Conservative Ambiguity Detection in Context-Free Grammars

Sylvain Schmitz

Laboratoire I3S, Université de Nice - Sophia Antipolis & CNRS, France

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Ambiguity in CFGs

- natural
- an issue
  - in language acquisition [Cheung and Uzgalis, 1995]
  - in RNA analysis [Reeder et al., 2005]
  - in computer languages
  - need a conservative test: no false negatives
- undecidable problem [Cantor, 1962, Chomsky and Schützenberger, 1963]

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An Ambiguity

Example

"She saw the man with a telescope ."

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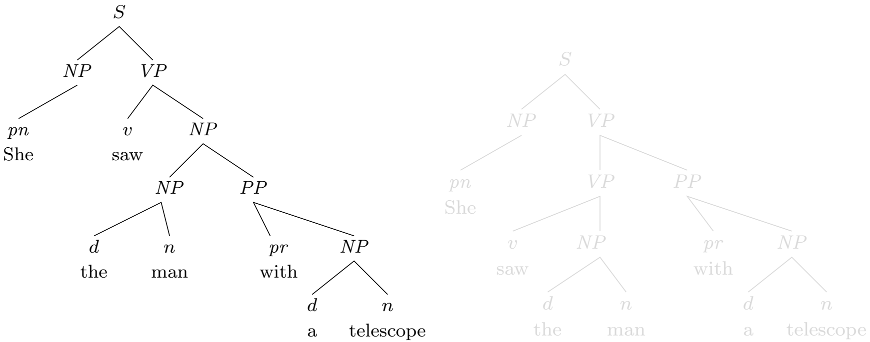

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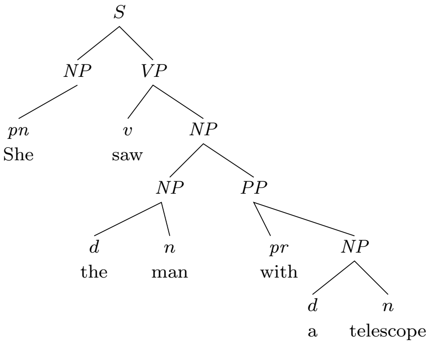

Bracketed Grammars

$$\mathcal{G} = \langle N, T, P, S \rangle,\ V = N \cup T$$

$$\begin{aligned}
S &\xrightarrow{2} NP\ VP \\
NP &\xrightarrow{3} d\ n \\
NP &\xrightarrow{4} pn \\
NP &\xrightarrow{5} NP\ PP \\
VP &\xrightarrow{6} v\ NP \\
VP &\xrightarrow{7} VP\ PP \\
PP &\xrightarrow{8} pr\ NP
\end{aligned}$$

Bracketed Grammars

$$\mathcal{G}_b = \langle N, T_b, P_b, S \rangle,\ V_b = N \cup T_b$$

$$\begin{aligned}
S &\xrightarrow{2} d_2\ NP\ VP\ r_2 \\
NP &\xrightarrow{3} d_3\ d\ n\ r_3 \\
NP &\xrightarrow{4} d_4\ pn\ r_4 \\
NP &\xrightarrow{5} d_5\ NP\ PP\ r_5 \\
VP &\xrightarrow{6} d_6\ v\ NP\ r_6 \\
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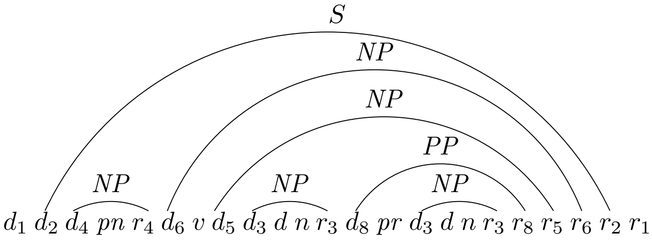
\end{aligned}$$

