Solvability in Call-by-Push-Value

Giulio Guerrieri (Dept. CS, Univ. of Bath) Delia Kesner (IRIF, Univ. de Paris)

Call-by-Push-Value. The Call-by-Push-Value (CBPV) paradigm, introduced by P. B. Levy [11, 12], distinguishes between values and computations under the slogan “a value is, a computation does”. It subsumes the $\lambda$-calculus by adding some primitives that allow to capture both the Call-by-Name (CBN) and Call-by-Value (CBV) semantics. Essentially, CBPV introduces unary primitives $\text{thunk}$ and $\text{force}$. The former freezes the execution of a term (i.e. it is not allowed to compute under a thunk) while the latter fires again a frozen term. Informally, $\text{force}(\text{thunk}(t))$ is semantically equivalent to $t$. Resorting to the paradigm slogan, thunk turns a computation into a value, while force does the opposite. Thus, CBN and CBV are captured by conveniently labelling a $\lambda$-term using force and thunk to pause/resume the evaluation of a subterm depending on whether it is an argument (CBN) or a function (CBV). In doing so, CBPV provides a unique formalism capturing two distinct $\lambda$-calculi strategies, thus allowing to study operational and denotational semantics of different evaluation strategies in a unified framework.

Bang calculus. T. Ehrhard [3] considered CBPV under a linear-logic perspective, showing that Levy’s primitives can be interpreted as well-known constructs of linear logic (LL, [5]). Ehrhard and Guerrieri [4] studied the bang calculus—the untyped version of the implicative fragment of CBPV—under this LL perspective. The bang calculus is essentially an extension of the $\lambda$-calculus with two new constructors, namely bang ($!$, corresponding to thunk) and dereliction (der, corresponding to force), together with the reduction rule $\text{der}(!t) \rightarrow t$. There are two interesting notions of reduction for the bang calculus, depending on whether reduction takes place under a bang constructor (strong reduction) or not (weak reduction). Ehrhard and Guerrieri [4] showed how (weak) CBPV captures the CBN and CBV semantics of the $\lambda$-calculus via Girard’s translations [5] of intuitionistic logic into LL. Other operational properties and intersection types for the weak bang calculus were studied in [6, 8].

CBN Solvability. The solvability notion denotes a term that can operationally interact with the environment by producing a given output when inserted into an appropriate context. Formally, in the framework of the CBN $\lambda$-calculus, a closed (i.e. without free variables) $\lambda$-term $t$ is solvable if there are $n \geq 0$ and some terms $u_1, \ldots, u_n$ such that $tu_1 \ldots u_n$ reduces to the identity function. Closed solvable terms represent meaningful programs: if $t$ is closed and solvable, then $t$ can produce any desired result when applied to a suitable sequence of arguments. This notion can be easily extended to open terms, through head contexts, which do the job of both closing the term and then applying it to an appropriate sequence of arguments. Thus, a $\lambda$-term $t$ is solvable if there is a head context $H$ such that, when $H$ is filled by $t$, then $H[t]$ is closed and reduces to the identity function.
Proof Techniques. There are different characterizations of solvability for the CBN \( \lambda \)-calculus. Indeed, the term \( t \) is solvable iff

1. \( H[t] \) reduces to the identity for an appropriate head context \( H \);
2. \( t \) has a head normal form;
3. \( t \) can be typed in a suitable intersection type system.

Statement (1) is the definition of solvability, Statement (2) (resp. (3)) is known as the syntactical (resp. logical) characterization of solvability. The syntactical characterization, i.e. \( (2) \iff (1) \), has been proved in an untyped setting using the standardization theorem (see for example [16]). The logical characterization, i.e. \( (3) \iff (1) \), uses the syntactical one: it is performed by building an intersection type assignment system characterizing terms having head normal form (see for example [17]). the implication \( (3) \Rightarrow (2) \) corresponds to the soundness of the type system (proved by means of a subject reduction property), while \( (2) \Rightarrow (3) \) states its completeness (proved by subject expansion).

CBV Solvability. In the CBV \( \lambda \)-calculus, the definition of solvability is analogous to the one in CBN, but the notion is trickier, as explained in [18, 19]. Moreover, it is not possible to provide a syntactical characterization of solvability in Plotkin’s original formulation of the CBV \( \lambda \)-calculus [22], because reduction is somehow too restricted. Syntactical characterizations of CBV solvability have been recently found out in some extensions of Plotkin’s CBV \( \lambda \)-calculus [20, 21], which are still encompassed into the bang calculus. Here some subtleties arise: to characterize syntactically CBV solvability is necessary to go beyond Plotkin’s original syntax, but CBV solvability in those extensions is equivalent to CBV solvability in Plotkin’s CBV \( \lambda \)-calculus. This is a sign that good frameworks to characterize CBV solvability are these extensions rather than Plotkin’s original calculus. Moreover, a logical characterization of CBV solvability is still an open question.

Goal of the internship. The goal of this internship is to study different characterizations of solvability in the call-by-push-value setting (bang calculus). It is expected that the obtained results translate to solvability results in CBN and CBV \( \lambda \)-calculi.

Tools. Mathematical tools such as rewriting, lambda-calculus, linear logic, and type systems will be used and studied during this work.

Contacts

- Delia Kesner, Professor, Université de Paris.  
  E-mail: kesner@irif.fr  
  URL: http://www.irif.fr/~kesner

- Giulio Guerrieri, Research Associate, University of Bath (United Kingdom).  
  E-mail: g.guerrieri@bath.ac.uk  
  URL: https://www.irif.fr/~giuliog
Duration
Six months.

Place
IRIF, Bâtiment Sophie Germain, 5 rue Thomas Mann. 75205 Paris CEDEX 13.

Collaborations
Many collaborations in the domain are carried out with S. Ronchi Della Rocca (Université de Turin, Italie), A. Bucciarelli (IRIF, Université Paris-Diderot), D. Ventura (Université de Goiás, Brazil), B. Accattoli (LIX Ecole Polytechnique), and A. Viso (INRIA, France). A PhD thesis could be considered after the internship.

References


[22] G. D. Plotkin: