The Complexity of Coverability in ν-Petri Nets

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Outline

ν-Petri nets (νPN)
Petri nets with data management and creation
(Rosa-Velardo and de Frutos-Escrig, 2008, 2011)

coverability

- decidable by classical backward coverability algorithm (Abdulla et al., 2000)
- dual view using downwards-closed sets (Lazić and S., 2015)

complexity νPN coverability is complete for double Ackermann ($\mathbf{F}_{\omega\cdot 2}$-complete)

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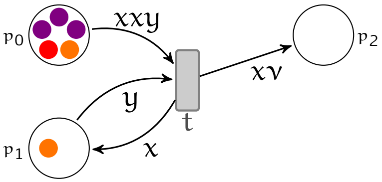
ν-Petri Nets

Tokens carry data from an infinite countable domain $\mathbb{D}$



Configurations in $(\mathbb{N}^P)^{\circledast}$: multisets of markings

$$\left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right]$$

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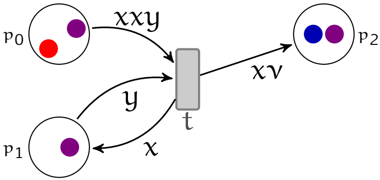

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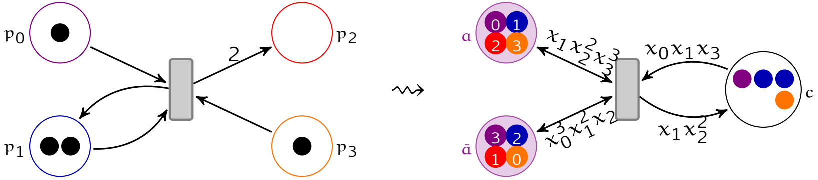
$$\left[\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}3\\0\\0\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}\right] \xrightarrow{t} \left[\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}0\\0\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\right]$$

Petri Nets as ν-Petri Nets



- a and $\bar{a}$ are complementary addressing places
- c holds the actual token counts

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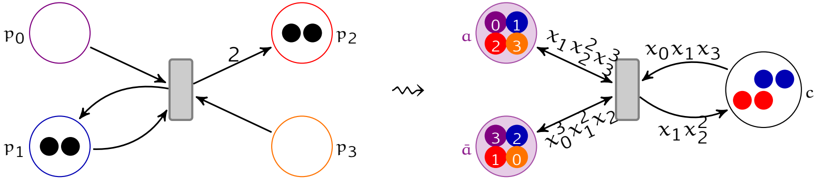

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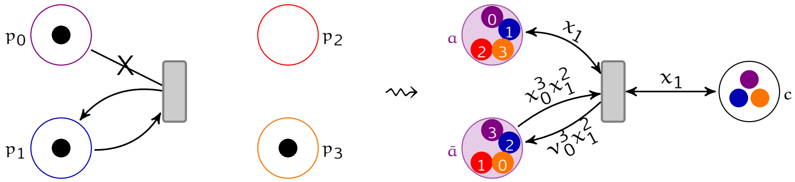
Reset Petri Nets as ν-Petri Nets



- $\mathfrak{a}$ and $\bar{\mathfrak{a}}$ are complementary addressing places for active tokens
- $\mathfrak{c}$ holds both the active and inactive tokens

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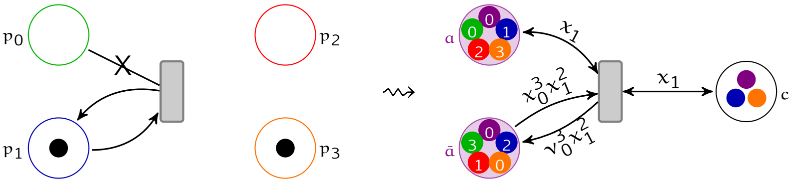

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- c holds both the active and inactive tokens

