Multi-Energy Games

Sylvain Schmitz

based on joint works with Th. Colcombet, J.-B. Courtois, M. Jurdziński, and R. Lazić

GandALF, September 3, 2019

Outline

multi-dimensional energy (parity) games

complexity through perfect half-space games
(Colcombet et al., LICS '17)

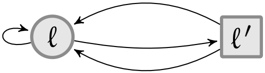
related problems:

- multi-dimensional mean-payoff parity games
(Chatterjee et al., Concur '12)
- VASS games (too many references!)
- regular VASS simulations (Courtois and S., MFCS '14)
- $(!,\oplus)$-Horn linear logic (Kanovich, APAL '95)
- μ-calculus on VASS (Abdulla et al., Concur '13)
- resource-bounded agent temporal logic RB$\pm$ATL*
(Alechina et al., RP '16)

Multi-weighted Game Graphs

- locations $\mathcal{L}$
- edges E
- weights $\mathbf{w}\colon E \to \mathbb{Z}^d$

Multi-weighted Game Graphs



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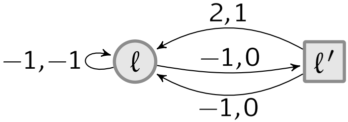

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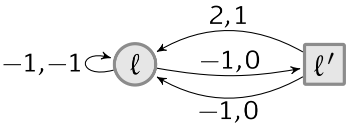

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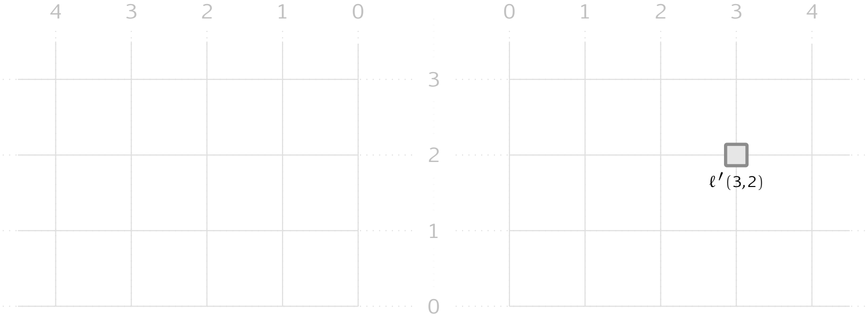
Example

Play with initial credit $\mathbf{c} = (3,2) \in \mathbb{N}^2$ starting in ℓ'



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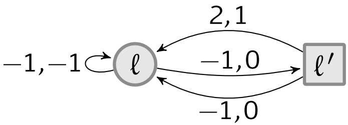

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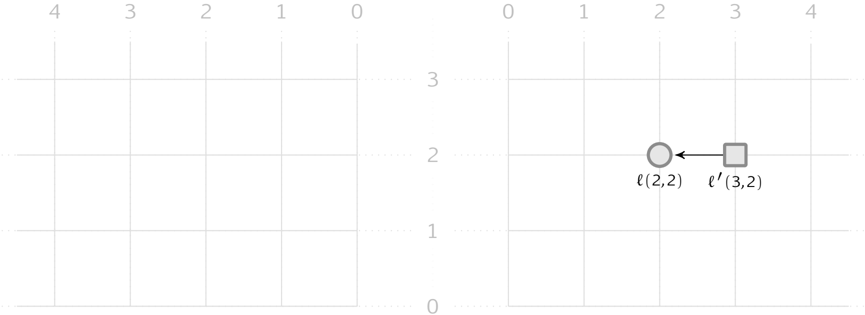

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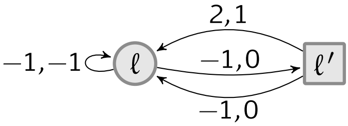

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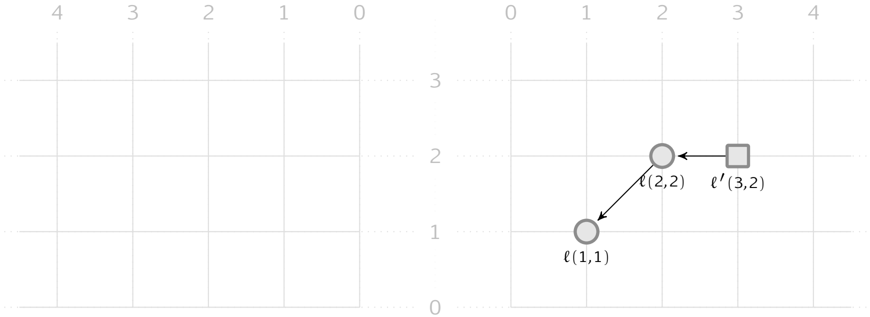

