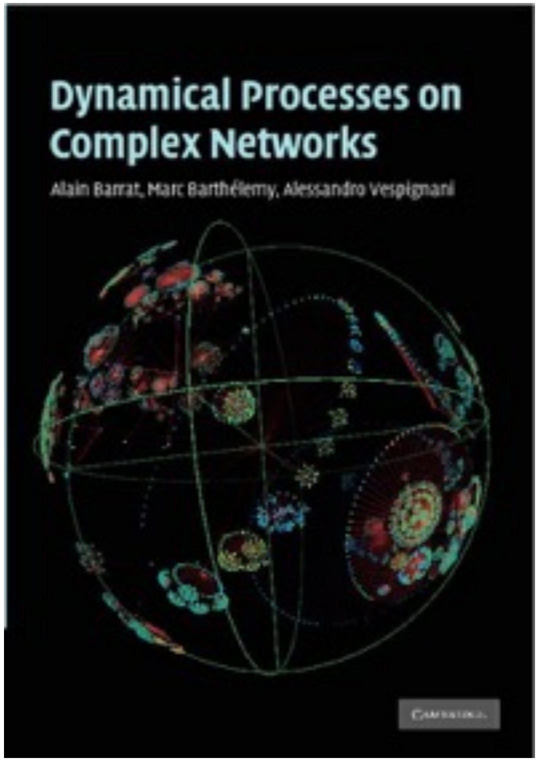
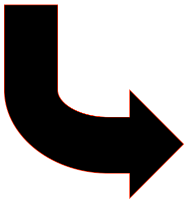
Processes on networks

- Robustness, resilience
- Random walks
- Diffusion, spreading
- Rumor propagation
- Opinion/consensus formation
- Cooperative phenomena
- Synchronization



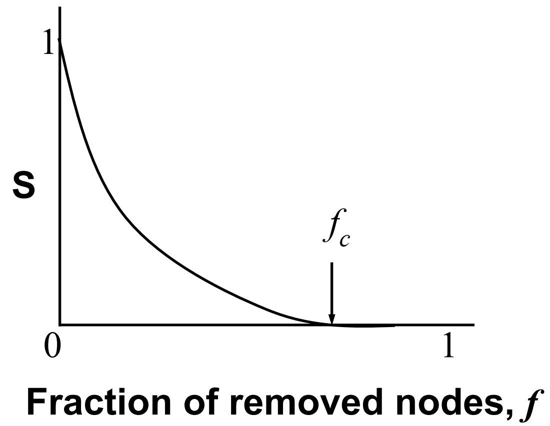


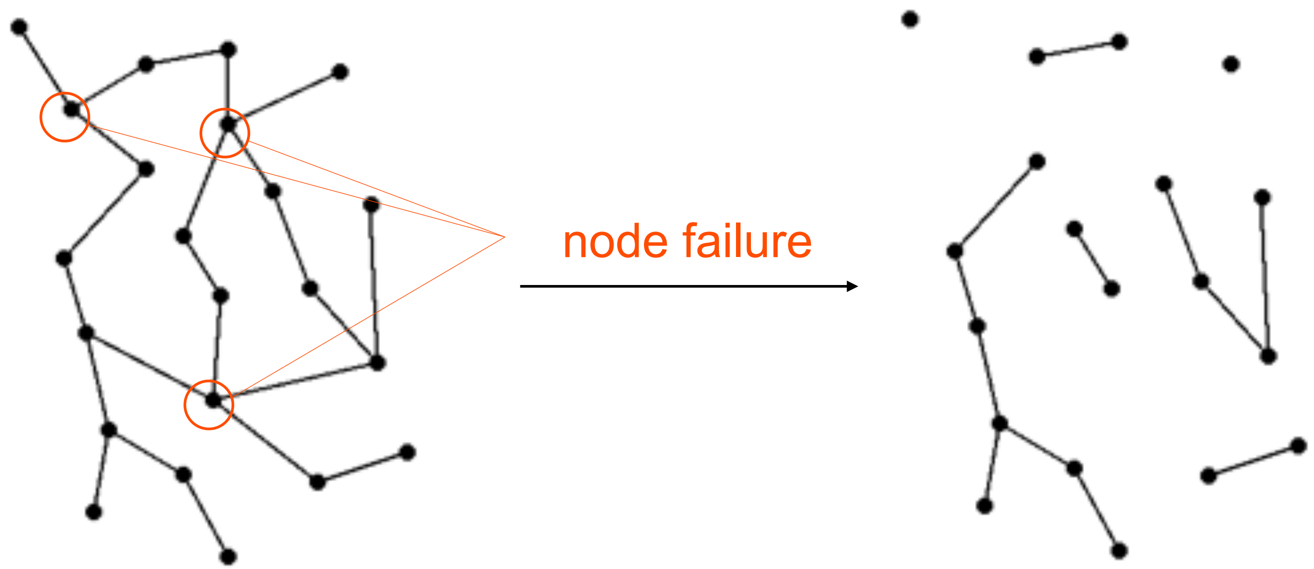
Studies of the role of the topology of the network