Coverability Problem

verification of safety properties "nothing bad happens"

ordering of configurations by multiset embedding

$$[\mathbf{u}_1,\ldots,\mathbf{u}_n] \sqsubseteq [\mathbf{v}_1,\ldots,\mathbf{v}_p]$$

iff $\exists f:\{1,\ldots,n\}\to\{1,\ldots,p\}$ injective ,
$\forall 1 \leqslant i \leqslant n, u_i \leqslant v_{f(i)}$
Example:

$$\left[\begin{pmatrix}1\\0\\1\end{pmatrix}\begin{pmatrix}3\\0\\0\end{pmatrix}\right] \sqsubseteq \left[\begin{pmatrix}10\\1\\0\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}\begin{pmatrix}2\\0\\3\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix}\right]$$

input a νPN, a source configuration src, and a "bad" configuration tgt

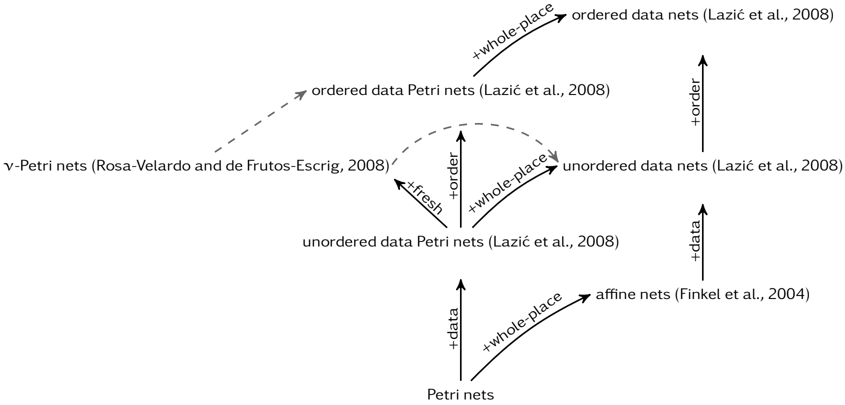
question $\exists m, \mathrm{tgt} \sqsubseteq m$ and $\mathrm{src} \to^* m$?

Polyadic ν-Petri Nets

(Rosa-Velardo and Martos-Salgado, 2012)

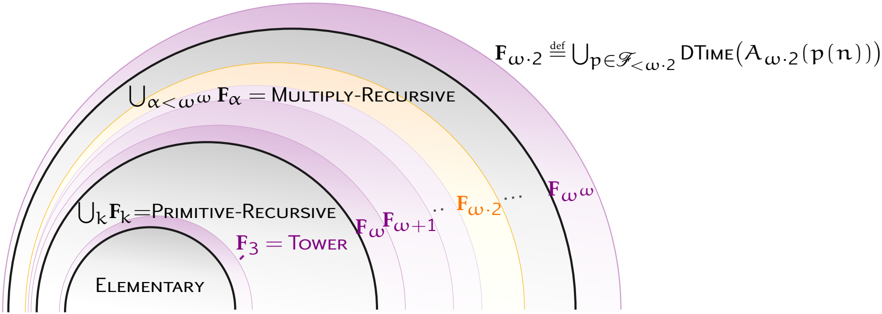
- hold tuples of tokens in places
- equivalent to the full π-calculus
- model of dynamic database systems with existential positive guards
- undecidable coverability

Taxonomy of Petri Net Extensions



FAST-GROWING COMPLEXITY

(S., 2016)



- Ackermann: "Ackermannian in" $x \mapsto 2x$

$$A_1(x) \stackrel{\text{def}}{=} 2x \qquad A_{k+2}(x) \stackrel{\text{def}}{=} A_{k+1}^x(1) \qquad A_\omega(x) \stackrel{\text{def}}{=} A_{x+1}(x)$$

- double Ackermann: "Ackermannian in" $A_\omega(x)$

$$A_{\omega+k+1}(x) \stackrel{\text{def}}{=} A_{\omega+k}^x(1) \qquad A_{\omega\cdot 2}(x) \stackrel{\text{def}}{=} A_{\omega+x+1}(x)$$

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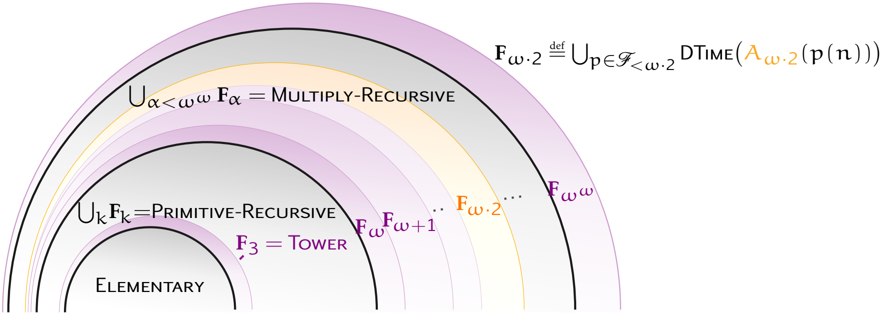