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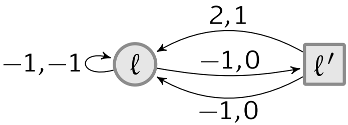

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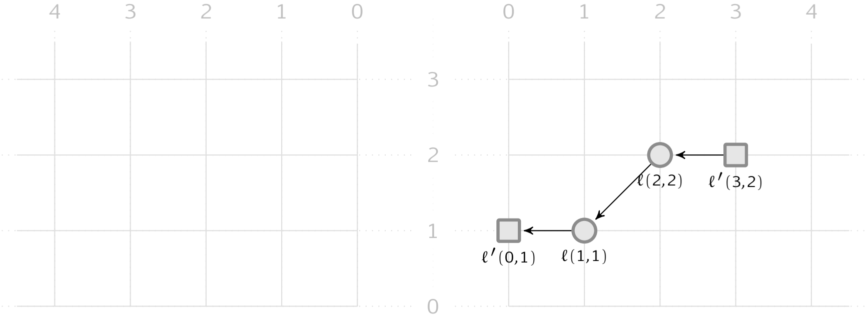

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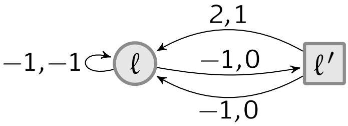

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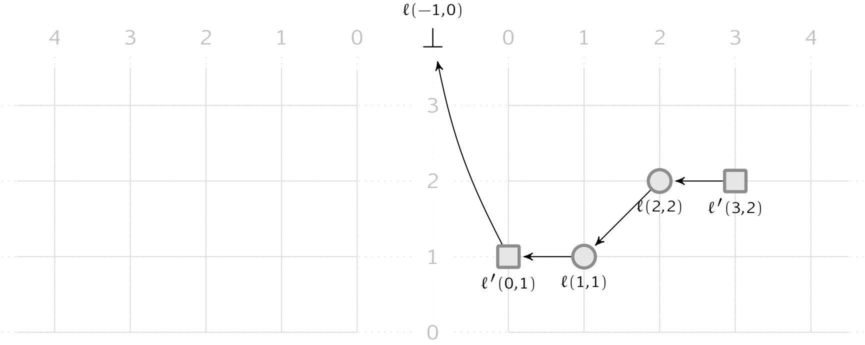

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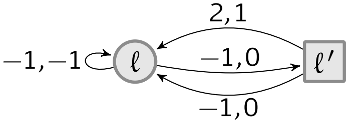

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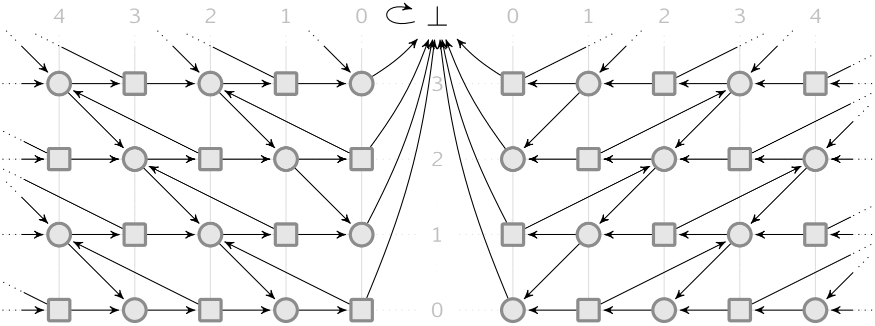
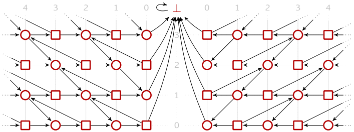
Energy Arena



Objectives: Non-Termination

The "usual" objective for energy games:
Player 1 wants to avoid ⊥.

Example (Winning Region)

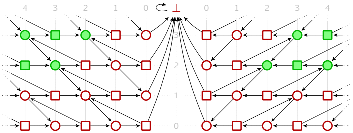


Objectives: Coverability

Given a target configuration $\ell_f(\mathbf{v}_f) \in \mathcal{L} \times \mathbb{N}^d$:
Player 1 wants to reach any $\ell_f(\mathbf{v})$ with $\mathbf{v} \geqslant \mathbf{v}_f$.

