

①

Ex 1:

1. $(a+ba)^*$

Mots de taille 1: a

_____ 2: ba, aa

_____ 3: aba, baa, aaa

_____ 4: haba, aaba, baaa, abaa, aaaa

2. $a(aa+b(ab)^*a)^*a$

Mots de taille 1: aucun

_____ 2: aa

_____ 3: aucun

_____ 4: aaaa, abaa

Ex 2: 1. Après avoir lu bbabbb, on arrive dans l'état 2.

Non ce mot n'est pas reconnu par l'automate

Après avoir lu bababba, on arrive dans l'état 5 qui est final \rightarrow ce mot est donc reconnu par l'automate.

2. baa n'est pas reconnu car il n'y a pas de chemin partant de l'état initial 1, étiqueté par baa

Exercice 3.

(2)

1. Mots de taille 1: b

Mots de taille 2: ba, ab, aa

_____ 3: aaa, aab, aba, abb, baa

_____ 4: aaaa, aaab, aaba, aabb,
abaa, abab, abba, abbb,
baaa

2. Mots de taille 1: a

_____ 2: aa, bb

_____ 3: aaa, abb, bba

_____ 4: aaaa, abba, aabb, bbaa, bbab

Exercice 4.

1. $(a+tb)(a+tb)$

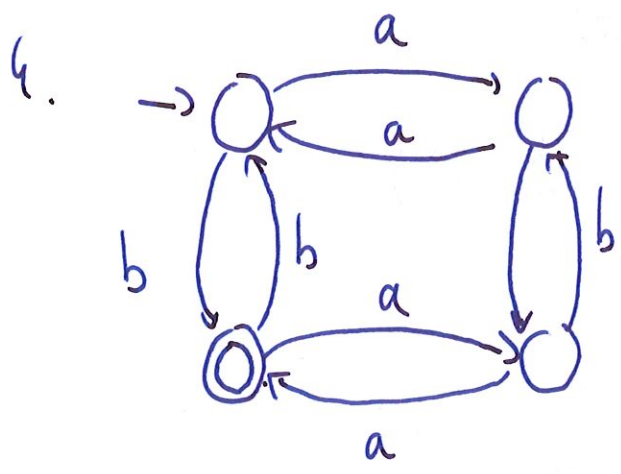
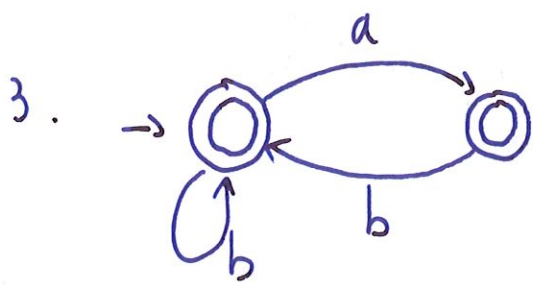
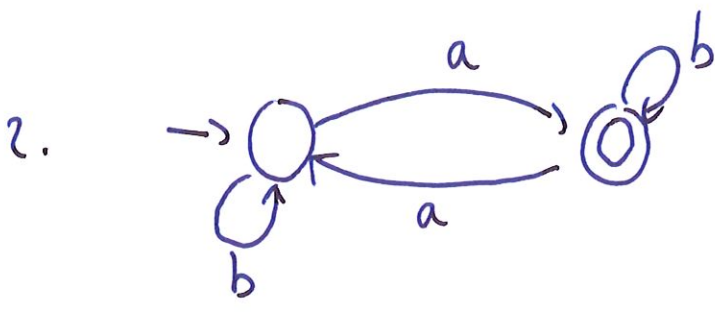
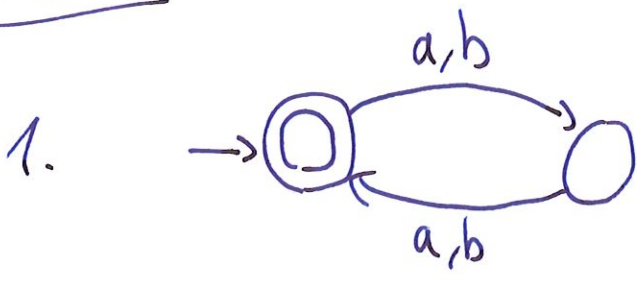
2. $((a+tb)(a+tb))^+(a+tb)$

