Robust Bell inequalities from communication complexity

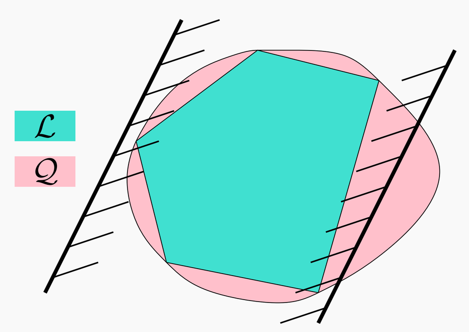
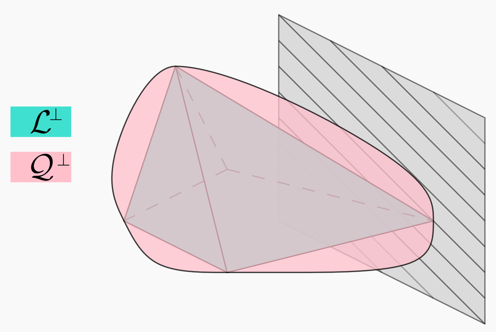
Sophie Laplante⋆, Mathieu Laurière⋆†, Alexandre Nolin⋆, Jérémie Roland‡, Gabriel Senno§

⋆ IRIF - Université Paris Diderot, † NYU Shanghai, ‡ QuIC - Université Libre de Bruxelles, § CONICET - FCEN - Universidad de Buenos Aires