Example (Winning Region)

With target $\ell(2,2)$:



Objectives: Configuration Reachability

Given a target configuration $\ell_f(\mathbf{v}_f) \in \mathcal{L} \times \mathbb{N}^d$:
Player 1 wants to reach $\ell_f(\mathbf{v}_f)$.

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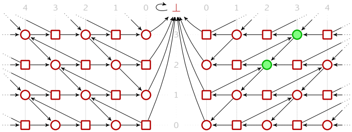

Decision Problems

Objectives

- non-termination
- coverability
- configuration reachability
- parity (with priorities on locations)

Two Variants

Does Player 1 have a winning strategy starting from

- a given initial credit $\mathbf{c} \in \mathbb{N}^d$ as part of the input
- existential: some initial credit

Undecidability of Config. Reachability

Theorem (Lincoln et al., 1992)

Configuration reachability, with given or existential initial credit, is undecidable in dimension $k \geqslant 2$.

Proof idea.

□

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det. Minsky machine $\mapsto$ multi-weighted game graph

with target $\ell_f(\mathbf{0})$ □

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PROOF IDEA.

det. Minsky machine $\mapsto$ multi-weighted game graph

$$\ell \xrightarrow{\mathbf{e}_i} \ell'$$

$$\ell \xrightarrow{i \stackrel{?}{=} 0} \ell' \qquad \ell \xrightarrow{-\mathbf{e}_i} \ell''$$

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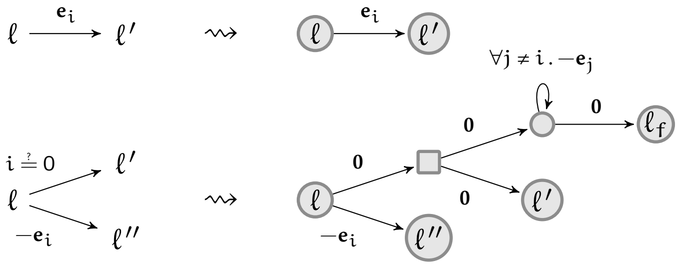

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Complexity

Multi-Dimensional Non-Termination Games

	lower bound	upper bound
given init. cred.		
∃ init. cred.		

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Complexity

Multi-Dimensional Non-Termination Games

	lower bound	upper bound
given init. cred.	EXPSPACE (Lasota, IPL '09)	TOWER (Brázdil et al., ICALP '10)
$\exists$ init. cred.	coNP (Chatterjee et al., FSTTCS '10)	coNP (Chatterjee et al., FSTTCS '10)

Complexity

Multi-Dimensional Non-Termination Games

	lower bound	upper bound
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Multi-Dimensional Non-Termination Games

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Complexity

Multi-Dimensional Coverability Games

	lower bound	upper bound
given init. cred.	2-EXP (Courtois and S., MFCS '14)	2-EXP (Courtois and S., MFCS '14)
$\exists$ init. cred.	P	P

Complexity

Multi-Dimensional Parity Games

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given init. cred.	2-EXP (Courtois and S., MFCS '14)	
$\exists$ init. cred.	coNP (Chatterjee et al., Concur '12)	coNP (Chatterjee et al., Concur '12)

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Multi-Dimensional Parity Games

	lower bound	upper bound
given init. cred.	2-EXP (Courtois and S., MFCS '14)	decidable (Abdulla et al., Concur '13)
$\exists$ init. cred.	coNP (Chatterjee et al., Concur '12)	coNP (Chatterjee et al., Concur '12)

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Multi-Dimensional Parity Games

	lower bound	upper bound
given init. cred.	2-EXP (Courtois and S., MFCS '14)	TOWER (Jančar, RP '15)
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Multi-Dimensional Parity Games

	lower bound	upper bound
given init. cred.	2-EXP (Courtois and S., MFCS '14)	2-EXP (Colcombet et al., LICS '17)
∃ init. cred.	coNP (Chatterjee et al., Concur '12)	coNP (Chatterjee et al., Concur '12)

Complexity

Fixed-Dimensional Fixed-Parity Games

	lower bound	upper bound
given init. cred.	EXP for $d \geqslant 2$ (Jurdziński et al., LMCS '08)	pseudoP (Colcombet et al., LICS '17)
$\exists$ init. cred.		pseudoP (Colcombet et al., LICS '17)