3. $(a+tb)^*a(a+tb)^*$

4. $(b+ab+aab)^*(a+aa+\epsilon)$

5. ~~$(b+aa+bb)^*$~~ $(b+aa^*bb)^*(aa^*a+aa^*b+\epsilon)$

Exercises:

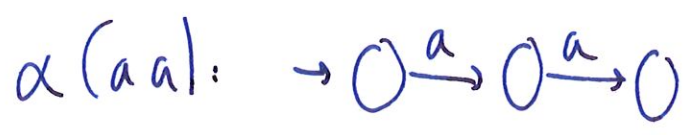
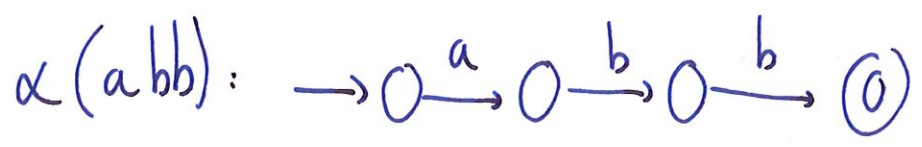
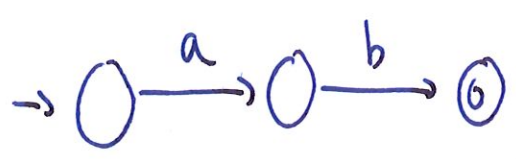


Exercice 6 :

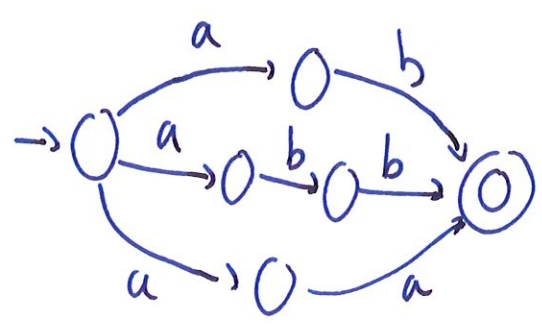
1. $(a+b)^* a b b a (a+b)^*$
2. $a^* a b^*$
3. $a (ab)^* (a+\epsilon)$
4. $a (aaaa)^* a$

Exercice 7 :

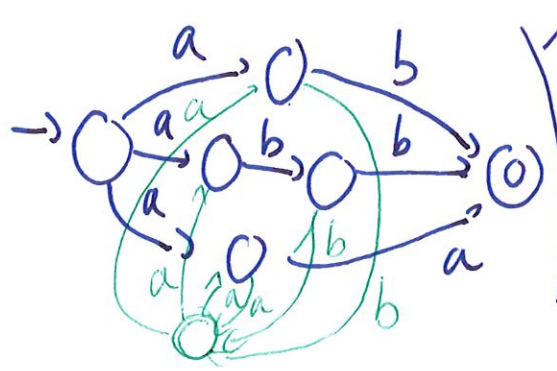
1. Automate pour $\alpha(ab) = \alpha(a)\alpha(b) + \epsilon(a)\alpha(b) + \alpha(a)\epsilon(b)$
 $= \alpha(a)\alpha(b)$