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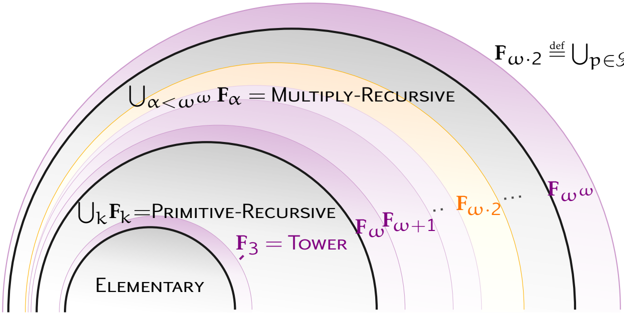

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Main Result

Theorem
Coverability in νPNs is $\mathbf{F}_{\omega\cdot 2}$-complete.

lower bound extends Lipton's "object-oriented" programming in Petri nets

- basic block: Ackermann counters using Schnoebelen's construction for reset Petri nets
- pushed to double Ackermann: composition and iteration operations

upper bound analyses a dual view of the backward coverability algorithm

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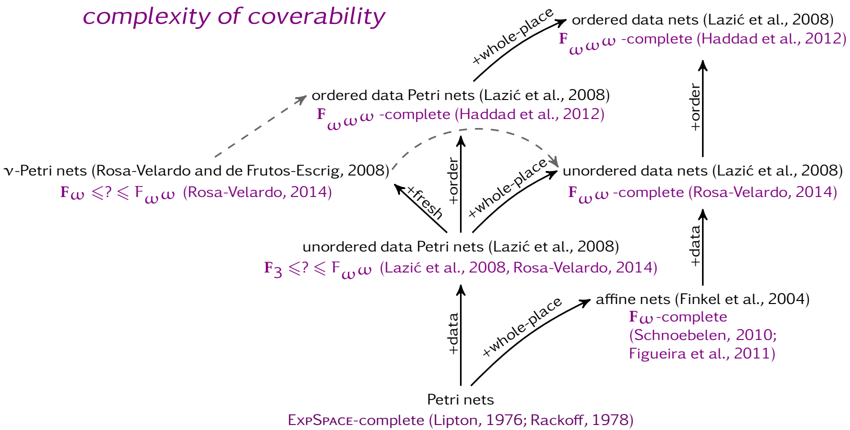

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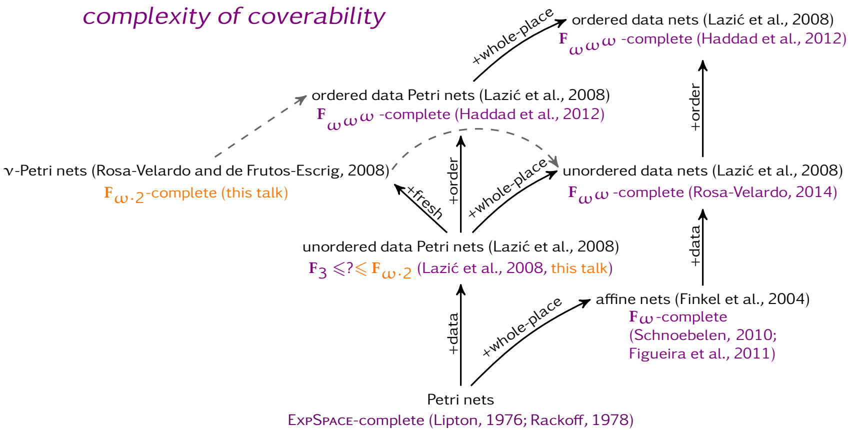

ν-Petri Nets are Well-Structured

(Finkel and Schnoebelen, 2001; Abdulla et al., 2000)

1. $((\mathbb{N}^P)^\circledast, \sqsubseteq)$ is a well-quasi-order (wqo), which entails

   finite bad sequences any sequence $m_0, m_1, m_2, \ldots$ with $\forall i < j, m_i \not\sqsubseteq m_j$, is finite

   finite basis property any upwards-closed subset U has a finite basis B such that $U = \uparrow B$

   ascending chain property all the ascending chains $U_0 \subsetneq U_1 \subsetneq U_2 \subsetneq \cdots$ of upwards-closed subsets are finite

