Perfect Half Space Games

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LSV, ENS Paris-Saclay & CNRS & Inria

LICS 2017, June 23rd, 2017

What to do this week-end?

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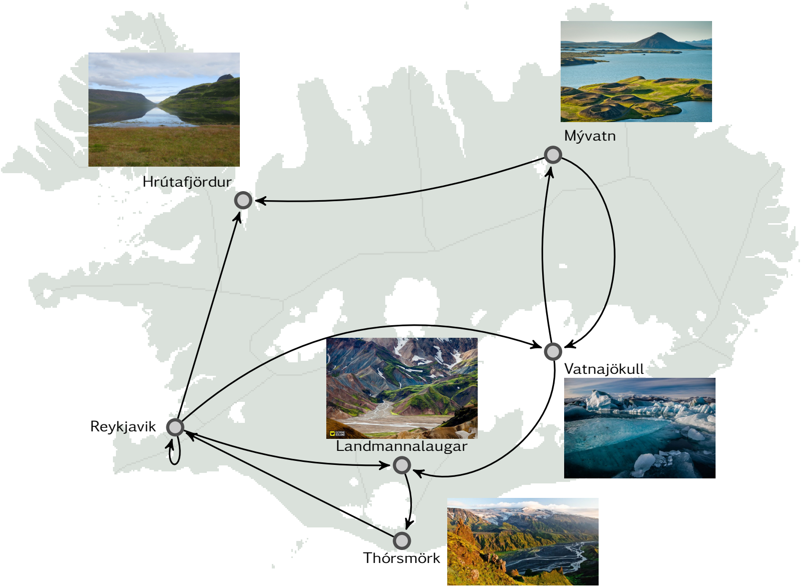
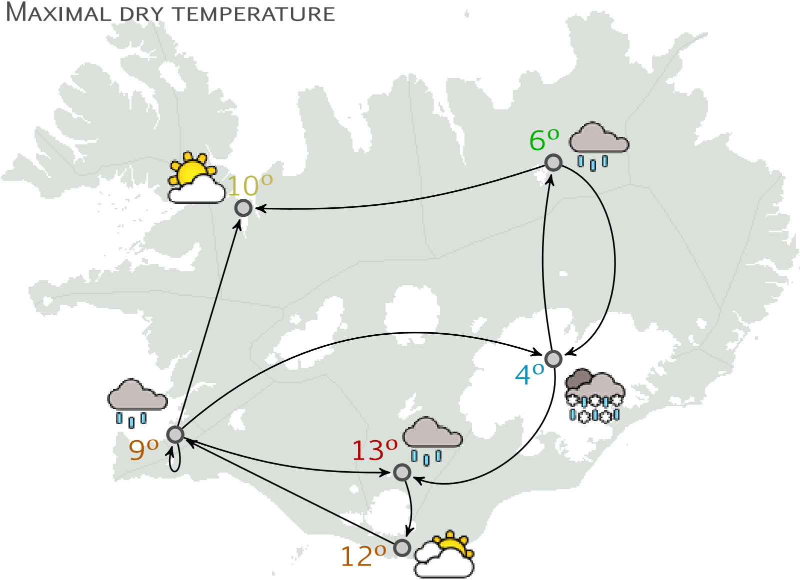


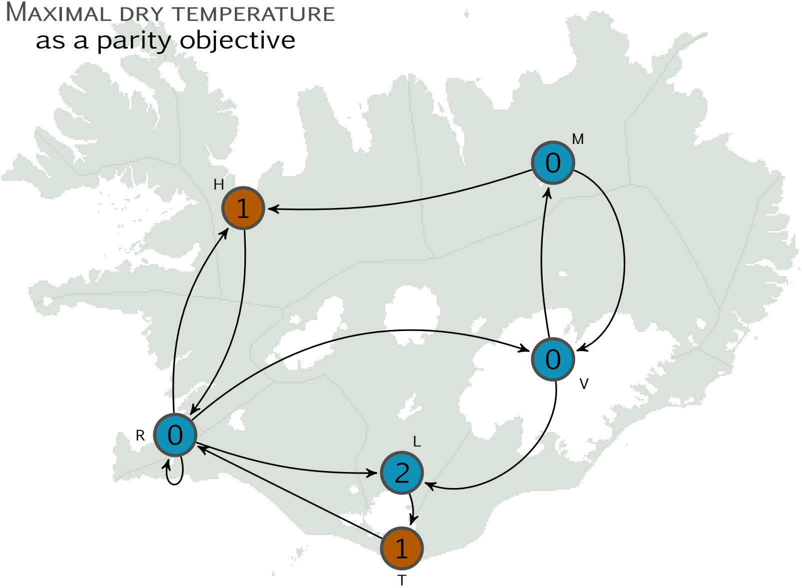
What to do this week-end?

