2-species exclusion processes and combinatorial algebras

Sylvie Corteel Arthur Nunge

IRIF, LIGM

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For all n, define

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$$S^I = S_{i_1} S_{i_2} \cdots S_{i_r}.$$

For example, $S_2(a_1, a_2, a_3) = a_1^2 + a_1 a_2 + a_1 a_3 + a_2^2 + a_2 a_3 + a_3^2$.

Ribbon basis

$$R_I = \sum_{J \preceq I} (-1)^{l(J)-l(I)} S^J.$$

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Polynomial realization

$$R_I = \sum_{\mathrm{Des}(w)=I} w.$$

For example, $R_{221}(a_1, a_2) = a_1 a_2 a_1 a_2 a_1 + a_2 a_2 a_1 a_2 a_1$.

Tevlin's bases

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Transition matrices

The transition matrices between the ribbon basis and the fundamental basis of size 3 and 4 are:

$$\mathfrak{M}_3 = \begin{pmatrix} 1 & . & . & . \\ . & 2 & 1 & . \\ . & . & 1 & . \\ . & . & . & 1 \end{pmatrix}$$

$$\mathfrak{M}_4 = \begin{pmatrix} 1 & . & . & . & . & . & . & . \\ . & 3 & 2 & . & 1 & 1 & . & . \\ . & . & 2 & . & 1 & . & . & . \\ . & . & 1 & 3 & . & 2 & 1 & . \\ . & . & . & . & 1 & . & . & . \\ . & . & . & . & . & 2 & 1 & . \\ . & . & . & . & . & . & 1 & . \\ . & . & . & . & . & . & . & 1 \end{pmatrix}$$

Statistics on permutations

- $\mathrm{Rec}(\sigma)$ is the composition associated with the values of recoils (*i.e.*, the values k such that $k+1$ is on the left).
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- $\mathrm{GC}(\sigma)$ is the composition associated with the values of descents (*i.e.*, the values $k = \sigma_i$ such that $\sigma_i > \sigma_{i+1}$) minus one.
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Combinatorial interpretation (F. Hivert, J.-C. Novelli, L. Tevlin, J.-Y. Thibon, 2009)

$$\begin{pmatrix} 1 & . & . & . & . & . & . & . \\ . & 3 & 2 & . & 1 & 1 & . & . \\ . & . & 2 & . & 1 & . & . & . \\ . & . & 1 & 3 & . & 2 & 1 & . \\ . & . & . & . & 1 & . & . & . \\ . & . & . & . & . & 2 & 1 & . \\ . & . & . & . & . & . & 1 & . \\ . & . & . & . & . & . & . & 1 \end{pmatrix}$$

GC \ Rec	4	31	22	211	13	121	112	1111
4	1234							
31		1243, 1423 4123	1342 3412		2341	2413		
22			1324 3124		2314			
211			3142	1432, 4132 4312		2431 4231	3241	
13					2134			
121						2143 4213	3421	
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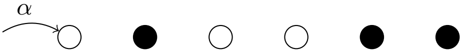

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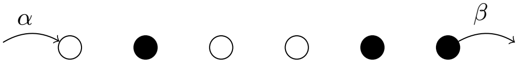

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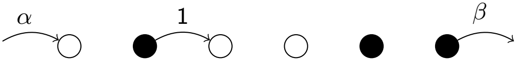

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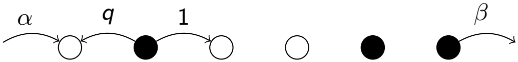

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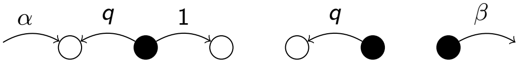

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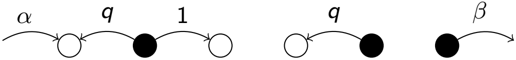

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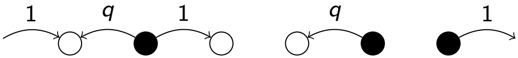

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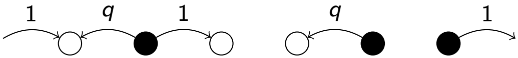

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Combinatorial study of the ASEP

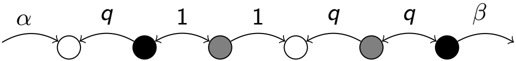
The ASEP is closely related with permutations. Let I be a composition associated to a state of the ASEP, the un-normalized steady-state probability of this state is given by

$$\sum_{\mathrm{GC}(\sigma)=I} q^{\#_{31-2}(\sigma)}$$

where $\#_{31-2}(\sigma)$ count the number of $31-2$ patterns in σ.

2-ASEP

The 2-ASEP is a generalization of the ASEP with two kinds of particles.



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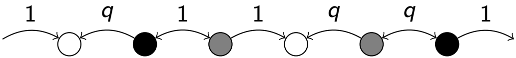

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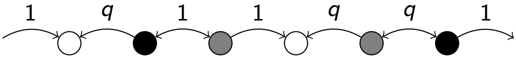

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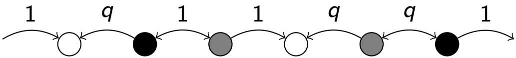

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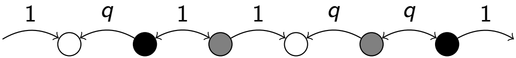

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What we want.

Let I be a segmented composition, the un-normalized steady-state probability of the state of the 2-ASEP associated with I is:

$$\sum_{\mathrm{GC}(\sigma)=I} q^{\#_{31\text{–}2}(\sigma)}$$

where the sum goes over all permutations.

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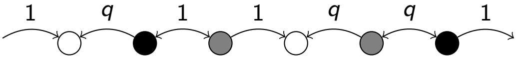

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What we have.

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$$\sum_{\mathrm{GC}(\sigma)=I} q^{\#_{31\text{–}2}(\sigma)+\#_{(31,\bar{2})}(\sigma)}$$

where the sum goes over all partially signed permutations.

Partially signed permutations

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- $\mathrm{Rec}(\sigma)$ is computed as previously, we add bars on the composition to retrieve the position of the overlined values in σ.
  For $\sigma = \overline{2}57836\overline{4}1$, $\mathrm{Rec}(\overline{2}57836\overline{4}1) =$.

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  For $\sigma = \overline{2}5{\color{blue}7}83{\color{red}6}\overline{4}1$, $\mathrm{Rec}(\overline{2}57836\overline{4}1) = 1|2|1{\color{red}2}$.

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- GC(σ) is computed as previously, we add bars on the composition to retrieve the position of the overlined values in σ.
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The algebra of segmented compositions

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Ribbon basis

Again we have

$$R_I = \sum_{J \preceq I} (-1)^{l(J)-l(I)} S^J.$$

For example, $R_{22|41} = S_{22|41} - S_{4|41} - S_{22|5} + S_{4|5}$.

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Transition matrix

The coefficients in the transition matrices from the ribbon basis to the fundamental basis are

$$(\mathcal{M}_n)_{I,J} = \#\{\sigma \mid \mathsf{GC}(\sigma) = I, \mathrm{Rec}(\sigma) = J\}$$

$$\mathcal{M}_3 = \left(\begin{array}{cccc|cc|cc|c}
1 & . & . & . & . & . & . & . & . \\
. & 2 & 1 & . & . & . & . & . & . \\
. & . & 1 & . & . & . & . & . & . \\
. & . & . & 1 & . & . & . & . & . \\
\hline
. & . & . & . & 3 & 1 & . & . & . \\
. & . & . & . & . & 2 & . & . & . \\
\hline
. & . & . & . & . & . & 2 & . & . \\
. & . & . & . & . & . & 1 & 3 & . \\
\hline
. & . & . & . & . & . & . & . & 6
\end{array}\right)$$

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- Definition of q-analogs of the bases of **SCQSym** and study of the transition matrices.

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Perspectives

- find α and β statistics on partially signed permutations.
- Understand the refinement $(\mathrm{GC}, \mathrm{Rec})$ on the 2-ASEP.