Complexity of Multi-Energy Parity Games

Theorem (Colcombet et al., LICS '17)

1. *The given initial credit problem for multi-dimensional energy parity games is in 2-EXP.*
2. *With fixed dimension and number of priorities, it is in pseudo polynomial time.*

- series of reductions using notably perfect half-space games
- fine understanding of Player 2's strategies:
  Player 2 can win by announcing in which perfect half space he will escape

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Reductions and Strategy Transfers

multi-dimensional energy parity games

↓ (Jančar, RP '15)

extended multi-dimensional energy games (Brázdil et al., ICALP '10)

↓

bounding games (Jurdziński et al., ICALP '15)

↓

perfect half space games (Colcombet et al., LICS '17)

↓

lexicographic energy games (Colcombet and Niwiński)

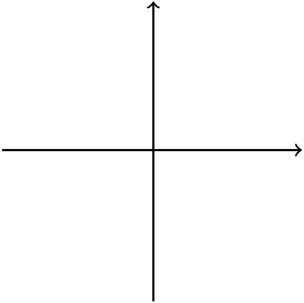
↓

mean-payoff games (Comin and Rizzi, Algorithmica '16)

Bounding Games

Player 1's Objective

existential energy



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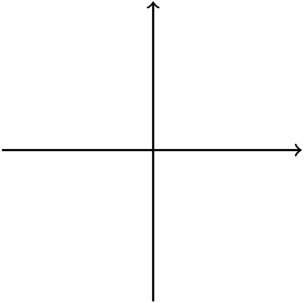

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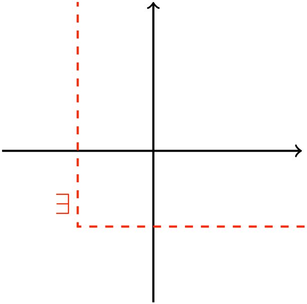

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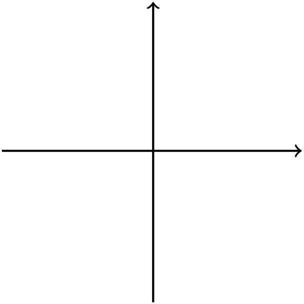

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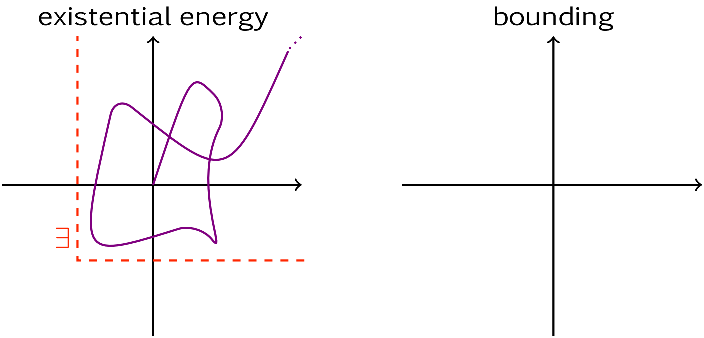

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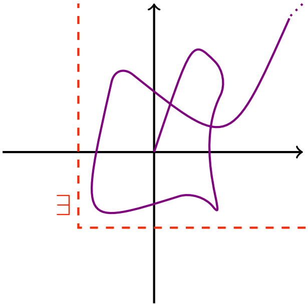

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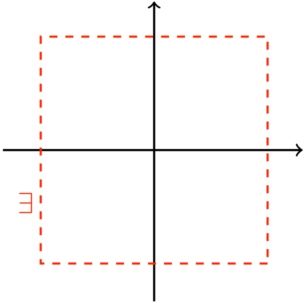

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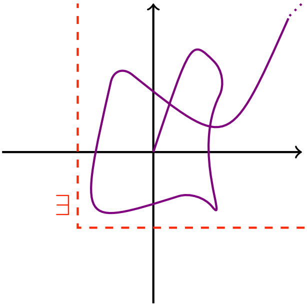
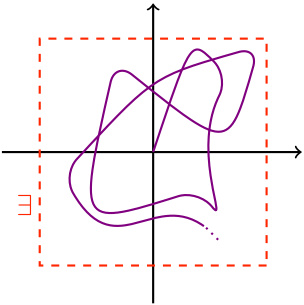
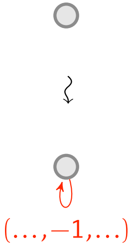