Robustness

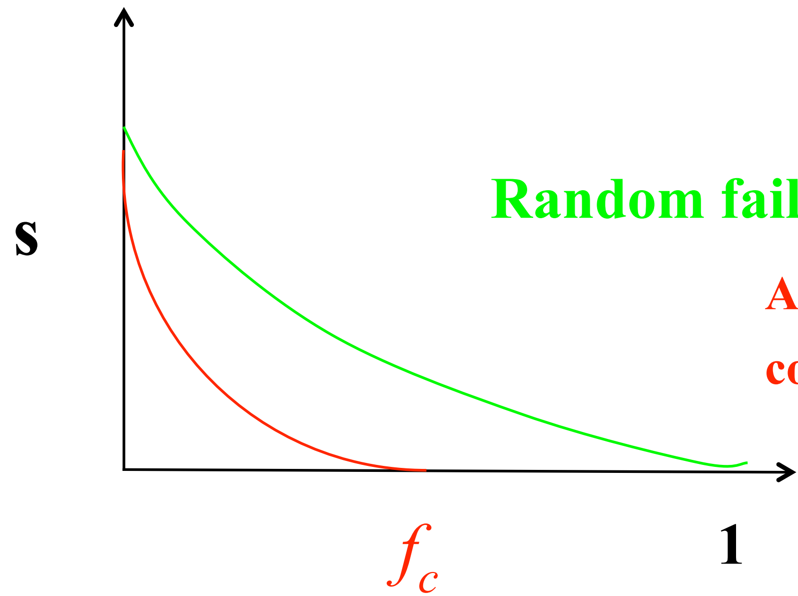
Complex systems maintain their basic functions
even under errors and failures
(cell → mutations; Internet → router breakdowns)