The Approach

- a bracketed sentence = a derivation tree



- ambiguity = more than one tree with the same yield
- construct a FSA $\mathcal{A}$ such that $\mathcal{L}(\mathcal{G}_b) \subseteq \mathcal{L}(\mathcal{A})$, and look for bracketed sentences with the same yield

The Approach

- a bracketed sentence = a derivation tree
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  Let h be the bracket-erasing homomorphism $V_b^* \to V^*$ with $h(X) = X$ for X in V and $h(d_i) = h(r_i) = \varepsilon$; the yield of w_b is $w = h(w_b)$.
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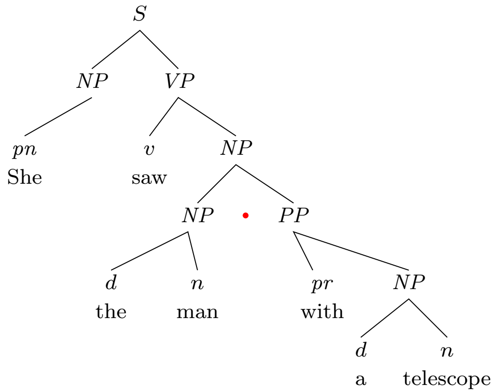
Positions



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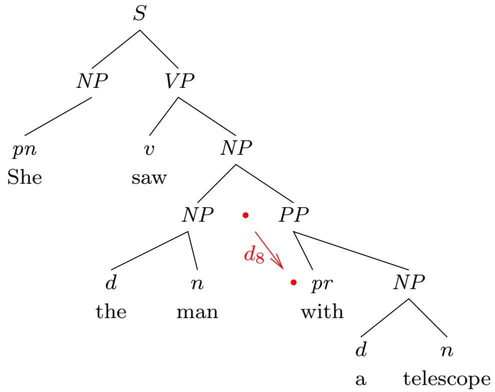
Position Graph Γ

Left-to-right Walks in Trees



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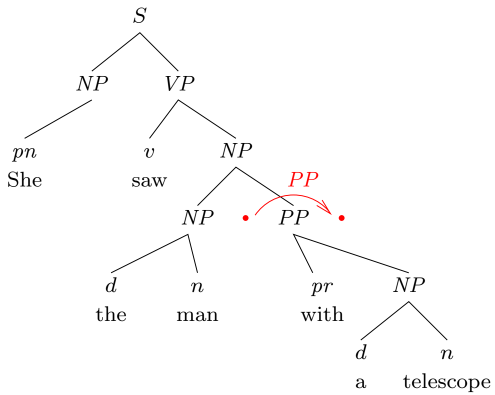

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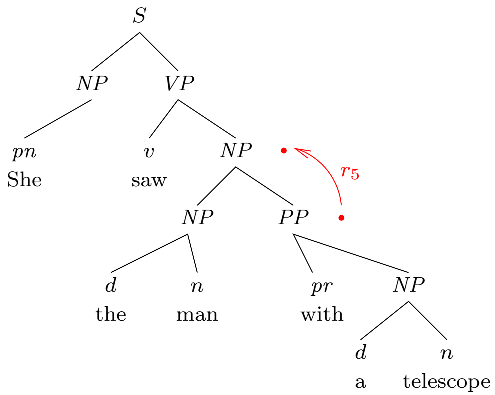

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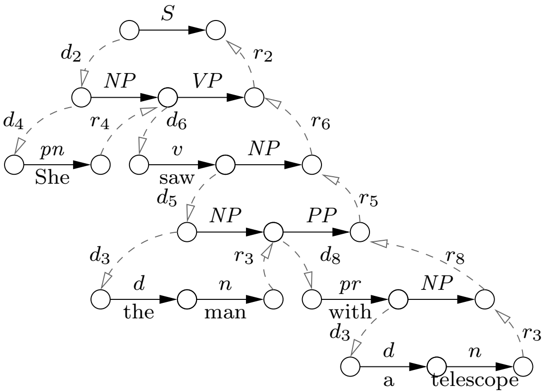

