(m,n)}(G) \leq 3(2m+n)^2+(2m+n)+1,$$

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Theorem: For the family P of planar graphs
$$\omega_{a(m,n)}(P) = 3(2m+n)^2+(2m+n)+1, \text{ for all } (m,n)\neq(0,1).$$

Proof (sketch)

For triangulated planar (m, n)–absolute clique G with domination number 2,

Lemma: $2 \leq |C| \leq 3(2m+n)^2$, where C is the common neighbor of x, y and $D=\{x, y\}$.

Lemma: If $3 \leq |C| \leq 3(2m+n)^2$ then,
$$\omega_{a(m,n)}(G) \leq 3(2m+n)^2+(2m+n)+1,$$
for all $(m,n)\neq(0,1)$.

Lemma: If $|C| = 2$ then,
$$\omega_{a(m,n)}(G) \leq 3(2m+n)^2+(2m+n)+1,$$
for all $(m,n)\neq(0,1)$.

Theorem: For the family P of planar graphs

$$\omega_{a(m,n)}(P) = 3(2m+n)^2+(2m+n)+1 \text{ , for all } (m,n)\neq(0,1).$$

Proof (sketch)

Lemma: For triangulated planar (m, n)-absolute clique G with domination number 2

$$\omega_{a(m,n)}(G) \leq 3(2m+n)^2+(2m+n)+1 \text{ ,}$$
$$\text{for all } (m,n)\neq(0,1).$$

Theorem: For the family P of planar graphs $\omega_{a(m,n)}(P) = 3(2m+n)^2+(2m+n)+1$, for all $(m,n)\neq(0,1)$.

Proof (sketch)

Lemma: For triangulated planar (m, n)-absolute clique G with domination number 1

$$\omega_{a(m,n)}(G) \leq 3(2m+n)^2+(2m+n)+1 ,$$

for all $(m,n)\neq(0,1)$.

Theorem: For the family P of planar graphs $\omega_{a(m,n)}(P) = 3(2m+n)^2+(2m+n)+1$, for all $(m,n)\neq(0,1)$.

Proof (sketch)

Lemma: For triangulated planar (m, n)-absolute clique G with domination number 1

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Lemma: For triangulated planar (m, n)-absolute clique G with domination number 2

$$\omega_{a(m,n)}(G) \leq 3(2m+n)^2+(2m+n)+1,$$

for all $(m,n)\neq(0,1)$.

Theorem: For the family P of planar graphs

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Theorem: For the family P of planar graphs

$$\omega_{a(m,n)}(P) = 3(2m+n)^2+(2m+n)+1, \text{ for all } (m,n)\neq(0,1).$$

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for all $(m,n)\neq(0,1)$.

Theorem (Bensmail, Duffy and Sen, Graphs Combin., 2017):

For the family P of planar graphs

$$3(2m+n)^2+(2m+n)+1 \leq \omega_{a(m,n)}(P), \text{ for all } (m,n)\neq(0,1).$$

$\blacksquare$

Thank You