S: fraction of giant component





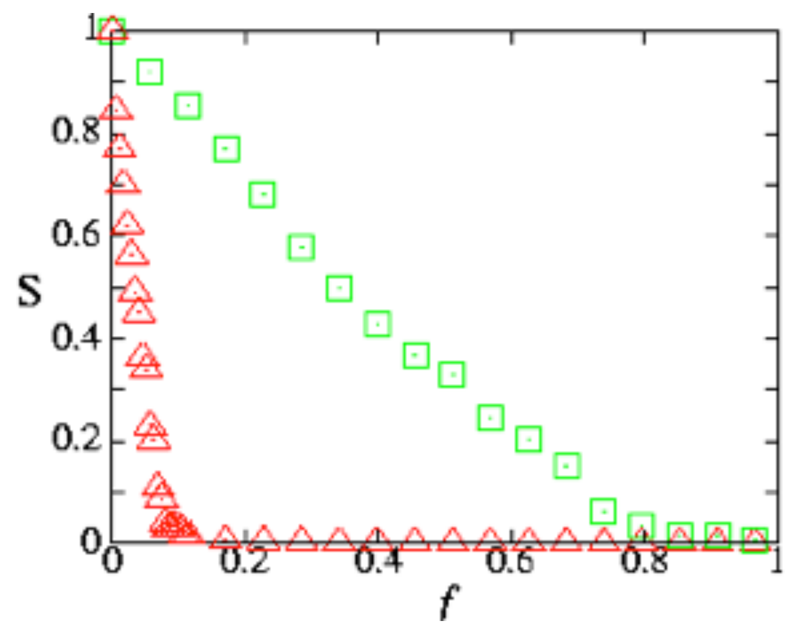
Case of Scale-free Networks



Random failure f_c =1 $(2 < \gamma \leq 3)$

Attack =progressive failure of the most connected nodes f_c <1

Internet maps

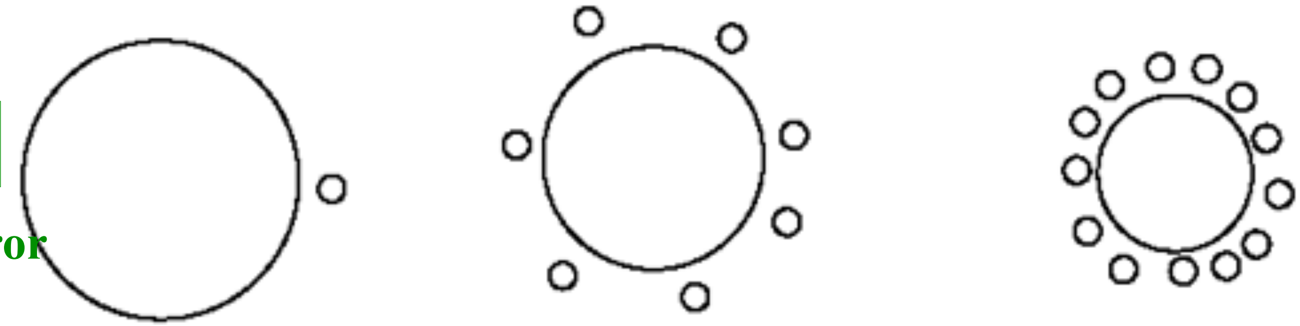


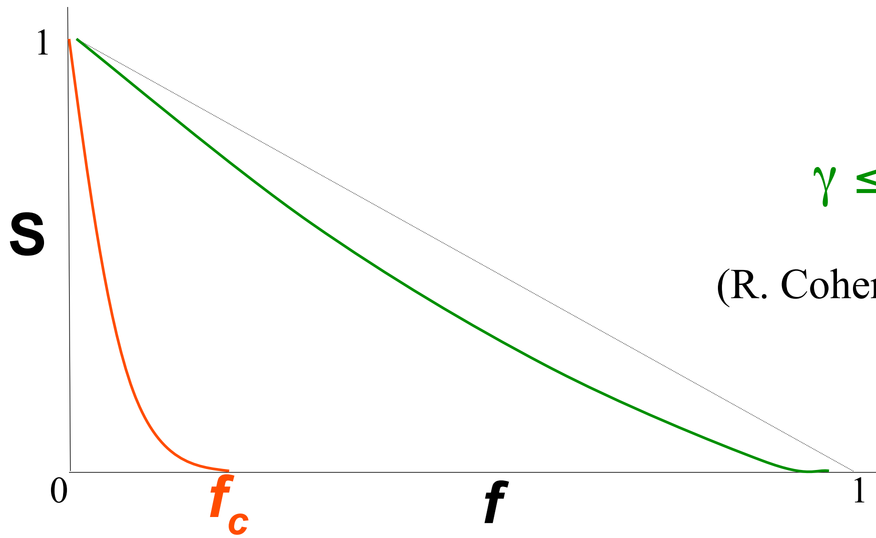
R. Albert, H. Jeong, A.L. Barabasi, Nature **406** 378 (2000)

Failures vs. attacks

Failures

Topological error tolerance



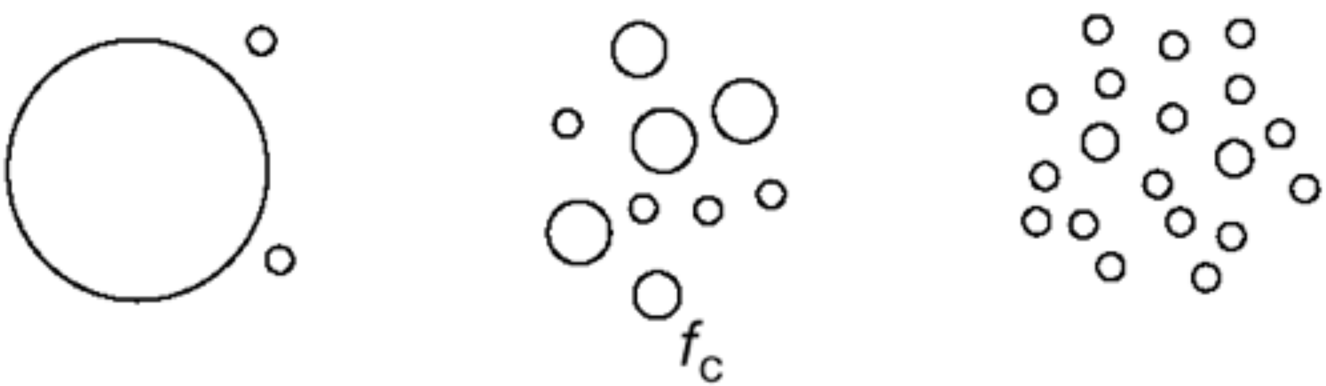


$\gamma \leq 3$: f_c=1

(R. Cohen et al PRL, 2000)