Bounding Games

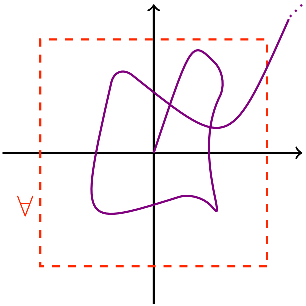
Encoding Energy Games

Bin excess energy



Perfect Half Space Games

Player 2's Objective in a Bounding Game



Key Intuition

Player 2 can escape in a perfect half space

Perfect Half Space Games

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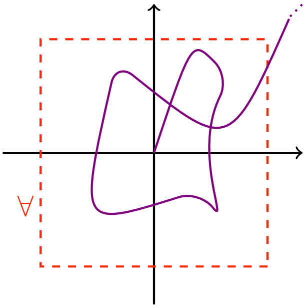

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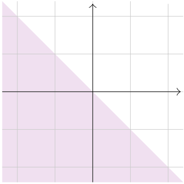

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Perfect Half Space Games

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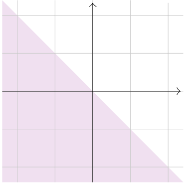

$$\{(x,y) : x + y < 0\}$$

boundary: $\{(x,y) : x + y = 0\}$

Perfect Half Space Games

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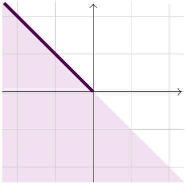
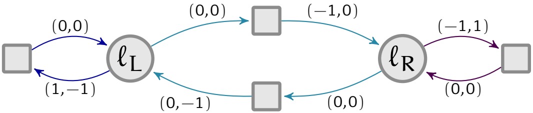
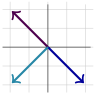


$$\{(x,y) : x + y < 0\} \cup \{(x,y) : x + y = 0 \wedge x < 0\}$$

Perfect Half Space Games





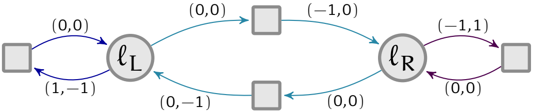
Plays

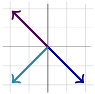
- pairs of vertices and perfect half spaces:

$$(\ell_0, \mathbf{H}_0) \xrightarrow{\mathbf{w}_1} (\ell_1, \mathbf{H}_1) \xrightarrow{\mathbf{w}_2} (\ell_2, \mathbf{H}_2) \cdots$$

- in his vertices, Player 2 chooses the current perfect half space

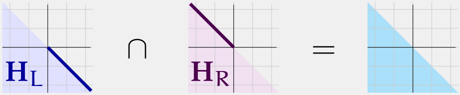
Perfect Half Space Games





- Player 2 wins if $\exists i$ s.t. $\sum_{j \geqslant 0} \mathbf{w}_j$ diverges into $\bigcap_{j>i} \mathbf{H}_j$

Example



Solving Perfect Half Space Games

Theorem

Perfect half space games on multi-weighted game graphs (V, E, d) *are solvable in* $(|V| \cdot \|E\|)^{O(d^3)}$.

Proof Idea

- reduce to a lexicographic energy game (Colcombet and Niwiński)
- $\approx$ perfect half space game with a single fixed $\mathbb{H}$
- itself reduced to a mean-payoff game

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Player 2 Strategies

Oblivious Strategy

Player 2 chooses the same $\mathbf{H}_\ell$ every time it visits location ℓ

Theorem

If Player 2 has a winning strategy in a perfect half space game, then it has an oblivious one.

"Counterless" Strategy

Corollary (Brázdil et al., ICALP '10)

If Player 2 has a winning strategy in an existential multi-dimensional energy parity game, then it has a positional one.

