

Computer-Aided Formal Reasoning

Tutorial

Exercise 1.

Prove each of the following formulas first by giving a derivation in natural deduction (NJ₀), then by giving a proof term:

- a) $(A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$
- b) $(A \wedge B \Rightarrow C) \Rightarrow (A \Rightarrow B \Rightarrow C)$
- c) $\neg\neg(A \vee \neg A)$
- d) $(A \vee \neg A) \Rightarrow (\neg\neg A \Rightarrow A)$
- e) $(A \vee B) \wedge C \Rightarrow (A \wedge C) \vee (B \wedge C)$

Remark: $\neg A$ is shorthand for $A \Rightarrow \perp$.

Reminder: Rules of natural deduction

$\frac{}{\Gamma, A \vdash A}(\text{Ax})$	$\frac{}{\Gamma \vdash \top}(\top\text{-I})$	$\frac{\Gamma \vdash \perp}{\Gamma \vdash A}(\perp\text{-E})$	$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B}(\Rightarrow\text{-I})$	$\frac{\Gamma \vdash A \Rightarrow B \quad \Gamma \vdash A}{\Gamma \vdash B}(\Rightarrow\text{-E})$
$\frac{\Gamma \vdash A_1 \quad \Gamma \vdash A_2}{\Gamma \vdash A_1 \wedge A_2}(\wedge\text{-I})$	$\frac{\Gamma \vdash A_1 \wedge A_2}{\Gamma \vdash A_1}(\wedge\text{-E})$	$\frac{\Gamma \vdash A_1 \wedge A_2}{\Gamma \vdash A_2}(\wedge\text{-E})$		
$\frac{\Gamma \vdash A_1}{\Gamma \vdash A_1 \vee A_2}(\vee\text{-I})$	$\frac{\Gamma \vdash A_2}{\Gamma \vdash A_1 \vee A_2}(\vee\text{-I})$	$\frac{\Gamma \vdash A_1 \vee A_2 \quad \Gamma, A_1 \vdash B \quad \Gamma, A_2 \vdash B}{\Gamma \vdash B}(\vee\text{-E})$		

Reminder: Typing rules of the simply-typed λ -calculus

$\frac{}{\Gamma, x : T \vdash x : T}(\text{Ax})$	$\frac{}{\Gamma \vdash \langle \rangle : \text{Unit}}(\text{Unit-I})$	$\frac{\Gamma \vdash t : \text{Void}}{\Gamma \vdash \text{case } t \text{ of } \{ \} : T}(\text{Void-E})$	$\frac{\Gamma, x : U \vdash t : T}{\Gamma \vdash \lambda x. t : U \rightarrow T}(\rightarrow\text{-I})$
$\frac{\Gamma \vdash t : U \rightarrow T \quad \Gamma \vdash u : U}{\Gamma \vdash t u : T}(\rightarrow\text{-E})$	$\frac{\Gamma \vdash t : T \quad \Gamma \vdash u : U}{\Gamma \vdash \langle t, u \rangle : T \times U}(\times\text{-I})$	$\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash \pi_1 t : T_1}(\times\text{-E})$	$\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash \pi_2 t : T_2}(\times\text{-E})$
$\frac{\Gamma \vdash t : T_1}{\Gamma \vdash \text{in}_1 t : T_1 + T_2}(\text{+I-1})$	$\frac{\Gamma \vdash t : T_2}{\Gamma \vdash \text{in}_2 t : T_1 + T_2}(\text{+I-2})$		
$\frac{\Gamma \vdash t : T_1 + T_2 \quad \Gamma, x : T_1 \vdash u_1 : U \quad \Gamma, y : T_2 \vdash u_2 : U}{\Gamma \vdash \text{case } t \text{ of } \{ \text{in}_1 x \mapsto u_1 \mid \text{in}_2 y \mapsto u_2 \} : U}(\text{+E})$			

Exercise 2. (in Coq)

Go to perso.ens-lyon.fr/jeremy.ledent/coq/ and follow the instructions in the fill-in-the-blanks tutorial.