$\alpha(abtabbtaa)$



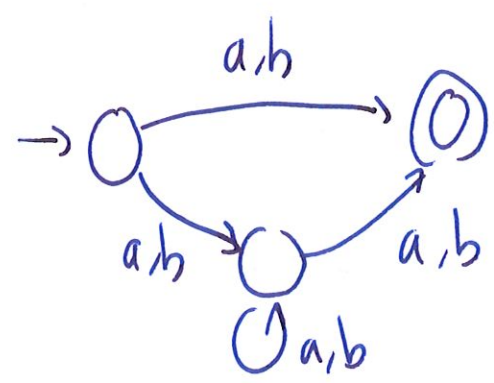
$\alpha((abtabbtaa)^*)$



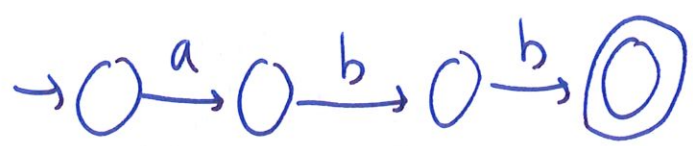
Comme $\epsilon \notin ((abtabbtaa)^*) = \{\epsilon\}$
 le lieu d'initial doit être final aussi à la fin!

$2 \propto ((atb)^+)$

(5)