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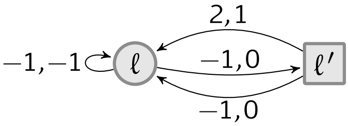

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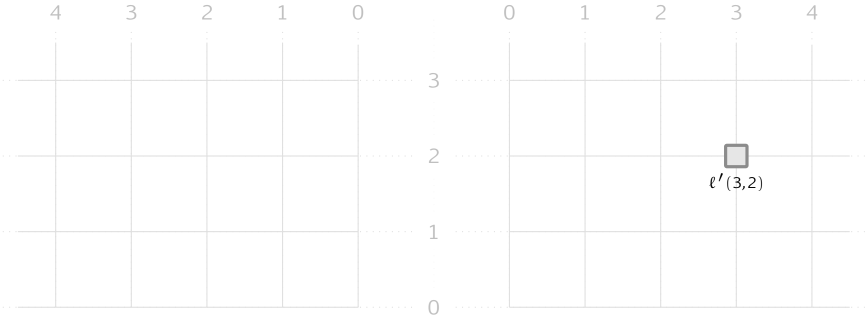

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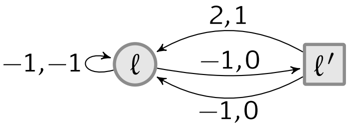

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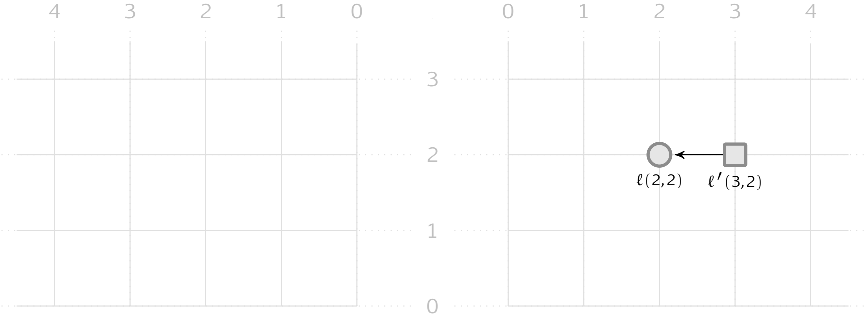
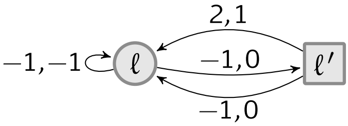
Play with initial credit $\mathbf{c} = (3,2) \in \mathbb{N}^2$ starting in ℓ'



VASS Games



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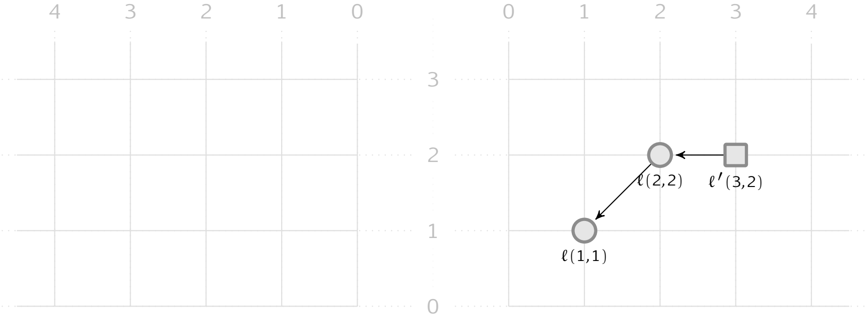

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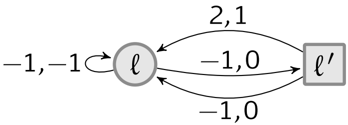

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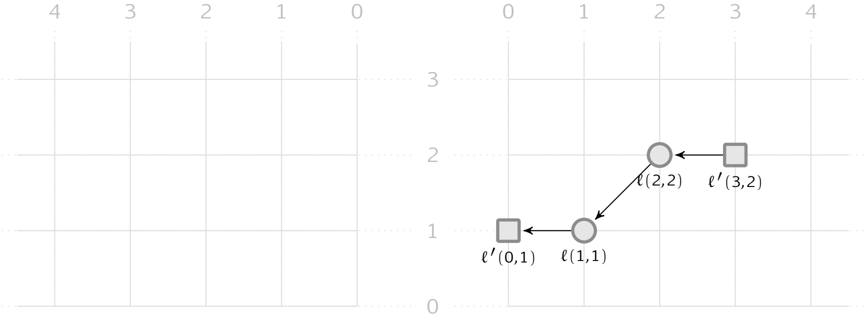

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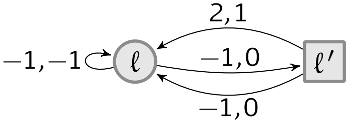

