Examen MPRI 2016/2017
"Modélisation par automates finis"
Automata and semigroups

Books and computers forbidden — Lecture and personal notes allowed

Exercise 1

We consider the two following monoids :

$$\mathbf{J} = \left\langle \mathtt{a}, \mathtt{b}, \mathtt{c} : \begin{array}{c} \mathtt{ab} = \mathtt{ba} \\ \mathtt{ac} = \mathtt{ca} \\ \mathtt{bc} = \mathtt{cb} \end{array} \right\rangle_+^1 \qquad \text{et} \qquad \mathbf{K} = \left\langle \mathtt{a}, \mathtt{b}, \mathtt{c} : \begin{array}{c} \mathtt{bab} = \mathtt{ba} \\ \mathtt{ac} = \mathtt{ca} \\ \mathtt{cbc} = \mathtt{cb} \end{array} \right\rangle_+^1 .$$

1. For each, compute explicitly its minimal Garside family. Is it finite ?
2. For each, explain what one can deduce : automaticity ? self-similarity ? etc.

We consider a third monoid :

$$\mathbf{L} = \left\langle \mathtt{a}, \mathtt{b}, \mathtt{c} : \begin{array}{c} \mathtt{bab} = \mathtt{ba} \\ \mathtt{ac} = \mathtt{ca} \\ \mathtt{bc} = \mathtt{cbc} \end{array} \right\rangle_+^1 .$$

3. Specify what distinguishes $\mathbf{L}$ from $\mathbf{J}$ and $\mathbf{K}$? Provide therefore an alternative approach.

Exercise 2

We aim to enumerate the quadratic normalisations over an alphabet $\mathcal{Q}$ of given size.

1. Give a simple algorithm to enumerate the idempotent functions from $\mathcal{Q}^2$ to $\mathcal{Q}^2$. Compute the number of such functions.
2. Give an algorithm to then check whether such a function define a quadratic normalisation, while computing its *complexity* whenever this is indeed the case.
3. Apply these algorithmes for $\mathcal{Q} = \{\mathtt{a}, \mathtt{b}\}$ while reducing by symmetry the number of cases to be treated.