Exercise 1

We consider the two following monoids:

\[ J = \left\langle a, b, c : \begin{array}{l} ab = ba \\ bc = cb \end{array} \right\rangle^1 + \text{ et } K = \left\langle a, b, c : \begin{array}{l} bab = ba \\ cbc = cb \end{array} \right\rangle^1. \]

1. For each, compute explicitly its minimal Garside family. Is it finite?
2. For each, explain what one can deduce: automaticity? self-similarity? etc.

We consider a third monoid:

\[ L = \left\langle a, b, c : \begin{array}{l} bab = ba \\ bc = cbc \end{array} \right\rangle^1. \]

3. Specify what distinguishes \( L \) from \( J \) and \( K \)? Provide therefore an alternative approach.

Exercise 2

We aim to enumerate the quadratic normalisations over an alphabet \( Q \) of given size.

1. Give a simple algorithm to enumerate the idempotent functions from \( Q^2 \) to \( Q^2 \). Compute the number of such functions.
2. Give an algorithm to then check whether such a function define a quadratic normalisation, while computing its complexity whenever this is indeed the case.
3. Apply these algorithms for \( Q = \{a, b\} \) while reducing by symmetry the number of cases to be treated.