Quantization of random walks: Search algorithms and hitting time

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Extended abstract

Many classical search problems can be cast in the following abstract framework: Given a finite set X and a subset $M \subseteq X$ of marked elements, *detect* if M is empty or not, or *find* an element in M if there is any. When M is not empty, a naive approach to the finding problem is to repeatedly pick a uniformly random element of X until a marked element is sampled. A more sophisticated approach might use a Markov chain, that is a random walk on the state space X in order to generate the samples. In that case the resources spent for previous steps are often reused to generate the next sample. Random walks also model spatial search in physical regions where the possible moves are expressed by the edges of some specific graph. The *hitting time* of a Markov chain is the number of steps necessary to reach a marked element, starting from the stationary distribution of the chain.

In this survey talk we discuss quantum walks, the discrete time quantization of classical Markov chains, together with some of their properties and applications to search problems. We proceed by analogy: Szegedy's method of quantization of Markov chains is presented as the natural analogue of a random walk on the edges of a graph. Grover search and the quantum walk based search algorithms of Ambainis, Szegedy, Magniez/Nayak/Roland/Santha and Magniez/Nayak/Richter/Santha are stated as quantum analogues of classical search procedures. The complexities of these search algorithms are analyzed in function of various parameters, including the quantum analogue of the eigenvalue gap of the classical chain, the phase gap, and the quantum hitting time.

Among the many applications of quantum walks we describe two in the query model of computation. In the ELEMENT DISTINCTNESS problem we are given a function f defined on $\{1, \ldots, n\}$, and we are looking for a pair of distinct elements $1 \leq i, j \leq n$ such that $f(i) = f(j)$. While the classical complexity of this problem is $\Theta(n)$, it can be solved by a quantum walk based algorithm due to Ambainis in time $O(n^{2/3})$, and this upper bound is thight. In the TRIANGLE problem the input is the adjacency matrix of a graph G on vertex set $\{1, \ldots, n\}$, and the output is a triangle if there is any. This problem can easily be solved by Grover's algorithm in time $O(n^{3/2})$, while the obvious quantum lower bound is $\Omega(n)$. We present the quantum walk based algorithm of Magniez, Santha and Szegedy whose time is $O(n^{13/10})$, the exact complexity of the problem is open.