Maximal dry temperature



What to do this week-end?

Maximal dry temperature
as a parity objective



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Maximal dry temperature

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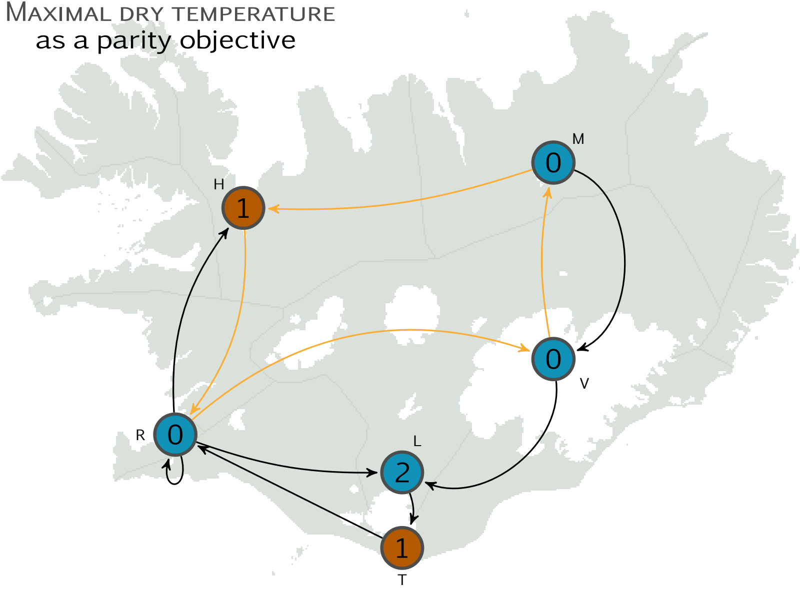


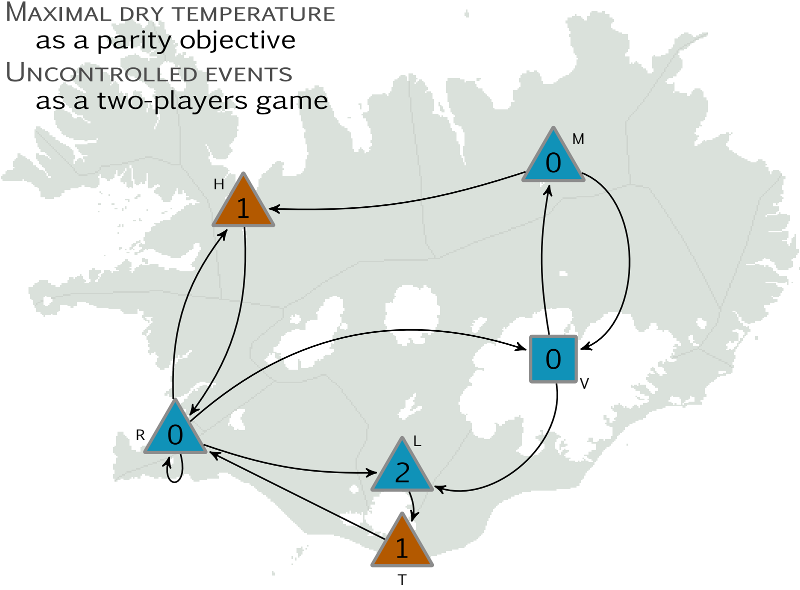
What to do this week-end?

Uncontrolled events

What to do this week-end?

