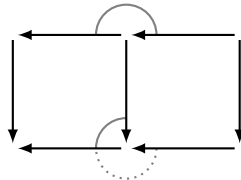

Examen MPRI 2017/2018
 "Modélisation par automates finis"
 Automata and semigroups

Books and computers forbidden — Lecture and personal notes allowed

Exercise 1

Recall that, for any family Q of a monoid M , a decomposition $q_2q_1 \in Q^2$ is said to be Q -normal, which is denoted by $\xleftarrow{q_1} \overset{q_2}{\curvearrowright}$ when, for any $q \in Q$, for any $f \in M$, if q right-divides q_2q_1f , then q right-divides q_1f .

0. Draw the diagram on which appear the arrows for q_1, q_2, q, f and some others.
1. Show that, if $q_2q_1 \in Q^2$ is Q -normal, so is p_2p_1 , for every right-multiple $p_1 \in Q$ of q_1 and every right-divisor $p_2 \in Q$ of q_2 .
2. Prove the *domino property* corresponding with the diagram below (where the hypotheses are indicated with solid lines and the conclusion with a dashed arc) :



3. Explain the connection between the domino property and the complexity $(4, 3)$ for a quadratic normalisation.

Exercise 2

Let us consider the monoids

$$\mathbf{L} = \langle \mathbf{a}, \mathbf{b} : \mathbf{ab} = \mathbf{ba} = \mathbf{aa} = \mathbf{bb} \rangle_+^1$$

$$\mathbf{W} = \langle \mathbf{a}, \mathbf{b}, \mathbf{c} : \mathbf{aa} = \mathbf{cc} = \mathbf{bc}, \mathbf{ba} = \mathbf{cb} = \mathbf{ab}, \mathbf{bb} = \mathbf{ca} = \mathbf{ac} \rangle_+^1.$$

Questions 1 and 2 below are independent. And, within both of them, Questions (b) and (c) can be also treated independently.

0. Show that \mathbf{L} and \mathbf{W} are infinite.
- 1.(a) Enumerate the quadratic normalisations over $\{\mathbf{a}, \mathbf{b}\}$ associated with \mathbf{L} . Let us notice that the chosen alphabet *is not* $\{1, \mathbf{a}, \mathbf{b}\}$.
Choose one of them.
 - (b) Compute its complexity. It can be convenient to build the associated graph.
 - (c1) Draw the Mealy automaton \mathcal{L} associated with this quadratic normalisation.
 - (c2) Draw its square \mathcal{L}^2 , then minimise \mathcal{L}^2 .
 - (d) Conclude with a one-sentence summary.
- 2.(a) Give a quadratic normalisation \mathbf{N} over $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ associated with \mathbf{W} for which \mathbf{bc} , \mathbf{ab} , and \mathbf{ac} are \mathbf{N} -normal. Let us notice that the chosen alphabet *is not* $\{1, \mathbf{a}, \mathbf{b}, \mathbf{c}\}$.
 - (b) Compute the complexity of \mathbf{N} . It can be convenient to build the associated graph.
 - (c1) Draw the Mealy automaton \mathcal{W} associated with the quadratic normalisation \mathbf{N} .
 - (c2) Draw the $\mathbf{m}\mathfrak{d}$ -reduction sequence for \mathcal{W} .
 - (d) Conclude with a one-sentence summary.
3. Based upon the argument given for Question 0 and the answers to Questions 1 and 2, explain the possible consequences of the lack of the unit 1 in the chosen alphabet.