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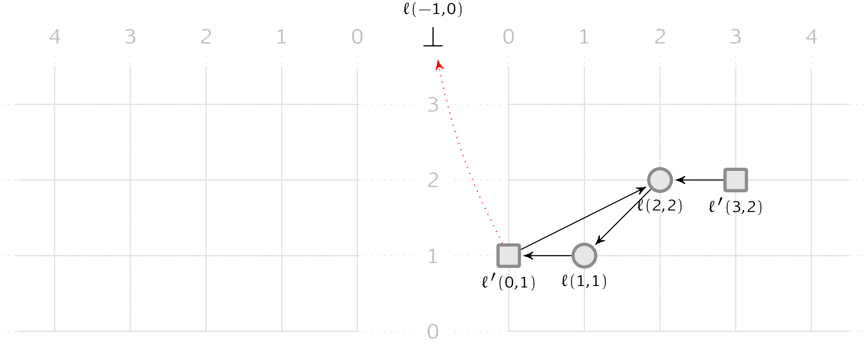

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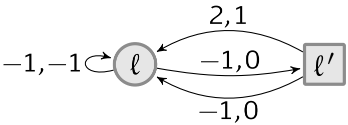

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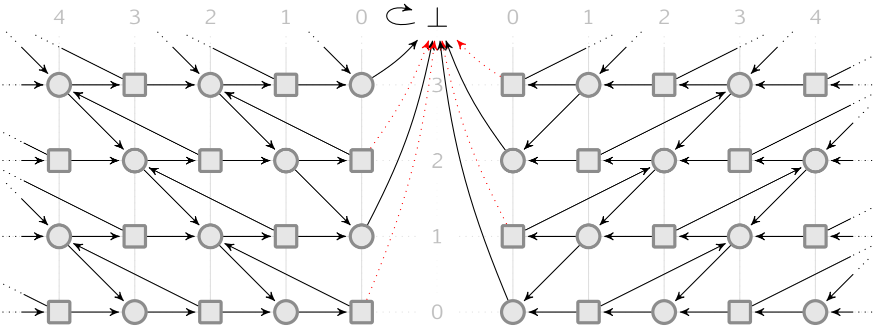
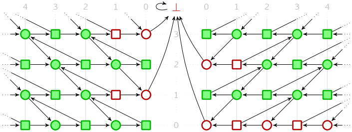
Natural Arena



Objectives

Non-Termination

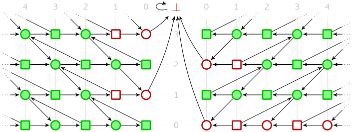


Coverability (of $\ell(2,2)$)

Configuration Reachability (of $\ell(2,2)$)

Objectives

Non-Termination



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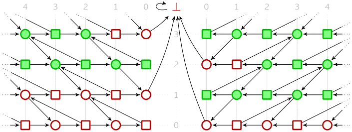

Configuration Reachability (of $\ell(2,2)$)

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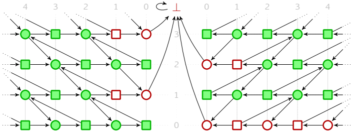

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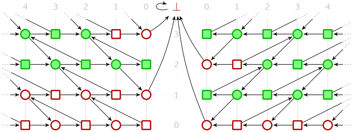

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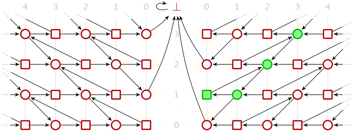


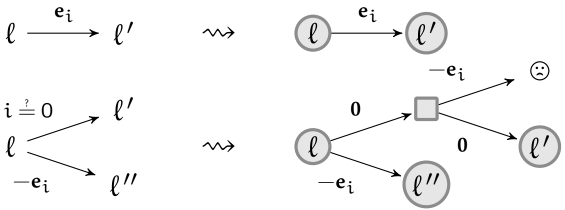
UNDECIDABILITY

THEOREM (multiple sources)

All the VASS games are undecidable in dimension $d \geqslant 2$.

PROOF IDEA.

det. Minsky machine $\mapsto$ multi-weighted game graph



□

Asymmetric VASS Games

Player 2 moves restricted to use the zero vector.

Theorem (Abdulla et al., 2013)

AVASS games and multi-energy games are logspace-equivalent.

