Project 5: Rock-Paper-Scissors Reaction Networks

ER02

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Abstract

Design reaction networks that will compete with the other team's networks to the Rock-Parer-Scissors game. During the defence, we will organise a championship to decide which is the champion network.

We consider a simplification of reaction network where the various chemicals involved act directly on each other and that these interactions can be summed up in a matrix $A = (a_{ij})$:

- If $a_{ij} > 0$, chemical i activates the production of chemical j;
- If $a_{ij} < 0$, chemical i inihibits the production of chemical j;
- Finally, if $a_{ij} = 0$, chemical i has no effect on chemical j;

when $a_{ii} > 0$, we speak of autocatalysis.

As far as the chemical interactions are concerned, the differential equations for the evolution of the concentration $x_i(t)$ of each chemical i is given by the differential equation:

$$\left(\frac{dx_i}{dt}\right)_{interaction} = \frac{\displaystyle\sum_{j:a_{ij}>0} a_{ij}x_j}{1 + \left(\displaystyle\sum_{j:a_{ij}>0} a_{ij}x_j\right) + 10\left(\displaystyle\sum_{j:a_{ij}<0} |a_{ij}|x_j\right)}$$

In order to limit the power of each player, we require the production of each chemical to consum a special chemical called *fuel*, indexed by 0, proper to each player. We also add a natural degradation of each chemical at contant rate 0.4. In order to ensure the numerical stability, we also add a negligible constant term $\epsilon = 10^{-6}$. The equation for the variation of each chemical $i > 0$ is thus given by the differential equation:

$$\frac{dx_i}{dt} = 10^{-6} - 0.4\,x_i + \frac{x_0}{2 + x_0} \cdot \left(\frac{dx_i}{dt}\right)_{interaction}$$

$$\text{that is: } \frac{dx_i}{dt} = 10^{-6} - 0.4\,x_i + \frac{x_0}{2 + x_0} \cdot \frac{\displaystyle\sum_{j>0\,:\,a_{ij}>0} a_{ij}x_j}{1 + \left(\displaystyle\sum_{j>0\,:\,a_{ij}>0} a_{ij}x_j\right) + 10\left(\displaystyle\sum_{j>0\,:\,a_{ij}<0} |a_{ij}|x_j\right)}$$

The fuel of each player is produced at a rate of 0.5 and its concentration x_0 verifies the differential equation:

$$\frac{dx_0}{dt} = 0.5 - \frac{x_0}{2+x_0} \cdot \sum_{i>0} \left(\frac{dx_i}{dt}\right)_{interaction}$$

$$\text{that is: } \frac{dx_0}{dt} = 0.5 - \frac{x_0}{2+x_0} \cdot \sum_{i>0} \frac{\sum_{j>0\,:\,a_{ij}>0} a_{ij}x_j}{1 + \left(\sum_{j>0\,:\,a_{ij}>0} a_{ij}x_j\right) + 10\left(\sum_{j>0\,:\,a_{ij}<0} |a_{ij}|x_j\right)}$$

The game. Each player will have n chemicals indexed by 1 to n plus its own fuel indexed by 0. Concentrations of the chemical for the first player will be denoted by $x_0, x_1, \ldots, x_n$ and $y_0, y_1, \ldots, y_m$ for the second player.

Chemicals indexed by 1,2, and 3 of each player will denote the movement they play: 1 for Rock, 2 for Paper and 3 for Scissors. Player one is considered to play a move as soon as one and only one of x_1, x_2, x_3 is above 1.

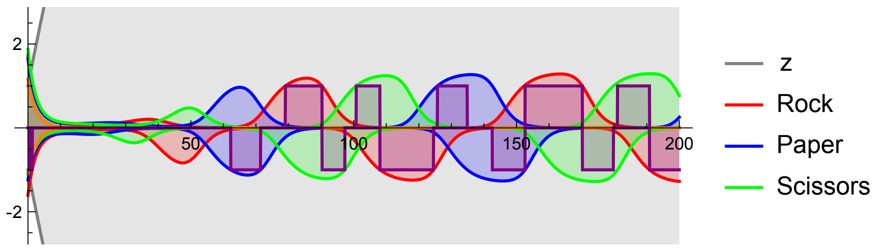
The referee provides a feed to both players thanks to two special chemicals, whose concentrations are denoted by z_1 and z_2, such that $z_k = 1.0$ at time t if and only if player k plays a winning move at time t (i.e. if player k plays a move and the other player either does not play or plays a move loosing against the move played by player k); $z_k = 0.0$ otherwise.

The game starts with all concentrations at 0.0 and is played until time $T = 200.0$. The winner is the player k for which $\int_0^T z_k(t)dt$ is maximum.

Write a simulator. You are first ask to write a simple simulator (using Euler method for instance) that displays the evolution of the concentrations of all chemicals over time for two players given by their interaction matrices. Note that each player k is allow to create its network using an arbitrary number of its own chemicals $(z_k, x_1, \ldots, x_n)$ but not on the chemicals of the other player $(z_{k'}, y_1, \ldots, y_m)$.

The problem. You are asked to design a reaction network for your player that will maximize its chance to win against other team's player.

You might first want to design a reaction network that plays each move one after the other. We might consider a mix of auto-catalysis and inhibition. Here is an example of such networks fighting against each other (player 1's concentration are drawn along the positive vertical axis, and player 2's along the negative vertical axis)



You may design your player by your self or try various evolutionary algorithms (like having different networks fighting each other to select which will survive to the next step of the genetic algorithm).