

## Le système $DN_{prop}$

$$\begin{array}{c}
 \frac{}{\Delta, A \vdash A} \text{ (ax)} \\
 \\
 \frac{\Delta, A \vdash B}{\Delta \vdash A \rightarrow B} (\rightarrow i) \quad \frac{\Delta \vdash A \quad \Delta \vdash A \rightarrow B}{\Delta \vdash B} (\rightarrow e) \\
 \\
 \frac{\Delta \vdash A \quad \Delta \vdash B}{\Delta \vdash A \wedge B} (\wedge i) \quad \frac{\Delta \vdash A \wedge B}{\Delta \vdash A} (\wedge e) \quad \frac{\Delta \vdash A \wedge B}{\Delta \vdash B} (\wedge e) \\
 \\
 \frac{\Delta \vdash A}{\Delta \vdash A \vee B} (\vee i) \quad \frac{\Delta \vdash B}{\Delta \vdash A \vee B} (\vee i) \quad \frac{\Delta \vdash A \vee B \quad \Delta, A \vdash C \quad \Delta, B \vdash C}{\Delta \vdash C} (\vee e) \\
 \\
 \frac{\Delta, A \vdash B \quad \Delta, A \vdash \neg B}{\Delta \vdash \neg A} (\neg i) \quad \frac{\Delta \vdash A \quad \Delta \vdash \neg A}{\Delta \vdash B} (\neg e) \quad \frac{\Delta \vdash \neg \neg A}{\Delta \vdash A} (\neg e)
 \end{array}$$

## Le Système $LK$

$$\begin{array}{c}
 \frac{}{A \vdash A} \text{ (ax)} \\
 \\
 \frac{\Delta, A, A \vdash \Gamma}{\Delta, A \vdash \Gamma} \text{ (cont g)} \quad \frac{\Delta \vdash \Gamma, A, A}{\Delta \vdash \Gamma, A} \text{ (cont d)} \quad \frac{\Delta \vdash \Gamma}{\Delta, A \vdash \Gamma} \text{ (aff g)} \quad \frac{\Delta \vdash \Gamma}{\Delta \vdash \Gamma, A} \text{ (aff d)} \\
 \\
 \frac{\Delta \vdash \Gamma, A}{\Delta, \neg A \vdash \Gamma} (\neg g) \quad \frac{\Delta, A \vdash \Gamma}{\Delta \vdash \Gamma, \neg A} (\neg d) \\
 \\
 \frac{\Delta \vdash A, \Gamma \quad \Pi, B \vdash \Lambda}{\Delta, \Pi, A \rightarrow B \vdash \Gamma, \Lambda} (\rightarrow g) \quad \frac{\Delta, A \vdash B, \Gamma}{\Delta \vdash A \rightarrow B, \Gamma} (\rightarrow d) \\
 \\
 \frac{\Delta, A, B \vdash \Gamma}{\Delta, A \wedge B \vdash \Gamma} (\wedge g) \quad \frac{\Delta \vdash A, \Gamma \quad \Pi \vdash B, \Lambda}{\Delta, \Pi \vdash A \wedge B, \Gamma, \Lambda} (\wedge d) \\
 \\
 \frac{\Delta, A \vdash \Gamma \quad \Pi, B \vdash \Lambda}{\Delta, \Pi, A \vee B \vdash \Gamma, \Lambda} (\vee g) \quad \frac{\Delta \vdash A, \Gamma}{\Delta \vdash A \vee B, \Gamma} (\vee d) \quad \frac{\Delta \vdash B, \Gamma}{\Delta \vdash A \vee B, \Gamma} (\vee d) \\
 \\
 \frac{\Delta \vdash \Gamma, A \quad A, \Pi \vdash \Lambda}{\Delta, \Pi \vdash \Gamma, \Lambda} \text{ (coupure)}
 \end{array}$$

## Le système $\mathcal{G}$

$$\begin{array}{c}
 \frac{}{\Delta, A \vdash \Gamma, A} \text{ (ax)} \\
 \\
 \frac{\Delta \vdash \Gamma, A}{\Delta, \neg A \vdash \Gamma} (\neg g) \quad \frac{\Delta, A \vdash \Gamma}{\Delta \vdash \Gamma, \neg A} (\neg d) \\
 \\
 \frac{\Delta \vdash A, \Gamma \quad \Delta, B \vdash \Gamma}{\Delta, A \rightarrow B \vdash \Gamma} (\rightarrow g) \quad \frac{\Delta, A \vdash B, \Gamma}{\Delta \vdash A \rightarrow B, \Gamma} (\rightarrow d) \\
 \\
 \frac{\Delta, A, B \vdash \Gamma}{\Delta, A \wedge B \vdash \Gamma} (\wedge g) \quad \frac{\Delta \vdash A, \Gamma \quad \Delta \vdash B, \Gamma}{\Delta \vdash A \wedge B, \Gamma} (\wedge d) \\
 \\
 \frac{\Delta, A \vdash \Gamma \quad \Delta, B \vdash \Gamma}{\Delta, A \vee B \vdash \Gamma} (\vee g) \quad \frac{\Delta \vdash A, B, \Gamma}{\Delta \vdash A \vee B, \Gamma} (\vee d) \\
 \\
 \frac{\Delta \vdash \Gamma, A \quad A, \Delta \vdash \Gamma}{\Delta \vdash \Gamma} \text{ (coupure)}
 \end{array}$$