
Examen MPRI 2015/2016
"Modélisation par automates finis"
Automata and semigroups

Books and computers forbidden — Lecture and personal notes allowed
Questions 1 and 2 are independant.

We consider the monoids

$$\mathbf{H} = \langle \mathbf{a}, \mathbf{b} : \mathbf{aba} = \mathbf{a} \rangle_+^1 \quad \text{and} \quad \mathbf{K} = \langle \mathbf{a}, \mathbf{b} : \mathbf{aba} = \mathbf{b} \rangle_+^1.$$

- 1.a For each, draw the fragment of the Cayley graph restricted to these elements of length at most 3.
 - b For each, give the first coefficients and a possible recurrence (at least conjectural) for its growth function.
 - c For each, exhibit, when possible, an infinite sequence $\cdots \prec f_2 \prec f_1 \prec f_0$ of proper left divisions (we have $f \prec g$ if there exists $h \neq 1$ satisfying $fh = g$).
 - d For each, describe a Garside family (strictly included in its monoid).
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- 2.a For each, give a system of (two) rewriting rules, each of the form $x_1x_2 \rightarrow x_3$ with $x_i \in \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ for $i \in \{1, 2, 3\}$, where \mathbf{c} is a new generator corresponding with the element represented by \mathbf{ab} .
 - b For each, deduce two quadratic rewriting systems over the alphabet $\{1, \mathbf{a}, \mathbf{b}, \mathbf{c}\}$ where 1 denotes the unit (one with two rules, the other with three rules).
 - c Provide a way to associate with any quadratic rewriting system some graph on which one can recognise whether it corresponds to a quadratic normalisation and, in which case, compute its complexity.
Describe some relevant fragments of those associated with the systems from Question 2.b.
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- 3.a For each, build a Mealy automaton over the alphabet $\{1, \mathbf{a}, \mathbf{b}, \mathbf{c}\}$.
 - b Explicit some criteria allowing to decide whether such a Mealy automaton generates (or does not generate) the original monoid.
Specify which ones apply to automata from Question 3.a.
 - c Explain the link between such Mealy automata (whether they generate the monoid or not) and the growth functions of the monoid (like the one from Question 1.b).