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Ramsey theory and coloring for signed graphs

The simplest form of Ramsey theorem is the following:

Theorem 1. *Given positive integers s, t, there exists a positive integer n such that for any 2-edge-coloring of the complete graph K_n with colors red and blue, either we have a K_s whose edges are all red or a K_t whose edges are all blue.*

Smallest value of n that works for this theorem is define to be $R(s,t)$. When $k, t \geq 3$, the question of determining $R(s,t)$ is known to be notoriously difficult and only a few pairs of s and k the exact value is known.

In this project we would like to study a problem of similar nature using the notion of signed graphs. A signed graph (G, σ) is a graph together with a mapping σ which assigns to each edge a sign ($+$ or $-$). A key notion here then is *switching*: that is to inverse the sign of each edge in an edge-cut. If (G, σ') is obtained from a switching on (G, σ), then they are said to be *equivalent*. A signed graph is said to be *balanced* (respectively *antibalanced*) if it can be switched so that all edge are positive (negative).

A general question then would be: given a signed graph what is its largest subgraph or induced subgraph which is balanced or antibalanced. In this regard, a Ramsey Theorem can be stated as follows.

Theorem 2. *Given positive integers s, t, there exists a positive integer n such that for any signature on complete graph K_n either we have an antibalanced s-clique or a balanced t-clique.*

The question to work on then would finding the best values of $n(s,t)$, and compare this to classic Ramsey numbers.

Depending on the progress, the work may develop in different directions. While the question of finding the largest balanced or antibalanced induced subgraph is similar in nature to find largest independent set or a largest clique, the question of partitioning vertices into such subgraphs is similar to coloring problems. A particular direction then would be to study coloring of signed graphs using complex number as defined in the following reference.

Reference R. Naserasr and L.A. Pham. Complex and homomorphic chromatic number of signed planar simple graphs. https://hal.archives-ouvertes.fr/hal-03000542