Preuve de programmes
Logique de Hoare

Exercice L'algorithme suivant calcule les entiers q et r t.q. $q = x/y$ et $q * y + r = x$. Prouver :

$\{x \geq 0 \wedge y > 0\}$

```
q:=0;
r:=x;
while r >= y do
  r := r - y;
  q := q + 1;
done
```

$\{q * y + r = x \wedge 0 \leq r < y\}$

Correction:
On déroule l'algorithme avec $x = 13$ et $y = 2$:

q	0	1	2	3	4	5	6
r	13	11	9	7	5	3	1

On pose $Inv = q * y + r = x \wedge r \geq 0$. On prouve :

1. Avant le while:

$$\begin{array}{l}
\{x \geq 0 \wedge y > 0\} \\
q := 0; \\
r := x; \\
\{Inv\} = \{q * y + r = x \wedge r \geq 0\}
\end{array}$$

$$\begin{array}{ll}
\{x \geq 0 \wedge y > 0\} & \Rightarrow \\
\{x \geq 0\} & \equiv \\
\{x = x \wedge x \geq 0\} & \equiv \\
\{0 * y + x = x \wedge x \geq 0\} & \\
q := 0; & \\
\{q * y + x = x \wedge x \geq 0\} & \\
r := x; & \\
\{q * y + r = x \wedge r \geq 0\} &
\end{array}$$

2. Le while:

$$
\begin{array}{l}
\{Inv\} = \{q * y + r = x \land r \geq 0\} \\
\textbf{while} \;\; r \geq y \textbf{ do} \\
r := r - y; \\
q := q + 1; \\
\textbf{done} \\
\{Inv \land r < y\} = \{q * y + r = x \land 0 \leq r < y\}
\end{array}
$$

$$
\begin{array}{ll}
\{q * y + r = x \land r \geq 0 \land r \geq y\} & \Rightarrow \\
\{q * y + r = x \land r \geq y\} & \equiv \\
\{(q * y + y + r - y = x \land r \geq y\} & \equiv \\
\{(q + 1) * y + r - y = x \land r - y \geq 0\} & \\
r := r - y; & \\
\{(q + 1) * y + r = x \land r \geq 0\} & \\
q := q + 1; & \\
\{q * y + r = x \land r \geq 0\} &
\end{array}
$$

3. On en conclut par la règle de conséquence

$$
\begin{array}{l}
\{x \geq 0 \land y > 0\} \\
Prog \\
\{q * y + r = x \land 0 \leq r < y\}
\end{array}
$$