You are asked to complete the exercise marked with a [★] and to send me your solutions at: 

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(or drop it in my mail box on the 4th floor) on Wed. 8/11 before 12:45.

[★] Homework: Solve questions 2.1 and 2.2 of exercise 2.

■ Exercise 1 (Triangle). We consider the family of shapes $S_{p}$, $p \geq 1$, of (discretized) right triangles whose horizontal and vertical sizes are respectively $np$ and $n$, for some $n \geq 1$. The figure below shows the shape in $S_{4}$ for which $n = 4$.

▶ Question 1.1) Give a tile set $T_{p}$ whose final productions are exactly the shapes in $S_{p}$ at temperature $T^{o} = 2$. How many tiles does it use? Give a generic assembly and indicate the glues, the seed tile and the assembly order on it. No proof asked.

■ Exercise 2 (Window Movie Lemma). We investigate the computation power of tile assembly at temperature $T^{o} = 1$. We allow mismatches, i.e. a tile can be added to the current aggregate as soon as it is attached by at least one side to the current aggregate for which the glues match (the other sides in contact can have mismatching glues). Unless specified explicitly otherwise, all assemblies take place at $T^{o} = 1$ in this exercise.

Let us first consider a (finite) tile set $T$ which only assembles unidimensional segments of size $1 \times \ell$ for some $\ell \geq 1$ starting from its seed tile. Let $\tau = |T|$ denote the number of tile types in $T$ in all of the following. Recall that the final productions of a tile set $T$ are the shapes corresponding to every possible assembly of tiles from $T$ starting from the seed tile of $T$ and where no more tile can be added.

▶ Question 2.1) Show (and explicit) that there is a constant $k(\tau)$, which depends only on $\tau$, such that if a segment of size $1 \times \ell$ with $\ell \geq k(\tau)$ is a final production of $T$, then there is an integer $1 \leq i < k(\tau)$ such that all the segments $1 \times (\ell + n \cdot i)$ are also final productions of $T$ for all $n \geq -1$. If so, we say that the tile set $T$ is pumpable.

Let us now consider a (finite) tile set $T$ whose final productions are 2-thick rectangles of size $2 \times \ell$ for some $\ell \geq 1$.

▶ Question 2.2) Show (and explicit) that there is a constant $k_{2}(\tau)$, which depends only on $\tau$, such that if a 2-thick rectangle of size $2 \times \ell$ with $\ell \geq k_{2}(\tau)$ is a final production of $T$, then $T$ is pumpable, i.e. that there is an integer $1 \leq i < k_{2}(\tau)$ such that all the 2-thick rectangles $2 \times (\ell + n \cdot i)$ are also final productions of $T$ for all $n \geq -1$. 

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Hint. Pay attention to the order in which the tiles are attached, make sure that the pumped structure can indeed self-assemble.

Let us now generalise and consider a (finite) tile set $T$ whose final productions are $q$-thick rectangles of size $q \times \ell$ for some $\ell \geq 1$.

**Question 2.3** Show (and explicit) that there is a constant $k_q(\tau)$, which depends only on $\tau$, such that if a $q$-thick rectangle of size $q \times \ell$ with $\ell \geq k_q(\tau)$ is a final production of $T$, then $T$ is pumpable, i.e. that there is an integer $1 \leq i < k_q(\tau)$ such that all the $q$-thick rectangles $q \times (\ell + n \cdot i)$ are also final productions of $T$ for all $n \geq -1$.

Consider the following tile set $U = \{\star, A, B, C, A', B', C', D\}$ at $T^\circ = 2$ for which $\star$ is the seed tile:

The final productions of $U$ at $T^\circ = 2$ consist of two arms which are either 1) of different lengths and then don’t touch each other; or 2) of equal length and then there is a tile $D$ that makes contact between them:

**Question 2.4** Show that no tile set can simulate intrinsically at $T^\circ = 1$, the dynamics of $U$ at $T^\circ = 2$.

> Hint. As a simplification, consider that in an intrinsic simulation, all megacell corresponding to an empty position in the simulated system must never be filled by more than 30% of tiles, and all megacell corresponding to a non-empty position in the simulated system must be filled at 100% by tiles. If you have time left: how would you waive these assumptions?