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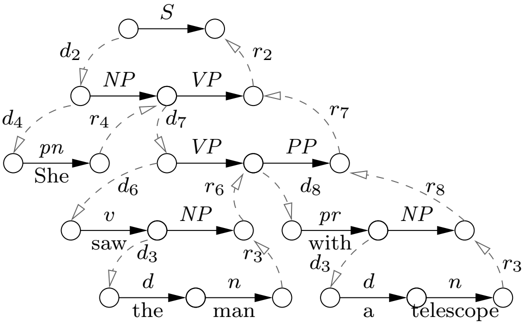


Position Automaton $\Gamma/\equiv$

Definition

$\Gamma/\equiv$ is the quotient of Γ by an equivalence relation $\equiv$ between positions.

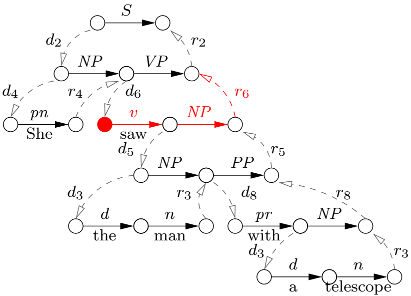
Lemma

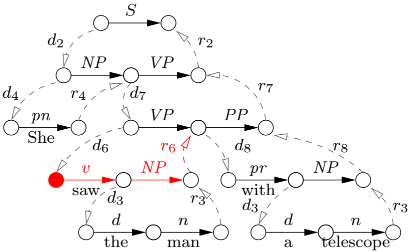
If $\nu \xrightarrow{\delta_b} \nu'$ *in* Γ, *then* $[\nu]_\equiv \delta_b \models^* [\nu']_\equiv$ *in* $\Gamma/\equiv$.

Theorem

$$\mathcal{L}(\mathcal{G}_b) \subseteq \mathcal{L}(\Gamma/\equiv) \cap T_b^*$$

Example: item_0 Equivalence





- This equivalence class: $[VP \overset{6}{\rightarrow} .v\ NP]$
- item_0 equivalence classes: LR(0) items
- Γ/item_0: nondeterministic LR(0) automaton

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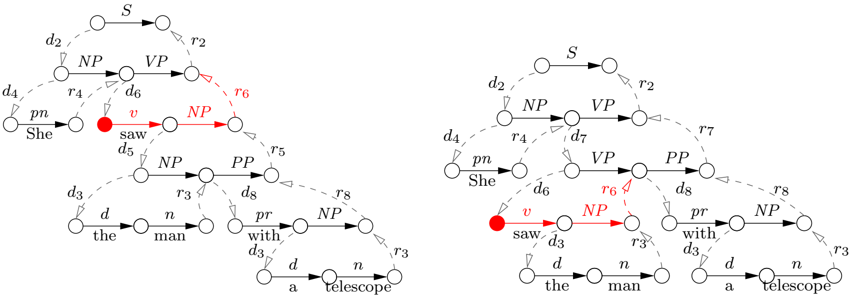

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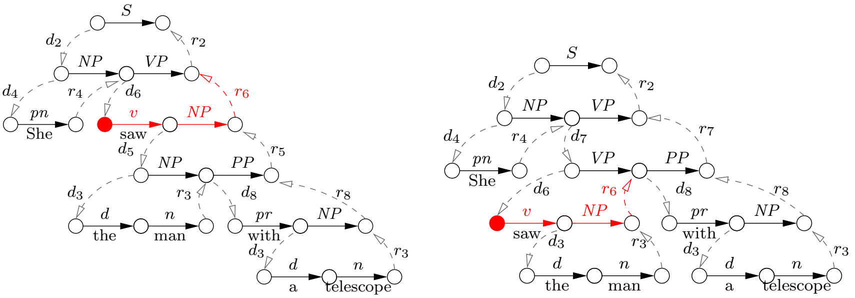