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A frequent assumption

- and-branching VASS (Lincoln et al., 1992)
- vector games (Kanovich, APAL 1995)
- B-games (Raskin et al., 2004)
- single-sided games (Abdulla et al., 2013)
- alternating VASS (Courtois and S., 2014)

Model-Checking Resource-Aware Logics

VASS models fragment of the μ-calculus on VASS executions
(Abdulla et al., Concur '13)

resource-bounded concurrent game structures RB±ATL*
(Alechina et al., RP '16)

Both are 2-EXP-complete by reduction to multi-energy parity games / parity AVASS games.

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Propositional (Intuitionistic) Linear Logic

$$\frac{}{A \vdash A}(\mathrm{I}) \qquad \frac{\Gamma, !A, !A \vdash B}{\Gamma, !A \vdash B}(\mathrm{C!}) \qquad \frac{\Gamma, A \vdash B}{\Gamma, !A \vdash B}(\mathrm{L!})$$

$$\frac{\Gamma \vdash A \quad \Delta, B \vdash C}{\Gamma, \Delta, A \multimap B \vdash C}(\mathrm{L}_{\multimap}) \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}(\mathrm{R}_{\multimap})$$

$$\frac{\Gamma, A \vdash C}{\Gamma, A \& B \vdash C} \quad \frac{\Gamma, B \vdash C}{\Gamma, A \& B \vdash C}(\mathrm{L}_{\&}) \qquad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \& B}(\mathrm{R}_{\&})$$

$$\frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \oplus B \vdash C}(\mathrm{L}_{\oplus}) \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \oplus B} \quad \frac{\Gamma \vdash B}{\Gamma \vdash A \oplus B}(\mathrm{R}_{\oplus})$$

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$(!,\oplus)$-Horn Programs (1/3)

connectives $\{\otimes, \multimap, \oplus, !\}$

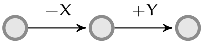
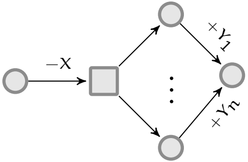
simple products $W, X, Y, Z ::= p_1 \otimes p_2 \otimes \cdots \otimes p_m$ for atomic p_i's

Horn implications $X \multimap Y$

$\oplus$-Horn implications $X \multimap (Y_1 \oplus \cdots \oplus Y_n)$

$(!,\oplus)$-Horn sequents $W, !\Gamma \vdash Z$ where Γ contains Horn and $\oplus$-Horn implications

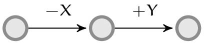
$(!, \oplus)$-Horn Programs (2/3)

Horn programs		AVASS
$X \multimap Y$	$\rightsquigarrow$	
$X \multimap (Y_1 \oplus \cdots \oplus Y_n)$	$\rightsquigarrow$	

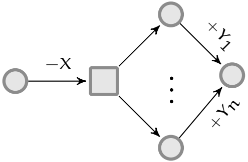
$(!, \oplus)$-Horn Programs (2/3)

Horn programs | AVASS

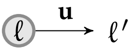
$X \multimap Y \quad \rightsquigarrow$



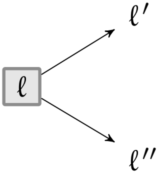
$X \multimap (Y_1 \oplus \cdots \oplus Y_n) \quad \rightsquigarrow$



$\ell \otimes \mathbf{u}^- \multimap \ell' \otimes \mathbf{u}^+ \quad \leftsquigarrow$



$\ell \multimap (\ell' \oplus \ell'') \quad \leftsquigarrow$



$(!,\oplus)$-Horn Programs (3/3)

Theorem (Kanovich, APAL '95)

Provability of $(!,\oplus)$-Horn sequents and configuration reachability AVASS games are PSPACE equivalent.

Corollary (Lincoln et al., APAL '92)

Provability in propositional linear logic is undecidable.

Corollary (Courtois and S., MFCS '14; Lazić and S., ToCL '15)

- *Provability of affine $(!,\oplus)$-Horn sequents is 2-EXP-complete.*
- *Provability of contractive $(!,\oplus)$-Horn sequents is ACKERMANN-complete.*

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Concluding Remarks

- tight 2-EXP bounds for multi-energy parity games
- impacts numerous problems
  - affine $(\oplus, !)$-Horn linear logic
    (Kanovich, APAL ’95)
  - (weak) simulation of finite-state systems by Petri nets
    (Abdulla et al., Concur ’13)
  - model-checking Petri nets with a fragment of μ-calculus
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  - resource-bounded agent temporal logic RB$\pm$ATL*
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- fine understanding of Player 2’s strategies:
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