Proving that project 4 is impossible

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Plan

Problem

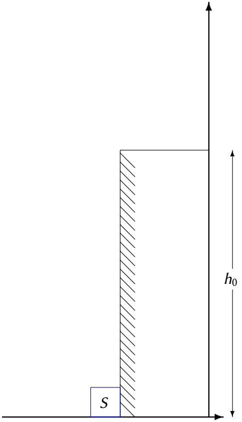

FIGURE – Initial state

Find a tile assembly system such that :

- Seed tile at position $S = (0, 0)$
- $\forall h$ there is a tile at $T = (10, h)$
- Finite size
- No tiles to the right and below the cut
- Possible presence of glues on the wall (infinite)

Different cases

- Case with no glues on the wall, more genera.
- Cases with odd or even h only.
- Succeed with probability $1 - \epsilon$

Tile and tile kind

Definition (Tile Kind)

A tile kind is a quadruplet of pairs (colour, strength).

Definition (Tile)

A tile is a pair (tile kind, coordinates)

Configuration and wall

Definition (Configuration)

A configuration $\mathcal{C}$ is a connected set of tiles that are joint by their colours.

It is relative to some set of tile kinds $\mathcal{T}$.

Definition (Wall)

A wall is a set of special tiles that occupy all the bottom-right corner of the plane.

It can have glues only in column 1. They must be lower than a given temperature τ.

Execution

Definition (Execution)

- *Sequence of tiles (added one after the other)*
- *Add a tile if its satisfies some temperature* τ
- *Build a configuration* $\mathcal{C}$ *over some tile kinds set* $\mathcal{T}$
- $\mathcal{C}$ *does not crash into a wall* $\mathcal{W}$

It is *ended* if we cannot add any new tile.
It is *finite* is the sequence is finite.
It is *valid* if it reaches $(10, h)$ (h is the height of the wall).

Impossibility

Theorem

There is no tile kinds set $\mathcal{T}$, temperature τ, seed σ, sequence of colours (c_i) and sequence of strengths (s_i) such that for any wall of any height (with respect to the sequences), any ended execution is finite and valid.

The proof within some images (1)

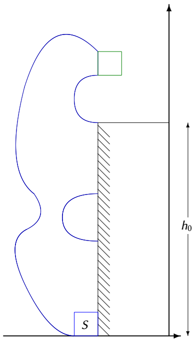

FIGURE – First growth of the tile algorithm

- h_0 arbitrary (different from 0)
- Stop the execution before posing the green tile
- h_1 = height of the green tile

The proof within some images (2)

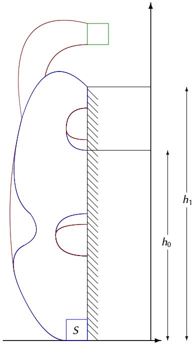

FIGURE – Second growth of the tile algorithm

- Stop the execution before posing the green tile
- The red tiles do not need the wall
- The red tiles between the blue ones and the wall are not important
- The wall is unchanged → the blue tiles can still be constructed

And so ?

$\Rightarrow (h_n)_{n \in \mathbb{N}}$ by recurrence

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- $h_n \underset{n\to\infty}{\longrightarrow} \infty$
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- Do not need the wall

And so ?

$\Rightarrow (h_n)_{n\in\mathbb{N}}$ by recurrence

- $h_n \underset{n\to\infty}{\to} \infty$
- Valid sequence of tiles
- Do not need the wall
- Goes to infinity

And so ?

$\Rightarrow (h_n)_{n \in \mathbb{N}}$ by recurrence

- $h_n \underset{n \to \infty}{\longrightarrow} \infty$
- Valid sequence of tiles
- Do not need the wall
- Goes to infinity

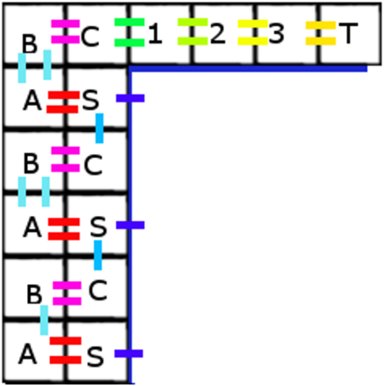
$\Rightarrow$ Contradiction !

Some tools we need

- each non-ended execution must be "endable"
- connexity must mean that each column and row is crossed between to points that are "connected"

Solution for a wall of odd or even height

- Use the wall to climb so we can't go up indefinitely.
- For that we go two row by two row, it limit to odd or even case.
- When above the wall we go to the right to reach the target.
- Merging impossible due to interaction.



Unprefix wall

- The constraint about prefix wall is needed
- Last tile of the wall could be different, we could start from it.

Conclusion (coin _o<)