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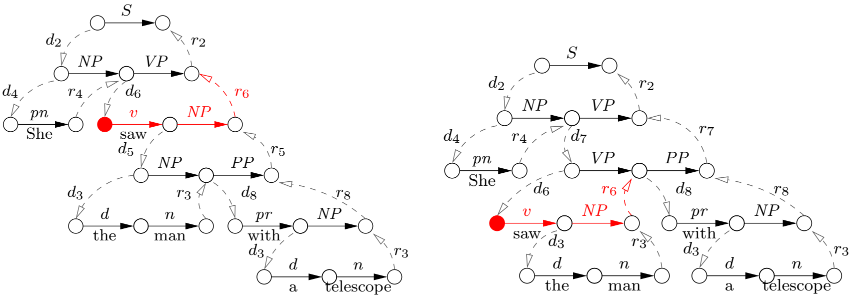

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Back to Ambiguity Detection

- Class of grammars: $\mathcal{G}$ is regular unambiguous for $\equiv$ of finite index, noted $\mathrm{RU}(\equiv)$, if there does not exist $w_b \neq w'_b$ in $\mathcal{L}(\Gamma/\equiv) \cap T_b^*$ with $h(w_b) = h(w'_b)$
- $\forall \equiv, \mathrm{RU}(\equiv) \subseteq \mathrm{UCFG}$
- $\mathrm{RU}(\mathrm{item}_0) \not\subseteq \mathrm{LR}(0)$
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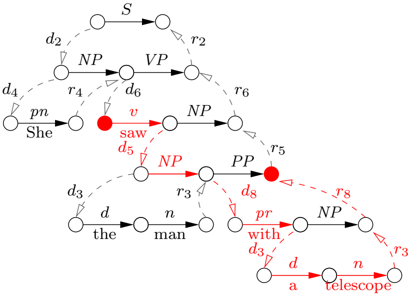

$$v\ d_5\ NP\ PP \Rightarrow_b^* v\ d_5\ NP\ d_8\ pr\ d_3\ d\ \ n\ r_3\ r_8$$

Lemma

If $\nu \xrightarrow{\delta_b} \nu'$ *in* Γ *and* $\gamma_b \Rightarrow_b^* \delta_b$ *in* $\mathcal{G}_b$*, then* $\nu \xrightarrow{\gamma_b} \nu'$ *in* Γ.

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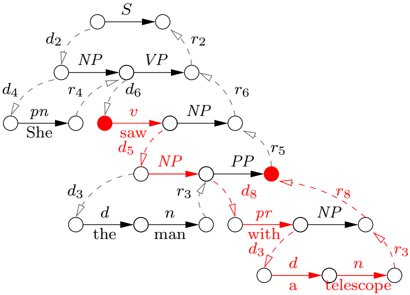

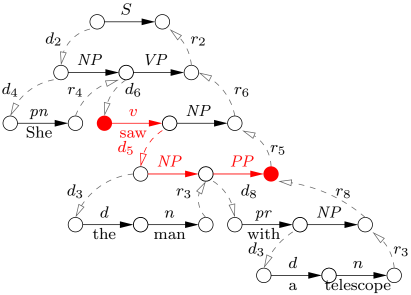

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Corollary

If $[\nu]_\equiv \delta_b \models^* [\nu']_\equiv$ *in* $\Gamma/\!\equiv$ *and* $\gamma_b \Rightarrow_b^* \delta_b$ *in* $\mathcal{G}_b$*, then* $[\nu]_\equiv \gamma_b \models^* [\nu']_\equiv$ *in* $\Gamma/\!\equiv$.

Common Prefixes with Conflicts

$$d_1\ d_2\ d_4\ \ pn\ \ r_4\ d_6\ \ v\ \ d_5\ d_3\ \ d\ \ n\ \ r_3\ d_8\ \ pr\ \ d_3\ \ d\ \ n\ \ r_3\ r_8\ r_5\ r_6\ r_2\ r_1$$
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Common Prefixes with Conflicts