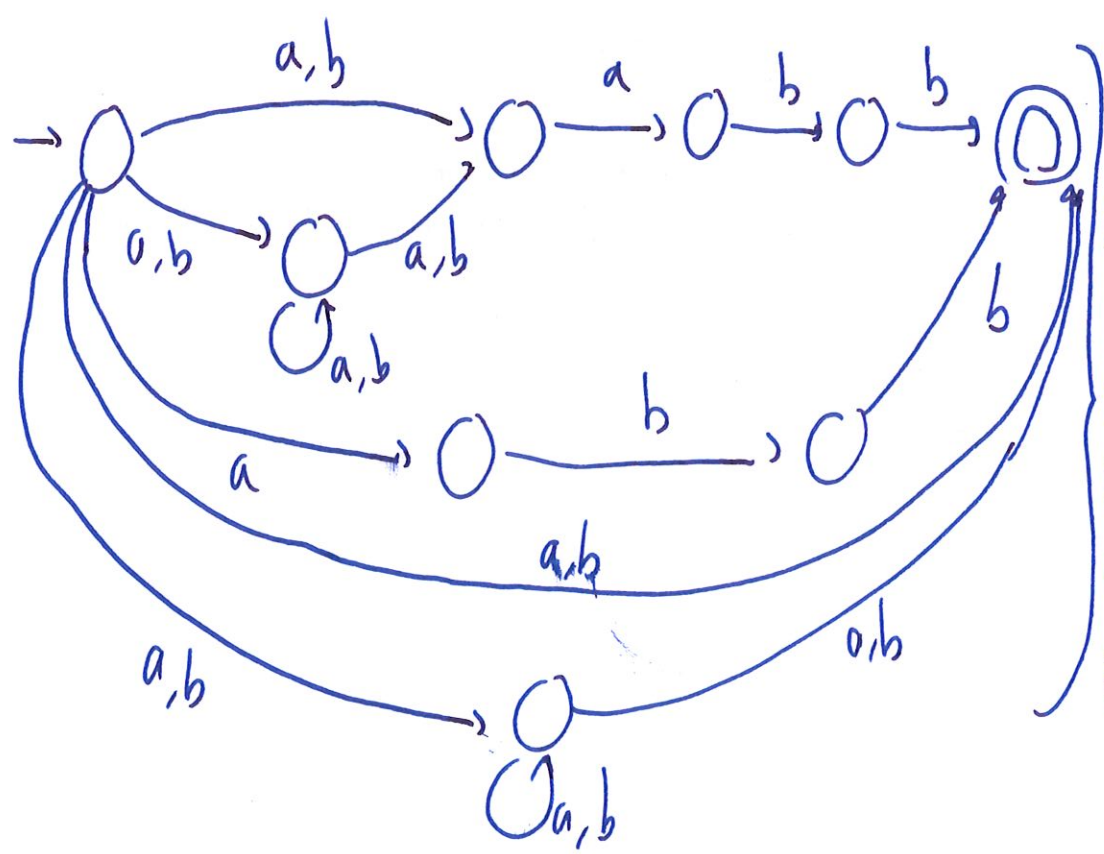


$\propto (abbt\epsilon)$



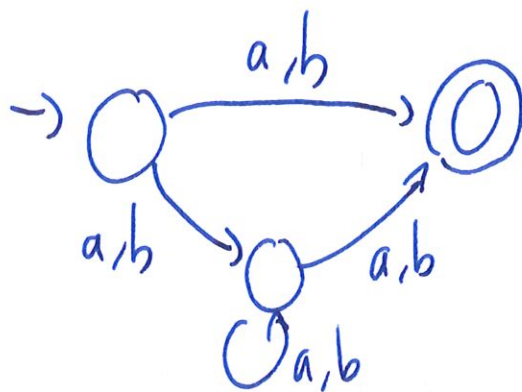
$$\begin{aligned} \propto ((atb)^+(abbt\epsilon)) &= \propto ((atb)^+) \propto (abbt\epsilon) + \epsilon ((atb)^+) \propto (abbt\epsilon) \\ &\quad + \cancel{\propto ((atb)^+)} \\ &\quad + \propto ((atb)^+) \epsilon (abbt\epsilon) \end{aligned}$$

$$= \propto ((atb)^+) \propto (abbt\epsilon) + \propto (abbt\epsilon) + \propto ((atb)^+)$$



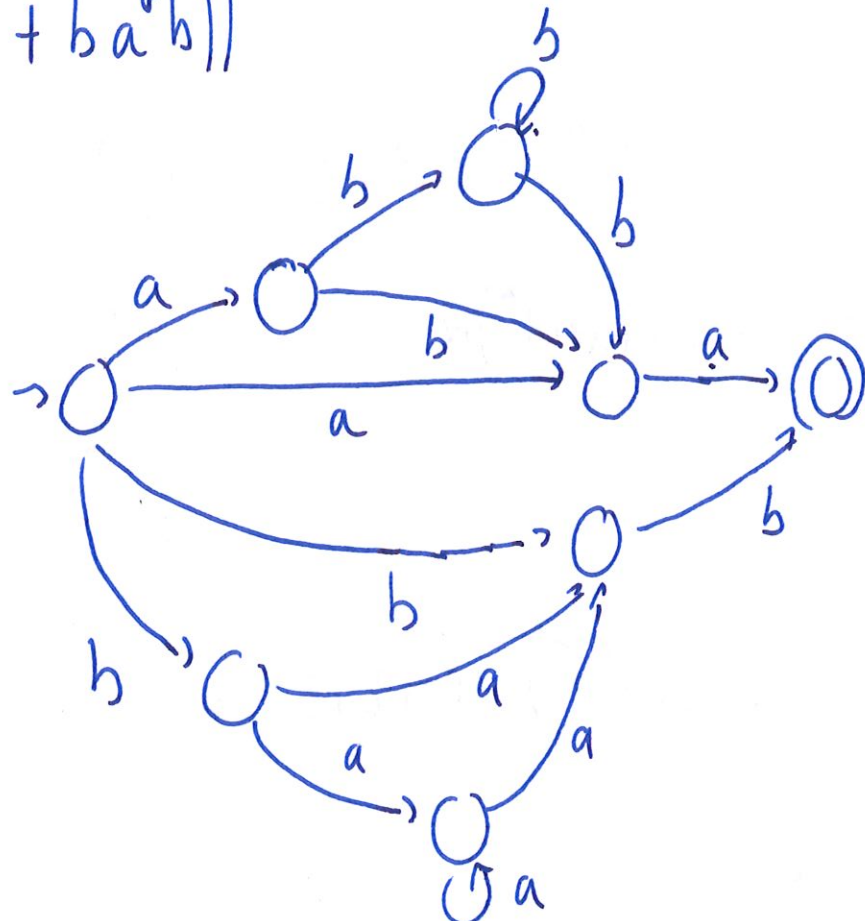
Comme $\epsilon((atb)^+(abbt\epsilon)) = \{\epsilon\}$ on doit mettre l'état initial comme final à la fin.