Maximal dry temperature
as a parity objective

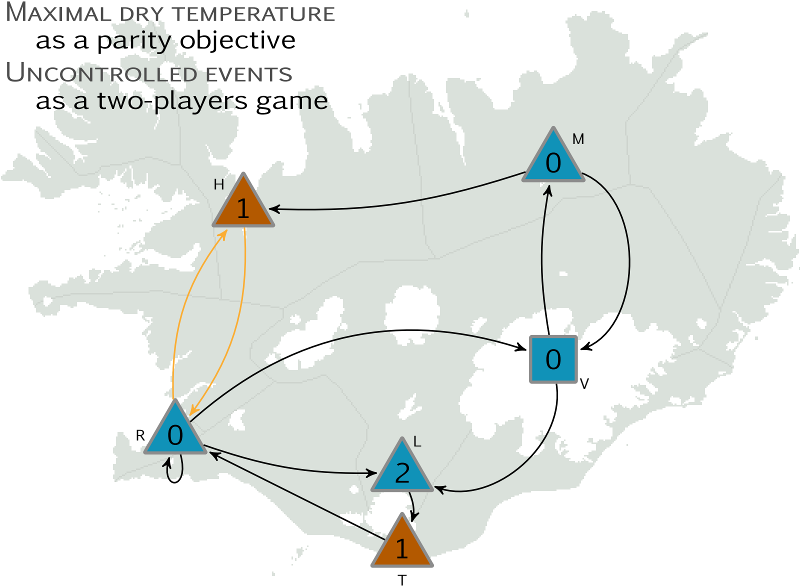
Uncontrolled events
as a two-players game



What to do this week-end?

Maximal dry temperature
as a parity objective

Uncontrolled events
as a two-players game

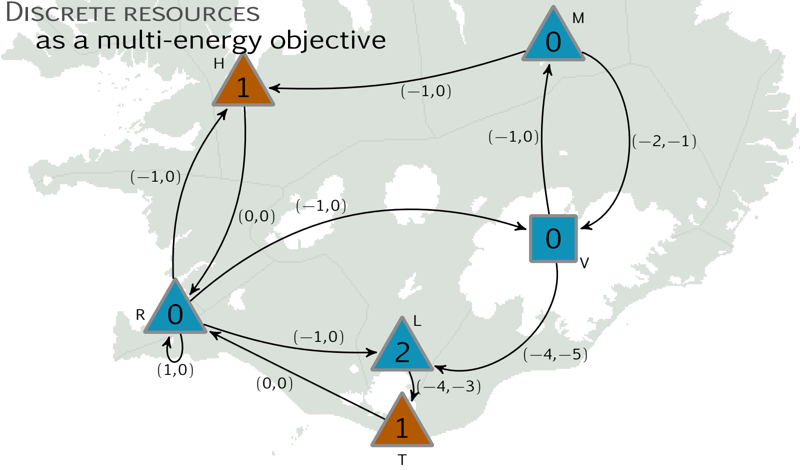


What to do this week-end?

Discrete resources

What to do this week-end?

- Maximal dry temperature
  - as a parity objective
- Uncontrolled events
  - as a two-players game
- Discrete resources
  - as a multi-energy objective

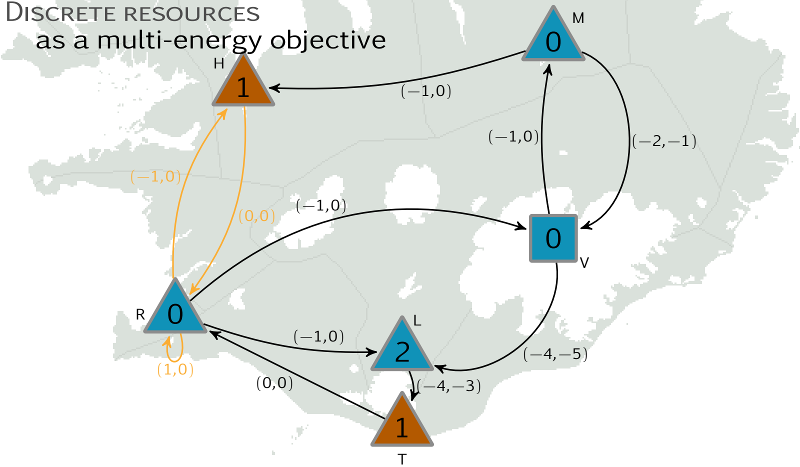


What to do this week-end?

Maximal dry temperature
as a parity objective

Uncontrolled events
as a two-players game

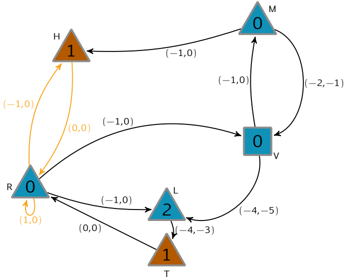
Discrete resources
as a multi-energy objective



Multi-Dimensional Energy Parity Games

Player 1 wins a play if both

- energy objective: no component goes negative
- parity objective: the maximal priority is odd



Example

$$R(0,0) \xrightarrow{(1,0)} R(1,0) \xrightarrow{(1,0)} R(2,0) \xrightarrow{(-1,0)} H(1,0) \xrightarrow{(0,0)} R(1,0) \to \cdots$$