NP

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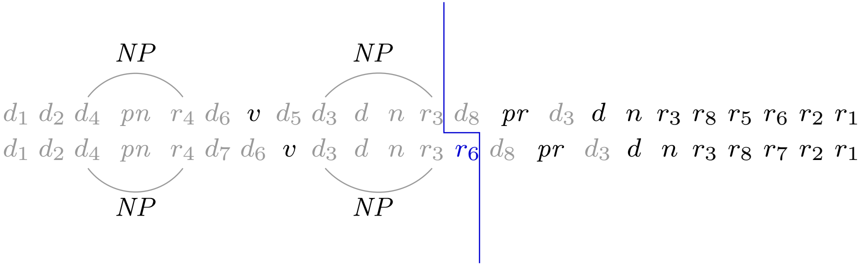

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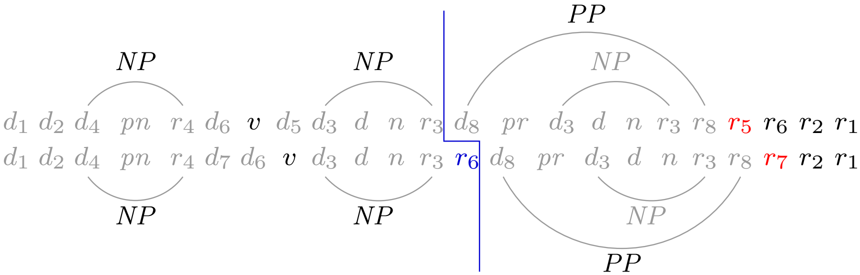

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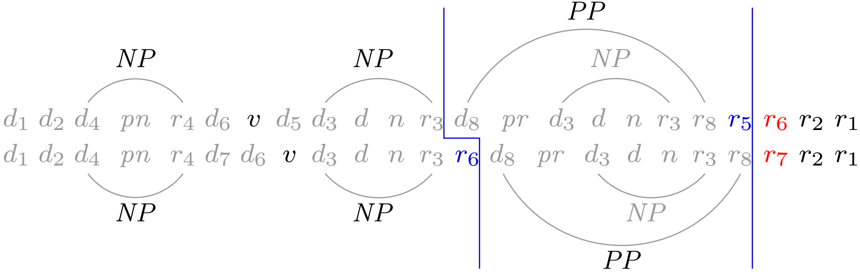

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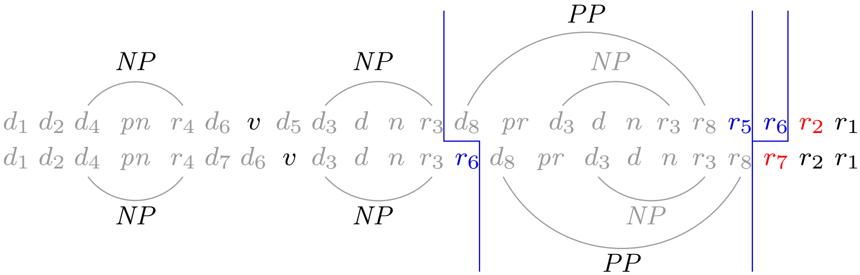

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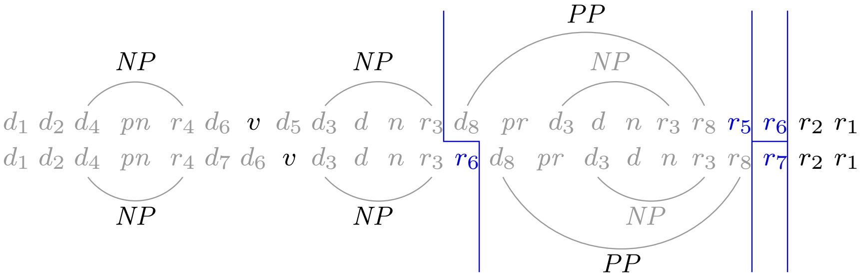