3. $\alpha((atb)^+)$

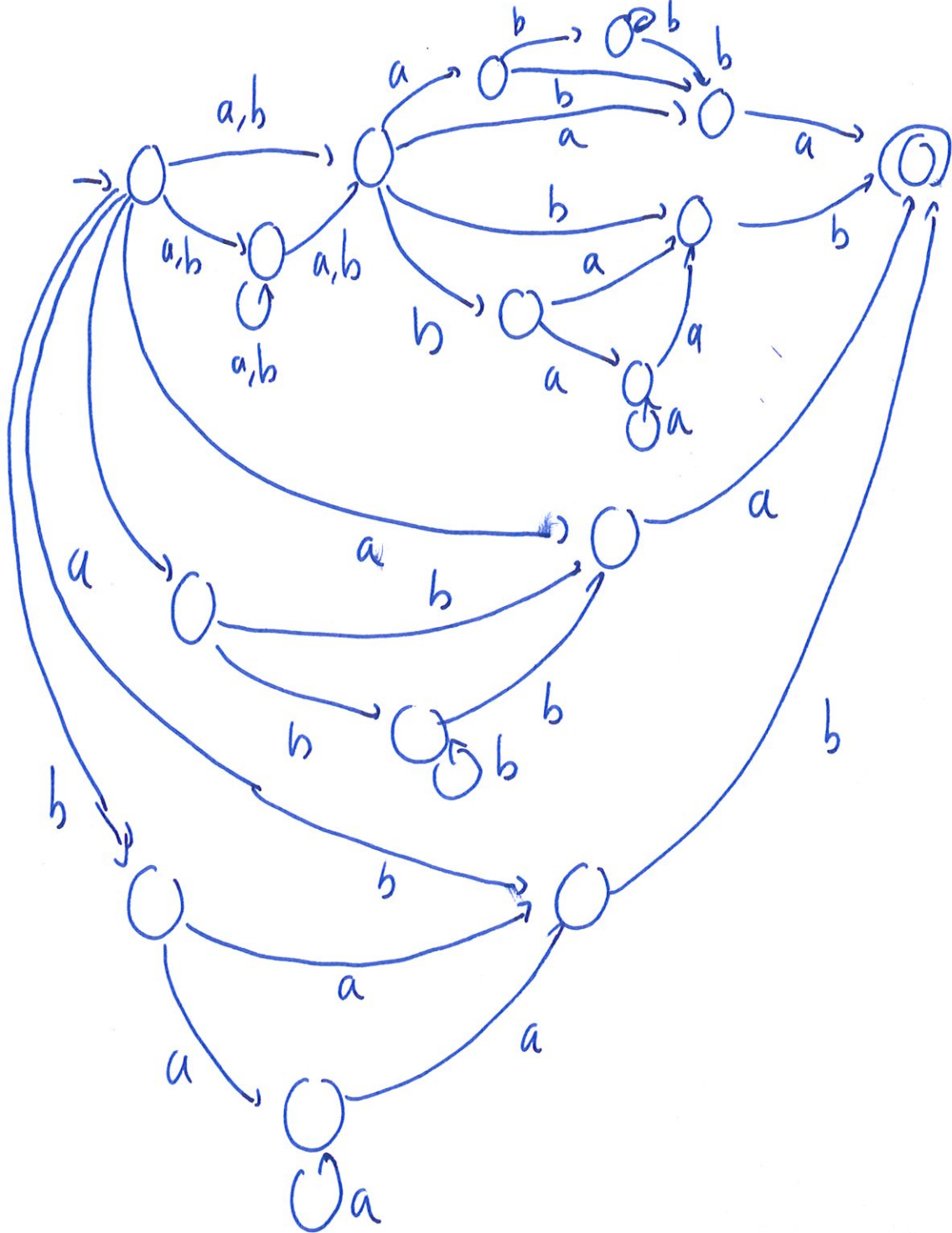


(6)