Multi-Dimensional Energy Parity Games

Applications

- contractive $(\oplus,!)$-Horn linear logic
  (Kanovich, APAL '95)
- (weak) simulation of finite-state systems by Petri nets
  (Abdulla et al., Concur '13)
- model-checking Petri nets with a fragment of μ-calculus
  (Abdulla et al., Concur '13)
- resource-bounded agent temporal logic RB$\pm$ATL*
  (Alechina et al., RP '16 & AI '17)

Multi-Dimensional Energy Parity Games

Complexity

	lower bound	upper bound
w. initial credit		
∃ initial credit		

Multi-Dimensional Energy Parity Games

Complexity

	lower bound	upper bound
w. initial credit	EXPSPACE (Lasota, IPL ′09)	TOWER (Brázdil et al., ICALP ′10)
$\exists$ initial credit	coNP (Chatterjee et al., FSTTCS ′10)	coNP (Chatterjee et al., FSTTCS ′10)

Multi-Dimensional Energy Parity Games

Complexity

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w. initial credit	2-EXP (Courtois and S., MFCS '14)	TOWER (Brázdil et al., ICALP '10)
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Multi-Dimensional Energy Parity Games

Complexity

	lower bound	upper bound
w. initial credit	2-EXP (Courtois and S., MFCS '14)	2-EXP (Jurdziński et al., ICALP '15)
$\exists$ initial credit	coNP (Chatterjee et al., FSTTCS '10)	coNP (Chatterjee et al., FSTTCS '10)

Multi-Dimensional Energy Parity Games

Complexity

	lower bound	upper bound
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Multi-Dimensional Energy Parity Games

Complexity

	lower bound	upper bound
w. initial credit	2-EXP (Courtois and S., MFCS ’14)	decidable (Abdulla et al., Concur ’13)
$\exists$ initial credit	coNP (Chatterjee et al., Concur ’12)	coNP (Chatterjee et al., Concur ’12)

Multi-Dimensional Energy Parity Games

Complexity

	lower bound	upper bound
w. initial credit	2-EXP (Courtois and S., MFCS '14)	TOWER (Jančar, RP '15)
$\exists$ initial credit	coNP (Chatterjee et al., Concur '12)	coNP (Chatterjee et al., Concur '12)

Multi-Dimensional Energy Parity Games

Complexity

	lower bound	upper bound
w. initial credit	2-EXP (Courtois and S., MFCS '14)	2-EXP this talk
$\exists$ initial credit	coNP (Chatterjee et al., Concur '12)	coNP (Chatterjee et al., Concur '12)

Fixed Dimensional Energy Fixed Parity Games

Complexity

	lower bound	upper bound
w. initial credit	EXP for $d \geqslant 4$ (Courtois and S., MFCS '14)	pseudoP this talk
$\exists$ initial credit		pseudoP this talk

Outline

multi-dimensional energy parity games

↓ (Jančar, RP '15)

extended multi-dimensional energy games (Brázdil et al., ICALP '10)

↓

bounding games (Jurdziński et al., ICALP '15)

↓

perfect half space games (this paper)

↓

lexicographic energy games (Colcombet and Niwiński)

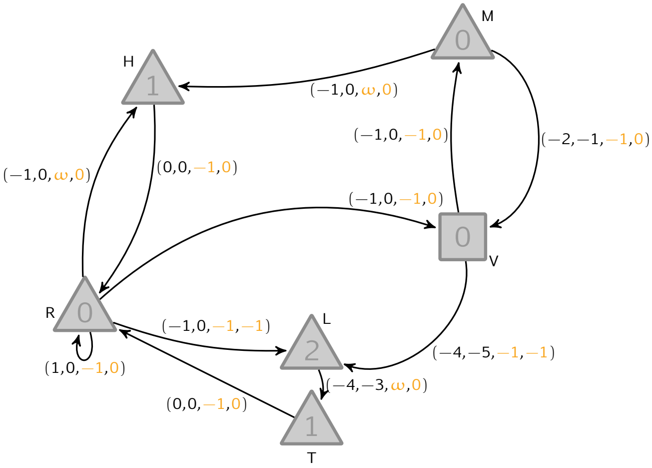
↓

mean-payoff games (Comin and Rizzi, Algorithmica '16)

