Proving that project 4 is impossible

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Plan

1. Introduction
2. Main Theorem
3. Extra features
4. Conclusion (you lost the game)
Problem

Find a tile assembly system such that:

- Seed tile at position $S = (0, 0)$
- $\forall h$ there is a tile at $T = (10, h)$
- Finite size
- No tiles to the right and below the cut
- Possible presence of glues on the wall (infinite)

**Figure** – Initial state
Different cases

- Case with no glues on the wall, more genera.
- Cases with odd or even $h$ only.
- Succeed with probability $1 - \epsilon$
**Definition (Tile Kind)**

*A tile kind is a quadruplet of pairs (colour, strength).*

**Definition (Tile)**

*A tile is a pair (tile kind, coordinates)*
Definition (Configuration)

A configuration $C$ is a connected set of tiles that are joint by their colours.

It is relative to some set of tile kinds $T$.

Definition (Wall)

A wall is a set of special tiles that occupy all the bottom-right corner of the plane.

It can have glues only in column 1. They must be lower than a given temperature $\tau$. 
Execution

**Definition (Execution)**

- Sequence of tiles *(added one after the other)*
- *Add a tile if its satisfies some temperature* $\tau$
- *Build a configuration* $C$ *over some tile kinds set* $\mathcal{T}$
- $C$ *does not crash into a wall* $\mathcal{W}$

It is *ended* if we cannot add any new tile.

It is *finite* if the sequence is finite.

It is *valid* if it reaches $(10, h)$ (*$h$ is the height of the wall*).
Impossibility

Theorem

There is no tile kinds set $\mathcal{T}$, temperature $\tau$, seed $\sigma$, sequence of colours $(c_i)$ and sequence of strengths $(s_i)$ such that for any wall of any height (with respect to the sequences), any ended execution is finite and valid.
The proof within some images (1)

- $h_0$ arbitrary (different from 0)
- Stop the execution before posing the green tile
- $h_1 =$ height of the green tile

**Figure** – First growth of the tile algorithm
The proof within some images (2)

- Stop the execution before posing the green tile
- The red tiles do not need the wall
- The red tiles between the blue ones and the wall are not important
- The wall is unchanged $\rightarrow$ the blue tiles can still be constructed

**Figure** – Second growth of the tile algorithm
And so?

\[ (h_n)_{n \in \mathbb{N}} \text{ by recurrence} \]
And so?

\[ \Rightarrow (h_n)_{n \in \mathbb{N}} \text{ by recurrence} \]

\[ h_n \xrightarrow[n \to \infty]{} \infty \]
And so?

$$\Rightarrow (h_n)_{n \in \mathbb{N}} \text{ by recurrence}$$

- $h_n \xrightarrow{n \to \infty} \infty$
- Valid sequence of tiles
And so?

⇒ $(h_n)_{n \in \mathbb{N}}$ by recurrence

- $h_n \xrightarrow{n \to \infty} \infty$
- Valid sequence of tiles
- Do not need the wall
And so?

$$\Rightarrow (h_n)_{n \in \mathbb{N}} \text{ by recurrence}$$

- $h_n \underset{n \to \infty}{\to} \infty$
- Valid sequence of tiles
- Do not need the wall
- Goes to infinity
And so?

\[ \Rightarrow (h_n)_{n \in \mathbb{N}} \text{ by recurrence} \]

- \[ h_n \xrightarrow[n \to \infty]{} \infty \]
- Valid sequence of tiles
- Do not need the wall
- Goes to infinity

\[ \Rightarrow \text{Contradiction!} \]
Some tools we need

- each non-ended execution must be "endable"
- connexity must mean that each column and row is crossed between to points that are "connected"
Solution for a wall of odd or even height

- Use the wall to climb so we can’t go up indefinitely.
- For that we go two row by two row, it limit to odd or even case.
- When above the wall we go to the right to reach the target.
- Merging impossible due to interaction.
The constraint about prefix wall is needed

Last tile of the wall could be different, we could start from it.
Conclusion (coin _o< )