$\alpha((ab^+a + ba^+b))$



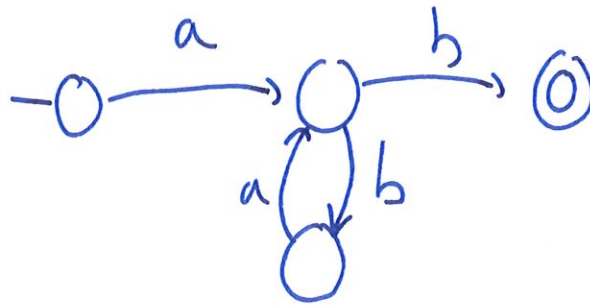
$$\alpha((atb)^+(ab^+a + ba^+b)) = \alpha((atb)^+) \alpha(ab^+a + ba^+b) + \alpha(ab^+a + ba^+b)$$



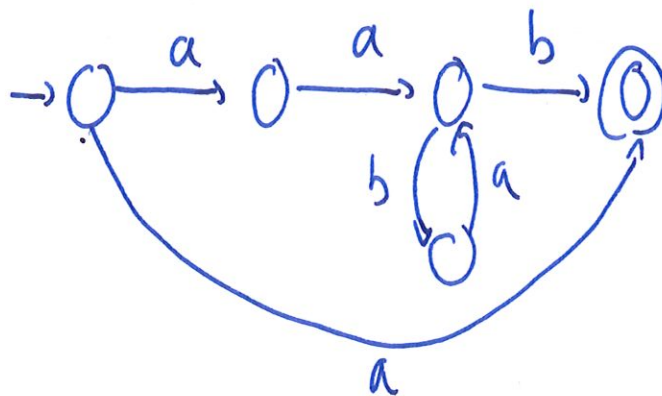
Ici on a $\epsilon((a+tb)^*(ab^*a + ba^*b)) = \emptyset$
donc l'état initial ne doit pas être final!

4. $\alpha((ab)^+)$

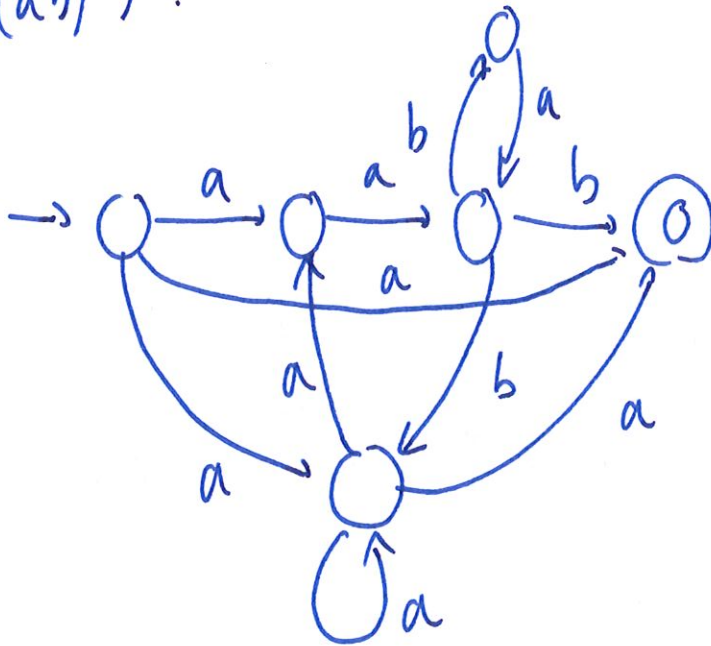
(8)



$$\alpha(a(ab)^+) = \alpha(a)\alpha((ab)^+) + \alpha(a)$$



$\alpha((a(ab)^+)^+)$:



Comme $\epsilon((a(ab)^+)^+) = \{\epsilon\}$
on doit mettre l'état initial comme final à la fin.