NB: mapping to percolation problem =>analytical solution

Attacks



Failures = percolation

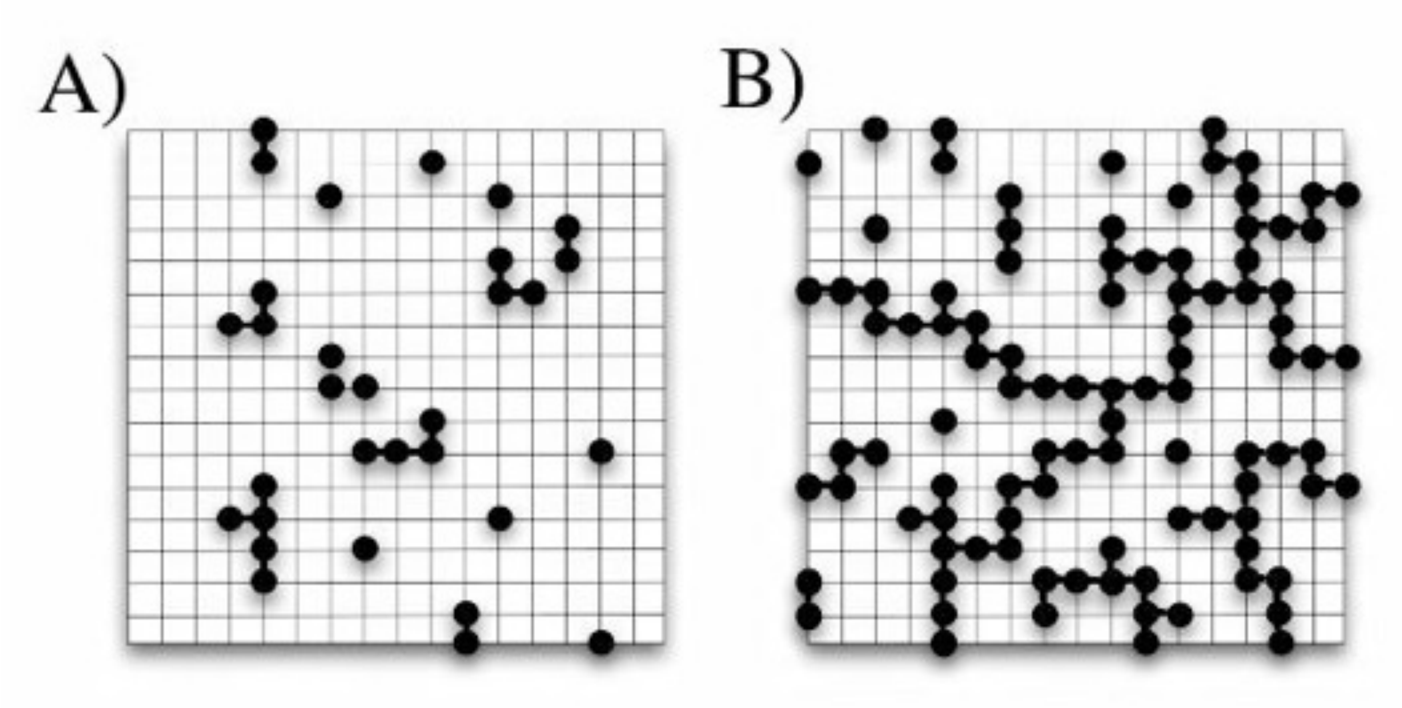
f=fraction of nodes removed because of failure

p=1-f

p=probability of a node to be present in a percolation problem

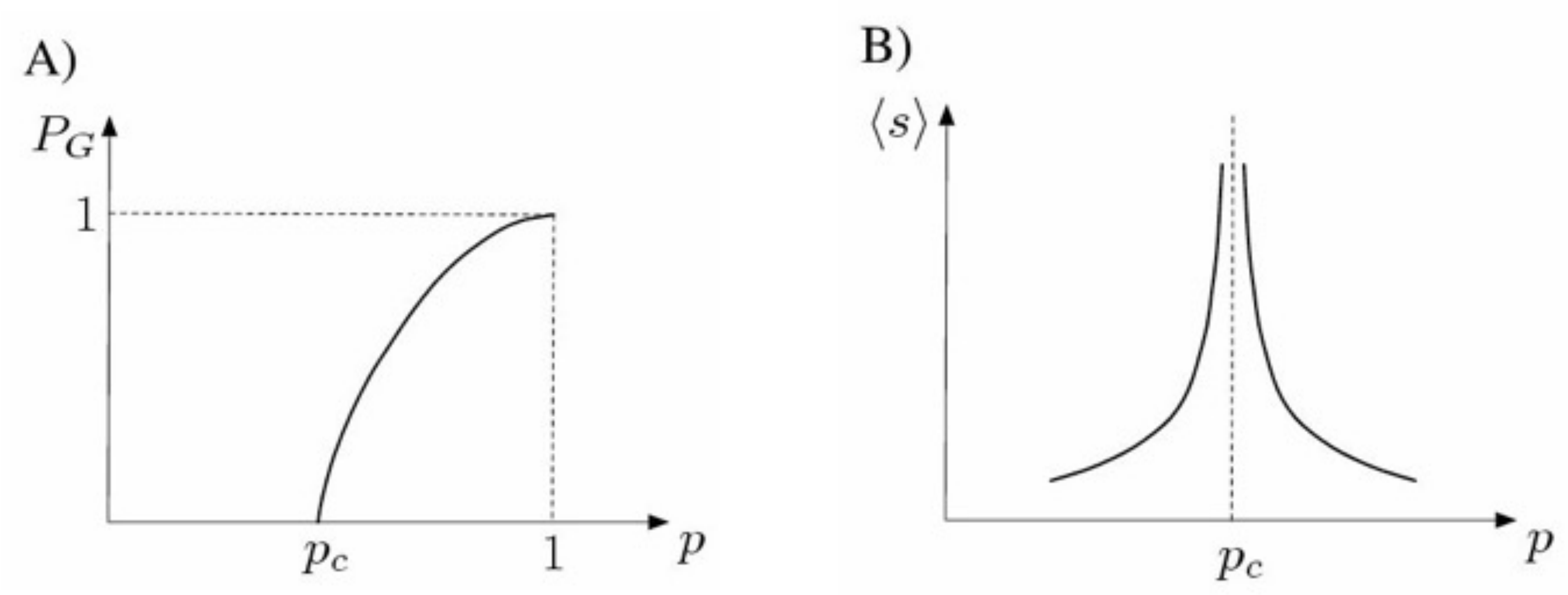
Question: existence or not of a giant/percolating cluster, i.e. of a connected cluster of nodes of size O(N)

Percolation



Question: existence or not of a giant/percolating cluster, i.e. of a connected cluster of nodes of size O(N)

Percolation



Question: existence or not of a giant/percolating cluster,
i.e. of a connected cluster of nodes of size O(N)

Analytical approach

Initial network: $P_0(k)$, $<k>_0$, $<k^2>_0$

existence of a giant cluster iff

$$f \leq f_c, \text{with } f_c = 1 - \frac{\langle k\rangle_0}{\langle k^2\rangle_0 - \langle k\rangle_0}$$

$$\langle k^2\rangle_0 \to \infty \qquad f_c \to 1$$

Robustness!!!

Attacks: various strategies

- Most connected nodes
- Nodes with largest betweenness
- Removal of links linked to nodes with large k
- Removal of links with largest betweenness
- Cascades
- ...