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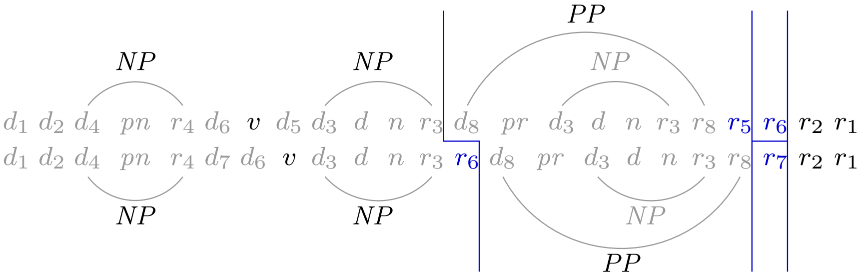

Two bracketed reduced sentential forms $\delta_b \neq \delta'_b$ with $h(\delta_b) = h(\delta'_b)$:

$$d_1\ d_2\ NP\ d_6\ v\ d_5\ NP\ PP\ r_5\ r_6\ r_2\ r_1$$

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Mutual Accessibility Relations

- between pairs of states of $\Gamma/\equiv$, (q_1, q_2)
- synchronized left-to-right walks from an initial pair (q_s, q_s)

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mae: read a d_i symbol from one state

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Shift

mas: read the same symbol of V from both states

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- synchronized left-to-right walks from an initial pair (q_s, q_s)

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Reduce

mar$^+$: read the same r_i from both states, if no mae was used since the last mac or mar

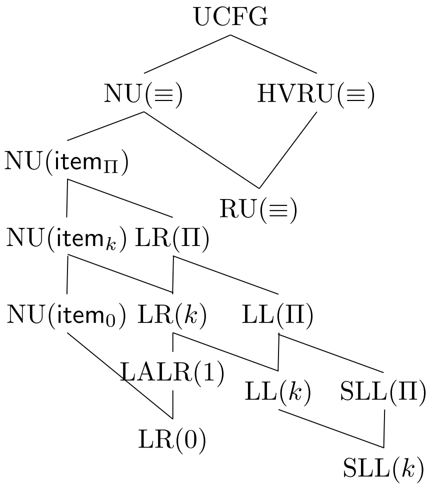
Noncanonical Unambiguity

- ma=mas ∪ mae ∪ mac ∪ mar
- $\mathcal{G}$ is noncanonically unambiguous, noted NU(≡), if there does not exist a relation (q_s, q_s) ma* (q_f, q_f) that uses mac at some step.
- Computation in $\mathcal{O}(|\Gamma/\equiv|^2)$ in space.

Comparisons

- Regular Unambiguity RU($\equiv$)
- Bounded-length detection schemes
- LR(k) and LR-Regular (LR(Π))
- Horizontal and vertical ambiguity (HVRU($\equiv$))

Comparisons



Bounded-length detection

[Gorn, 1963, Cheung and Uzgalis, 1995, Schröer, 2001, Jampana, 2005]

- generate sentences up to some length and find ambiguities
- not conservative
- define an equivalence relation prefix_m
- if $w_b \neq w'_b \in \mathcal{L}(\Gamma/\text{prefix}_m) \cap T_b^*$, $w = w'$ and $|w| \leq m$, then w_b and w'_b are in $\mathcal{L}(\mathcal{G}_b)$
- generation needs to construct the sentence a^{2^n+1} to find $\mathcal{G}_4^n$ ambiguous, but $\mathcal{G}_4^n \notin \text{NU}(\text{item}_0)$

$$S \to A \mid B_n a,\ A \to Aaa \mid a,\ B_1 \to aa,\ B_2 \to B_1 B_1,\ \ldots,\ B_n \to B_{n-1} B_{n-1} \qquad (\mathcal{G}_4^n)$$

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LR(k) and LR-Regular

[Knuth, 1965, Hunt III et al., 1975, Čulik and Cohen, 1973, Heilbrunner, 1983]

- conservative tests
- define item_Π equivalence relation
- mac corresponds to an LR(0) conflict
- subsequent ma^* implies that lookaheads match
- $\text{LR}(\Pi) \subset \text{NU}(\text{item}_\Pi)$
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  - also experimentally better
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