arXiv 1606.09514 - TQC2016 proceedings

Normalized and inefficiency-resistant Bell inequalities

We consider two kinds of Bell inequalities, differently constrained

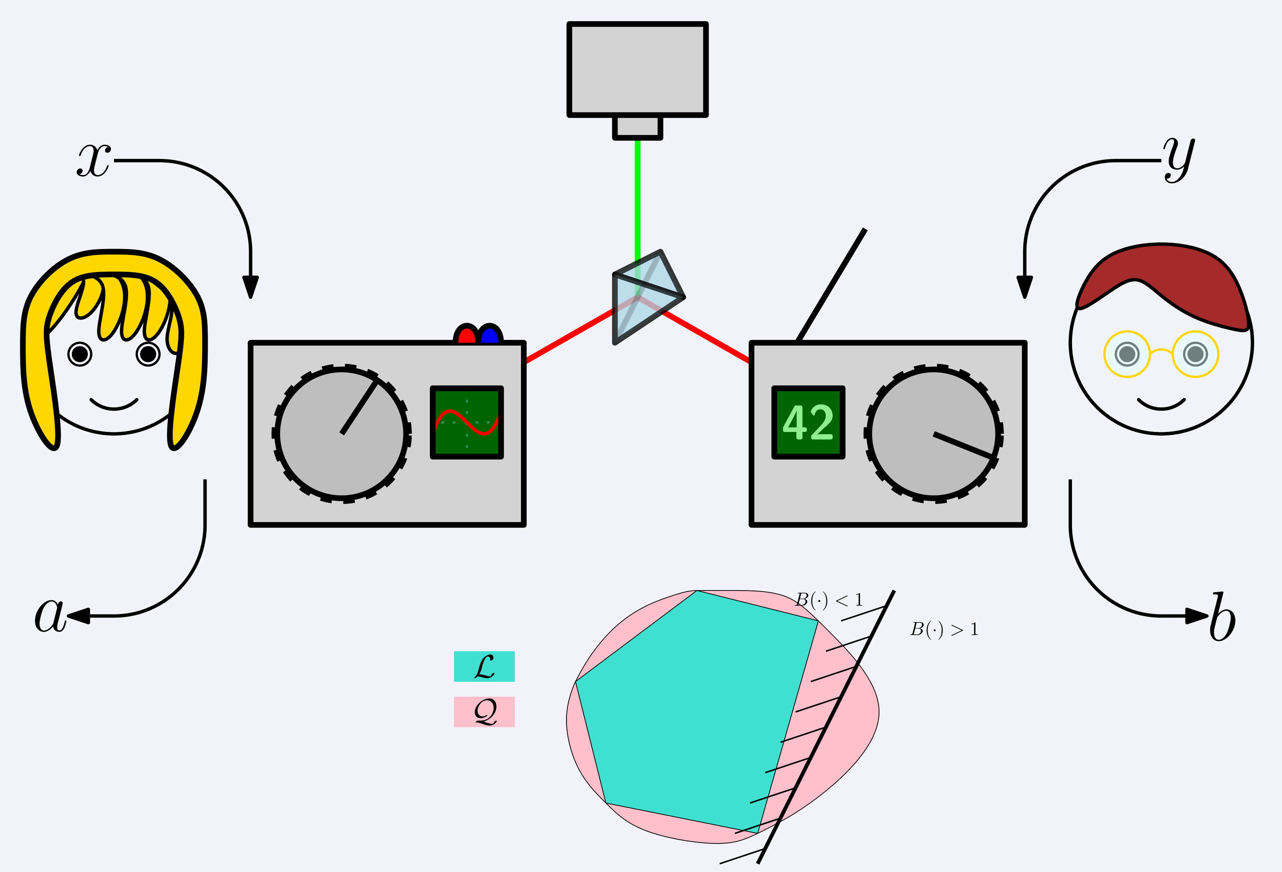
Normalized	Inefficiency-resistant
	
$\lvert B(\ell)\rvert \leq 1,\ \forall \ell \in \mathcal{L}$	$B(\ell) \leq 1,\ \forall \ell \in \mathcal{L}^{\perp}$ $a = \perp \vee b = \perp \Rightarrow B_{ab,xy} = 0$

Distributions in $\mathcal{L}^{\perp}$ have an additional special abort outcome $\perp$ per player which corresponds to the detectors not clicking.

Yes, provided all coefficients are divided by 2.

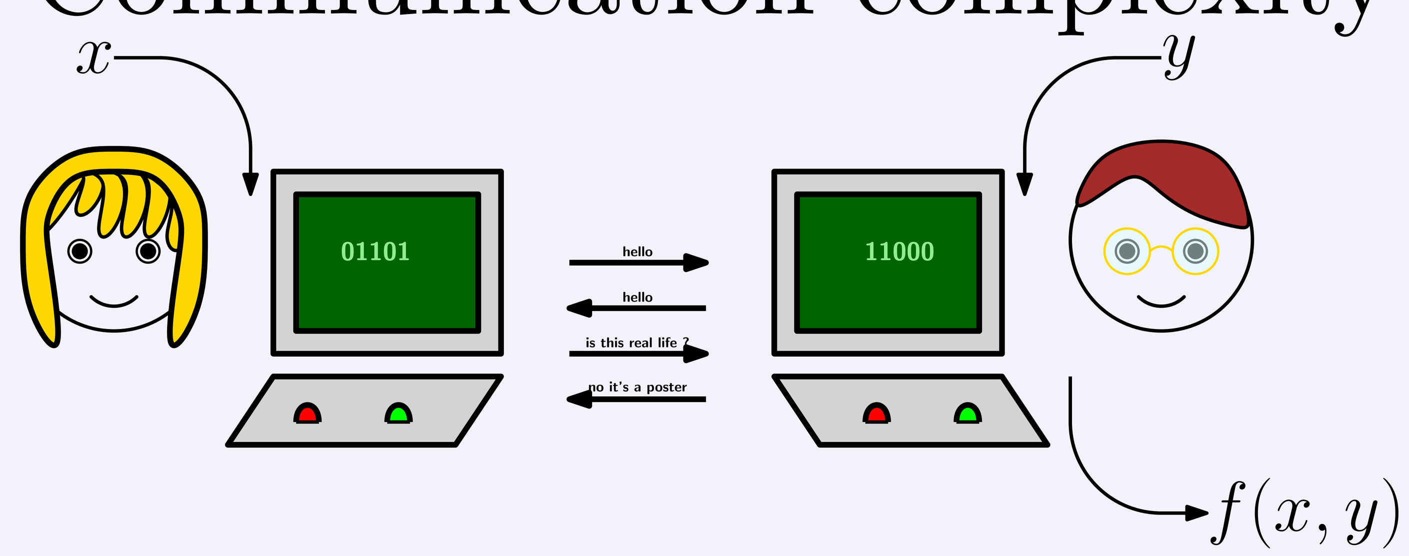
Yes! Even if allowed to abort, no local strategy can win with probability ≥ 0.75.

The Bell inequality setup



$\mathcal{L}$ (local set) : distributions achievable with shared randomness.
$\mathcal{Q}$ (quantum set) : distributions achievable with shared entanglement.
Bell functional : linear form over families of probability distributions $B(\mathbf{p}) = \sum_{abxy} B_{ab,xy} p(ab|xy)$
Bell inequality : Bell functional bounded on the local set $B,\ B(\ell) \leq c, \forall \ell \in \mathcal{L}$

Communication complexity



Given a 2-argument function f, how many *classical/quantum* bits do two players with inputs x and y need to exchange to know $f(x,y)$?