Extended Multi-Dimensional Energy Games

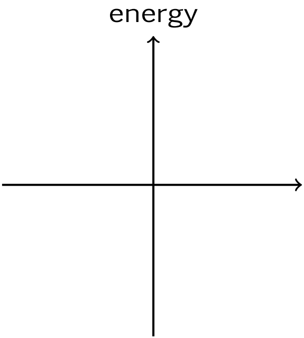
Encode Priorities as Energy (Jančar, RP '15)

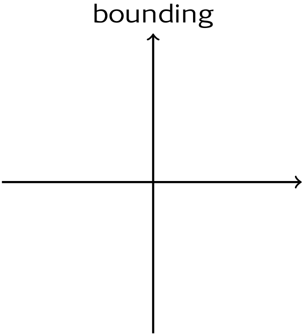
Two new dimensions: tolerance to humid low/high temperature



Bounding Games

Player 1's Objective





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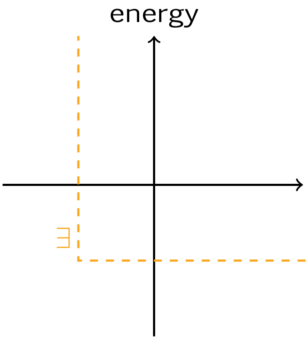

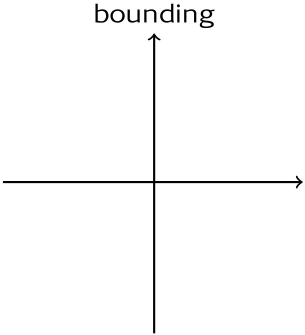

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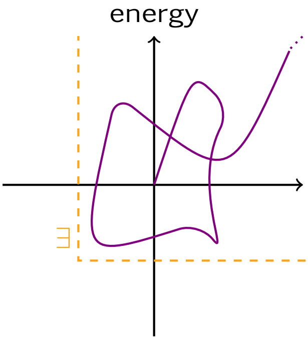

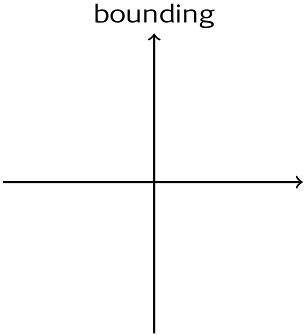

Bounding Games

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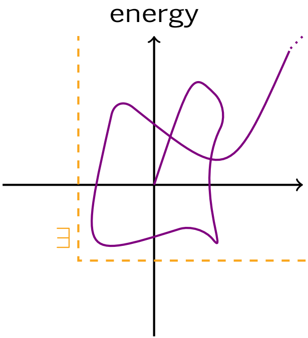

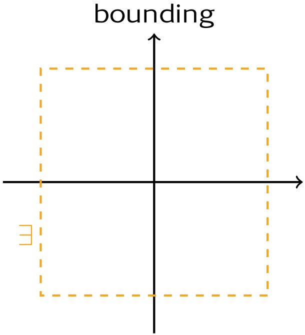

Bounding Games

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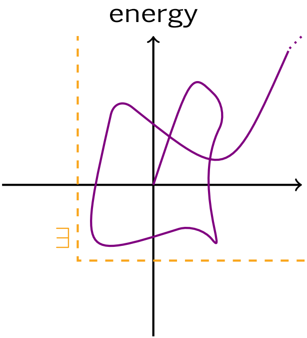
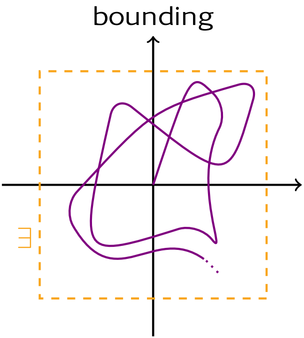
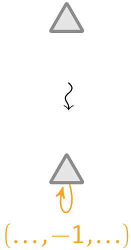




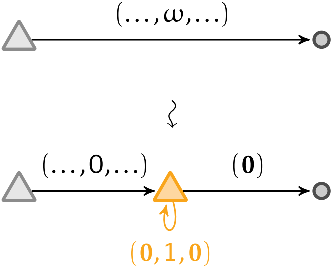
Bounding Games

Encoding Extended Energy Games

Bin excess energy



Unbounded replenishing



Bounding Games

Theorem (Jurdziński et al., ICALP '15)
Bounding games on multi-weighted game graphs (V, E, d) *are solvable in* $(|V| \cdot \|E\|)^{O(d^4)}$.