Attacks in weighted networks

Weighted quantities:

– For attack strategies

– For evaluating damage

Attacks in weighted networks

Example: transportation network

Centrality measures:

- Strength $s_i = \sum w_{ij}$
- Weighted betweenness centrality
- Distance strength $D_i = \sum_j d_{ij}$
- Outreach $O_i = \sum_j w_{ij} d_{ij}$

Attacks in weighted networks

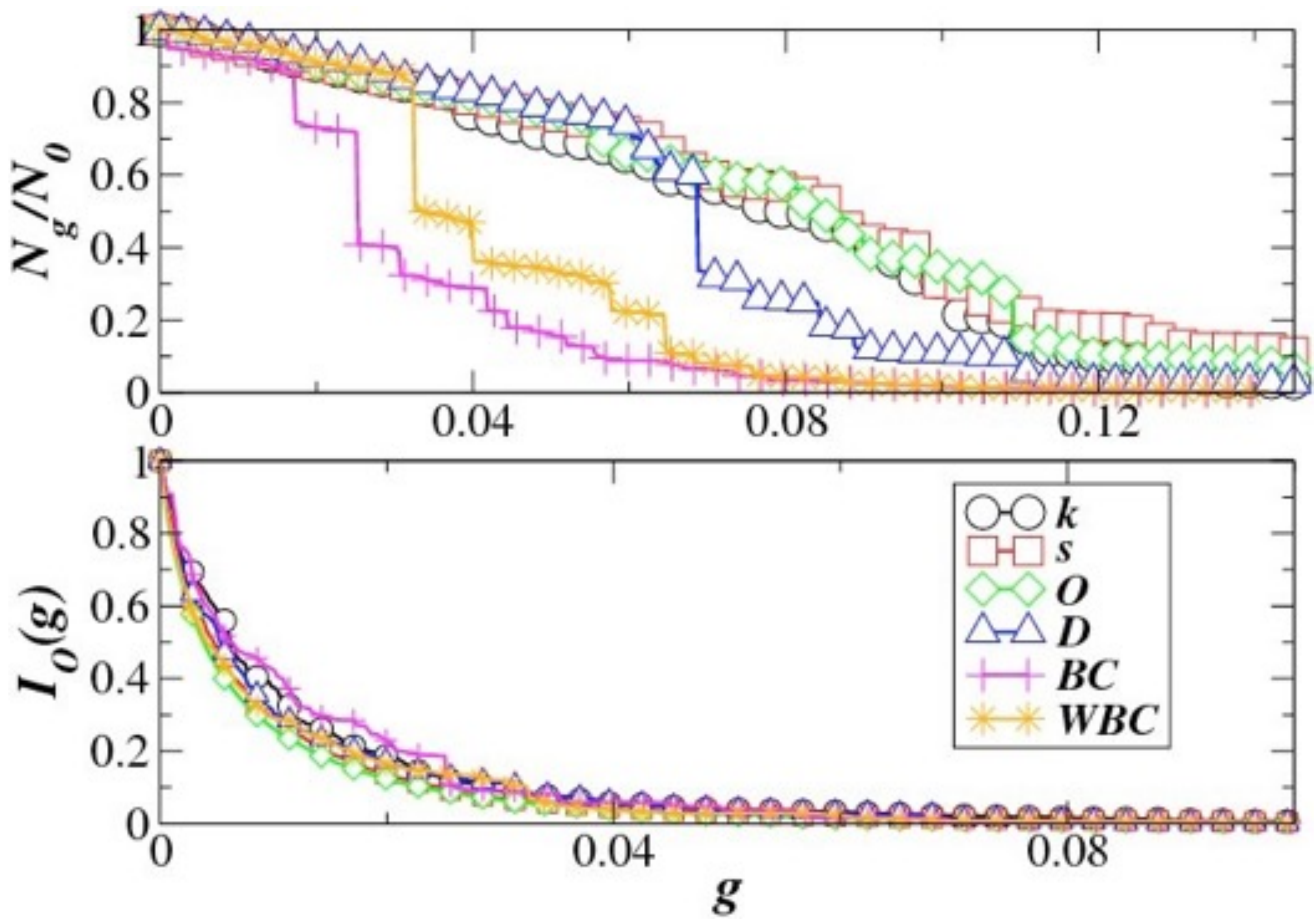
Example: transportation network

Damage measures:

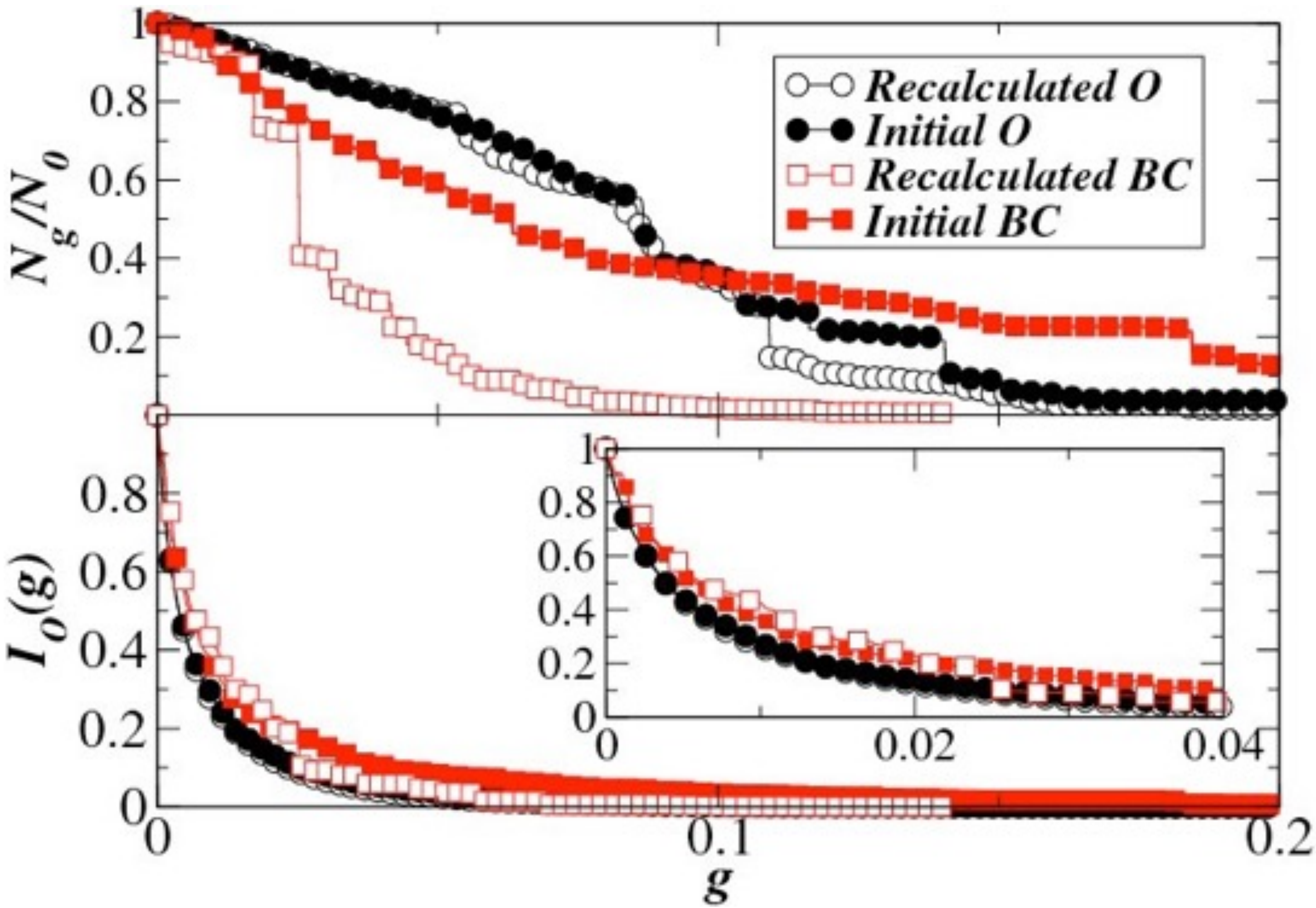
- Topological integrity N_g/N_0
- Weighted integrity measures:
  - Total strength S_g/S_0
  - Total distance strength, outreach…

Attacks in weighted networks

Example: world airport network



Note: recomputed quantities



Exercise

- Data:
  - http://www.cpt.univ-mrs.fr/~barrat/LYON_JAN2015/data.html
  - Create networks of $N=10^3$-10^5 nodes with same average degree (e.g., 5) according to various models (ER, WS, BA, UCM)
- Compute and plot basic properties (size, clustering coefficient, degree distribution, clustering vs degree, knn, shortest paths (sampling))
- Rank nodes according to degrees/betweenness
- Remove nodes one after the other
  - at random
  - by decreasing order of degree (/strength if weighted network)
  - by decreasing order of betweenness centrality
- After each removal, compute the size of the largest connected component
- Plot this size versus the number of nodes removed
- Do it again, but recomputing the ranking after each node removal
- Compare the results