CC : Classical communication complexity
Q : Quantum communication complexity

Prior work

Examples of normalized Bell inequality violations

Construction		#inputs	#outputs	dimension	violation
Hidden-Matching	[1]	n^n	n	n	$\sqrt{n}/\log n$
Khot-Vishnoi	[1]	$2^n/n$	n	n	$n/\log^2 n$
Low-entanglement	[2]	n	n	n	$\sqrt{n}/\log n$
General construction	[3]	2^n	2^Q	2^{2^Q}	$\sqrt{CC/Q}$

[1] BRSdW11 [2] JP11 [3] BCG+16

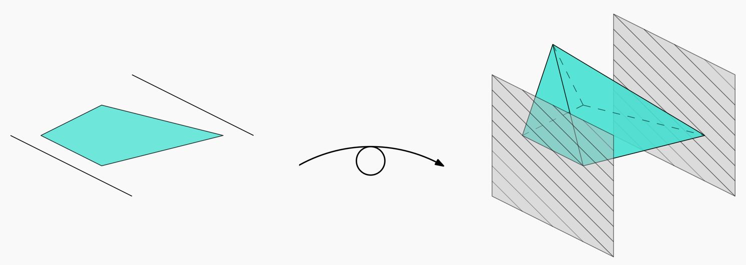
With N inputs, violation $\leq 2^{CC} \leq N$ [LS09, DKLR11, JPPG10]

With K outputs, violation $\leq O(K)$ [JP11]

With dimension d, violation $\leq O(d)$ [JPPG+10]

Our results

For any normalized Bell inequality, there is a similar Bell inequality that is also inefficiency-resistant.



Theorem 1
For B such that $\lvert B(\ell)\rvert \leq 1$ over $\mathcal{L}$, there exists B^* such that for any nonsignaling $\mathbf{p}$ such that $B(\mathbf{p}) \geq 1$, $B^*(\mathbf{p}) \geq \frac{1}{3}B(\mathbf{p}) - \frac{2}{3}$ and $\lvert B(\ell)\rvert \leq 1$ over $\mathcal{L}^{\perp}$

If a distribution can be simulated quantumly with less communication than classically, then there is a violation of an inefficiency-resistant Bell inequality.

Theorem 2
If a distribution $\mathbf{p}$ is such that $CC(\mathbf{p}) \geq \log(B(\mathbf{p})) = C$ and $QCC(\mathbf{p}) \leq Q$. Then there exists $\mathbf{q} \in \mathcal{Q}$ with one additional output per player compared to the distribution $\mathbf{p}$ such that $B(\mathbf{q}) \geq 2^{C-2Q}$

In particular, for most functions f for which there is a known gap between their classical and their quantum communication complexities, one can define a pair $(B_f, \mathbf{p}_f)$ such that $\log(B_f(\mathbf{p}_f)) = \Theta(CC(f))$
Go to the bottom right quadrant of this poster for a few examples.

Our violations in practice

Observed violations of inefficiency-resistant Bell inequalities obtained with Theorem 2 compared to the general construction in [1], assuming η detector efficiency.

Problem		Normalized Bell violations [1]	Inefficiency-resistant Bell violations (this work)	
VSP	[2,3]	$\Omega\left(\sqrt[6]{n}/\sqrt{\log n}\right) \cdot \eta^2$	$2^{\Omega(\sqrt[3]{n}) - O(\log n)} \cdot \eta^2$	$d = n^{O(1)}$
DISJ	[4,5,6]	N/A	$2^{\Omega(n) - O(\sqrt{n})} \cdot \eta^2$	$d = 2^{O(\sqrt{n})}$
TRIBES	[7,8]	N/A	$2^{\Omega(n) - O(\sqrt{n}\log^2 n)} \cdot \eta^2$	$d = 2^{O(\sqrt{n}\log^2 n)}$
ORT	[9,8]	N/A	$2^{\Omega(n) - O(\sqrt{n}\log n)} \cdot \eta^2$	$d = 2^{O(\sqrt{n}\log n)}$

[1] BCG+16
[2] Raz99
[3] KR11
[4] Razb92
[5] Razb03
[6] AA05
[7] JKS03
[8] BCW98
[9] She12