Corollary
The given initial credit problem with credit c *for energy parity games on multi-weighted game graphs* (V, E, d) *with* p *even priorities is solvable in*

$$O(|V| \cdot \|E\|)^{2^{O(d \log(d+p))}} + O(d \cdot \log \|c\|) .$$

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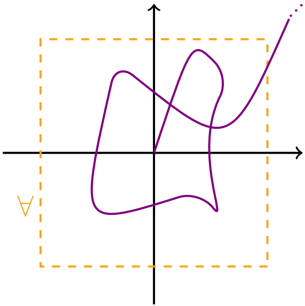
Perfect Half Space Games

Player 2's Objective in a Bounding Game

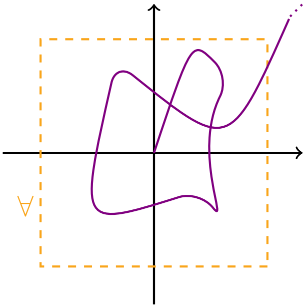


Key Intuition

Player 2 can escape in a perfect half space

Perfect Half Space Games

Player 2's Objective in a Bounding Game



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Perfect Half Space Games

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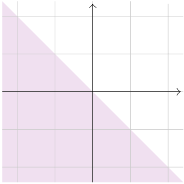
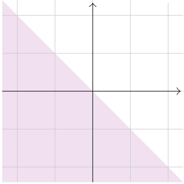


$$\{(x,y) : x+y<0\}$$

Perfect Half Space Games

Perfect Half Space

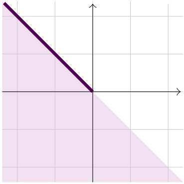


$$\{(x,y) : x+y<0\}$$

boundary: $\{(x,y) : x+y=0\}$

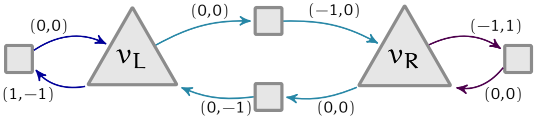
Perfect Half Space Games

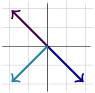
Perfect Half Space



$$\{(x,y) : x+y<0\}$$
$$\cup\{(x,y) : x+y=0 \wedge x<0\}$$

Perfect Half Space Games





Plays

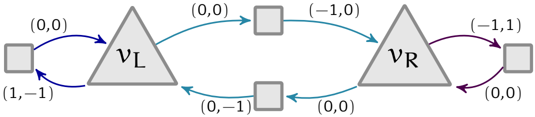
- pairs of vertices and perfect half spaces:

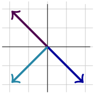
$$(v_0, \mathbf{H}_0) \xrightarrow{\mathbf{w}_1} (v_1, \mathbf{H}_1) \xrightarrow{\mathbf{w}_2} (v_2, \mathbf{H}_2) \cdots$$

- in his vertices, Player 2 chooses the current perfect half space

- Player 2 wins if $\exists i$ s.t. $\sum_{j \geqslant 0} \mathbf{w}_j$ diverges into $\bigcap_{j>i} \mathbf{H}_j$

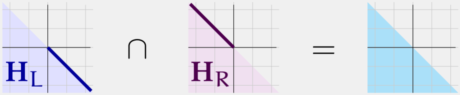
Perfect Half Space Games





- Player 2 wins if $\exists i$ s.t. $\sum_{j \geqslant 0} \mathbf{w}_j$ diverges into $\bigcap_{j > i} \mathbf{H}_j$

Example



Solving Perfect Half Space Games

Theorem

Perfect half space games on multi-weighted game graphs (V, E, d) are solvable in $(|V| \cdot \|E\|)^{O(d^3)}$.

Proof Idea

- reduce to a lexicographic energy game (Colcombet and Niwiński)
- $\approx$ perfect half space game with a single fixed $\mathbf{H}$
- itself reduced to a mean-payoff game

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Player 2 Strategies

Oblivious Strategy

Player 2 chooses the same $\mathbf{H}_v$ every time it visits vertex v

Theorem

If Player 2 has a winning strategy in a perfect half space game, then it has an oblivious one.

"Counterless" Strategy

Corollary (Brázdil et al., ICALP '10)

If Player 2 has a winning strategy in a multi-dimensional energy parity game, then it has a positional one.

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Concluding Remarks

- tight 2-EXP bounds for multi-energy parity games
- impacts numerous problems
- fine understanding of Player 2's strategies:
  Player 2 can win by announcing in which perfect half space he will escape

Disclaimer

The Icelandic Met Office does not endorse any of the information provided during this talk, and cannot be held liable for a ruined week-end subsequent to foolishly trusting these fabricated forecasts.

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