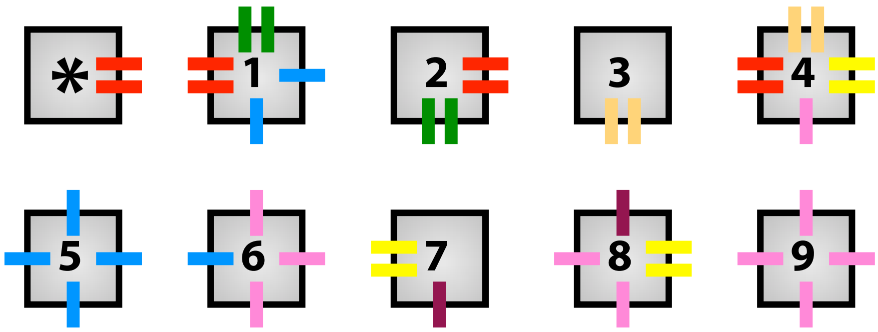
HW1 MPRI 2.11.1 Molecular Programming

18.10.2016 - **Due on Wed. 25/10 before 12:45**

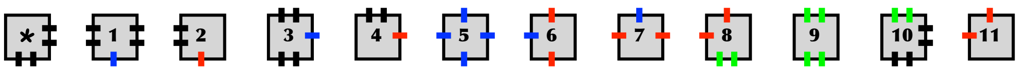
You are asked to complete the exercise marked with a [★] and to send me your solutions at:
nicolas.schabanel@cnrs.fr
(or drop it in my mail box on the 4th floor) on **Wed. 25/10 before 12:45**.

■ **Exercise 1 (Algorithmic Self-Assembly).** Recall that the self-assembly process consists in, given a finite tileset (with infinitely many tiles of each type), starting from the seed tile (marked with a ★), glueing tiles with matching colors to the current aggregate so that each new tile is attached by at least *two* links to the aggregate (either on the same border or on two borders). Recall that a shape is *final* if no tiles can be attached to it anymore.

► ***Question* 1.1)** *What is the exact family of final shapes self-assembled by the following tileset? (No proof nor justification is asked.) Indicate the local order of assembly by drawing arrows over the tiles of a generic final shape. Which are the two competing tiles that decide the size of the resulting final shape?*



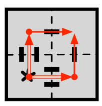
■ **Exercise 2.** What is the family of shapes built by this tileset at temperature $T^\circ = 2$? *(no justification asked)*



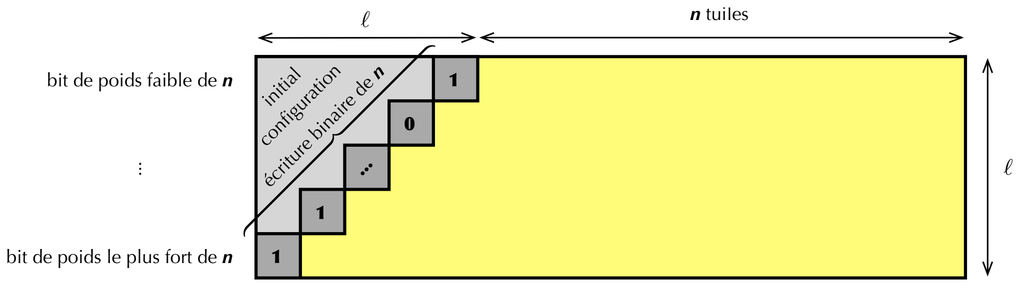
Indicate the assembly order with arrows[1] on a generic production. Is this a well-ordered tileset?

■ **Exercise 3 (Counter at $T^\circ = 2$ (★)).** Given an integer n, and an seed configuration consisting of an isosceles rectangle triangle isocèle of side $\ell = \lceil \log_2 n \rceil$ where the bits of n are encoded on the diagonal as shown in grey bellow.

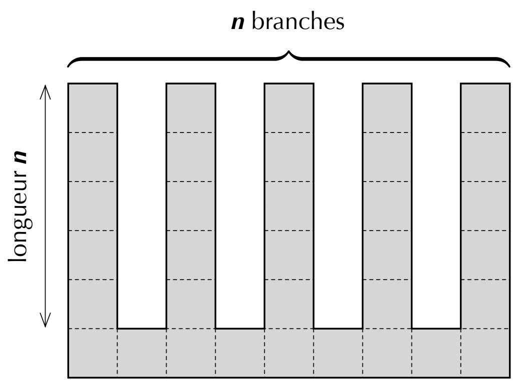
Propose a well-ordered (finite) tileset which assembles the yellow at $T^\circ = 2$ to realise a rectangle of size $\ell \times (n + \ell)$. Carefully indicate the position of the glue of strength 1 and 2

[1] Draw an arrow $(i, j) \rightarrow (i', j')$ iff tile (i, j) is attachedbefore tile (i', j'): for instance, 

on the diagonal of the seed configuration. Indicate the assembly order[1]. What does the tiles encode?



■ **Exercise 4.** Propose a staged assembly scheme at temperature $T^\circ = 1$ of the shape family E of candelabrums with n branches of length n.



Describe the tiles, glues, their number, the number of stages and the number of different bechers needed. Give an illustration of the stages to build a generic production.

■ **Exercise 5.** Assume a random Poisson model where the random time X between the appearance of a tile of a given type t at a given empty location follows an exponential law: $p(x) = c \cdot e^{-cx}$ where c is the concentration of the tiles of type t.

Theorem 1 (Adleman et al, 2001). *The expected time to build a shape P is:*

$$O(\gamma \times \text{rank}(P))$$

where γ only depends on the concentrations and rank(P) *is a highest rank in the shape P (i.e. the ℓ_1-eccentricity of the seed tile in P).*

■ **Exercise 6 (Minimum number of tile types).**

► ***Question 6.1)*** *Show by a careful case study that no tileset with < 5 tile types can assemble at $T^\circ = 2$ the $n \times n$-squares family for all $n \geqslant 2$.*

► ***Question 6.2)*** *Show by a careful case study that no tileset with < 6 tile types can assemble at $T^\circ = 2$ the $n \times m$-rectangles family for all $n, m \geqslant 2$.*