Course 2.18.1: Distributed Algorithms for Networks
Port-Numbering Model

Mikaël Rabie

1 Definition - Slides 2-7

Informally, the Port-Numbering model is a simplification of the LOCAL model where we remove the identifiers. It has the following properties:

- Graph structure G
- Each node knows its degree, and can differentiate its neighbors
- Communication is synchronous - At each step, you know from which neighbor you receive which message

2 3 Algorithms - Slides 8-14

2.1 Maximal Matching - Slide 11

Input: Bipartite Black/White Δ-regular graph (each node knows if it is black or white).
The idea of the algorithm is to loop as follows:

For $i = 1$ to Δ do:

1. unmatched White nodes send a proposal to Black neighbor through port number i
2. unmatched Black nodes accept the proposal they got from smaller port (if they received any)
3. White nodes know if their proposal got accepted

Theorem 1 *The algorithm provides a maximal matching.*

Proof: We need to prove that we have a matching, and that it is maximal.

If a White node is matched, it means that a Black node has accepted its proposal. Hence, both nodes are matched together. Moreover, White nodes send proposals one at a time and stop when they are matched, so they are not matched to several nodes. Black nodes accept only one proposal and then stop when they receive proposals in a round, so they are not matched to several nodes.

Let assume that we could have added one edge (b, w). w is linked to b through some port k. As it was not matched at round $i = k$, w has sent a proposal at round k. As b did not accept the proposal, it means it has accepted another one, hence b must be matched, which leads to a contradiction. $\square$

2.2 Vertex Cover - Slide 13

To compute the Vertex Cover of an arbitrary graph, we do the following steps:

1. Each node u virtually duplicates itself, by acting as it was two nodes, its own Black version u_B and its White version u_W.
2. If you had the edge (u, v) in G, you put edges (u_B, v_W) and (u_W, v_B) in the virtual graph.
   - One step in the virtual graph can be simulated in one step in G, as the neighbors are simulated by neighboring nodes.
   - The virtual graph is bipartite, and each node knows its partition.
3. We run the Maximal Matching in the virtual graph.
4. We put in the Vertex Cover nodes such that at least one of its virtual copy is matched in the Maximal Matching.

3 Impossibility - 15-17

The key element for impossibility to resolve a problem is indistinguishability. If an algorithm runs in T rounds, two nodes that have the same neighborhood after that amount of rounds must have the same output. In the first path (Slide 15), both nodes will have the same color, the graph will not be properly colored. Same for the two middle nodes on the second path. For Maximal Matchings on cycles, on the two examples, each node will have the same radius-T view, and hence will output the same thing, that cannot lead to a Maximal Matching. You can notice that in the Port Numbering Model, as there are no identifiers, nodes cannot even know the size of the cycle, as the radius-T view will be the same, for any cycle, as long as the ports are always of the form $1 - 2$ if we follow the cycle through the same direction.