2. compatibility: if $m_1 \sqsubseteq m_1'$ and $m_1 \to m_2$, then there exists m_2', $m_2 \sqsubseteq m_2'$ and $m_1' \to m_2'$

"Classical" Backward Coverability

(Abdulla et al., 2000)

compute $U_k = \{m' \mid \exists m \sqsupseteq tgt, m' \to^{\leqslant k} m\}\,;\, U_* = \bigcup_k U_k$:

initially $U_0 \stackrel{\text{def}}{=} \uparrow tgt$

step $U_{k+1} \stackrel{\text{def}}{=} Pre_\exists(U_k) \cup U_k$
where

$$Pre_\exists(S) \stackrel{\text{def}}{=} \{m \mid \exists s \in S, m \to s\}$$

representation of upwards-closed subsets U through their minimal elements thanks to finite basis property

termination guaranteed by ascending chain property

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Ideal Decompositions for a wqo $(X, \leqslant)$

(Bonnet, 1975; Finkel and Goubault-Larrecq, 2009; Goubault-Larrecq et al., 2016)

- a subset $\Delta \subseteq X$ is directed iff $\Delta \neq \emptyset$ and $\forall x, y \in \Delta, \exists z \in \Delta, x \leqslant z$ and $y \leqslant z$
- an ideal I is a downwards-closed and directed subset
- equivalently, I is downwards-closed and irreducible: if $I \subseteq D_1 \cup D_2$ for D_1, D_2 downwards-closed, then $I \subseteq D_1$ or $I \subseteq D_2$
- every downwards-closed subset $D \subseteq X$ is the union of a unique finite family of incomparable ideals: $D = I_1 \cup \cdots \cup I_n$, called its canonical ideal decomposition
- finite ideal representations for many wqos

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Dual Backward Coverability

(Lazić and S., 2015)

compute $D_k = \{m' \mid \forall m \sqsupseteq tgt, m' \not\to^{\leq k} m\}\,;\, D_* = \bigcap_k D_k$:

initially $D_0 \stackrel{\text{def}}{=} (\mathbb{N}^P)^{\circledast} \setminus (\uparrow tgt)$

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$$Pre_{\forall}(S) \stackrel{\text{def}}{=} \{m \mid \forall s, m \to s \implies s \in S\}$$

representation of downwards-closed subsets D through finite representations of their ideal decompositions

termination guaranteed by descending chain property

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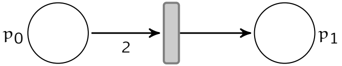
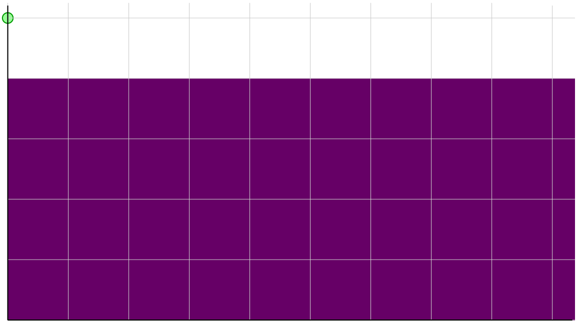
Dual Backward Coverability: Example



$\mathrm{tgt} = (0,5)$



$$D_0 = \downarrow(\omega, 4)$$

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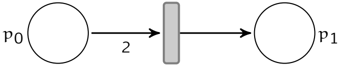

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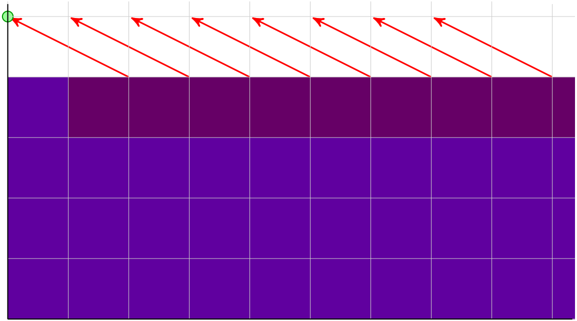

$$D_1 = \downarrow(1,4) \cup \downarrow(\omega,3)$$

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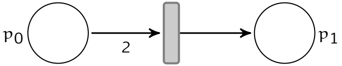

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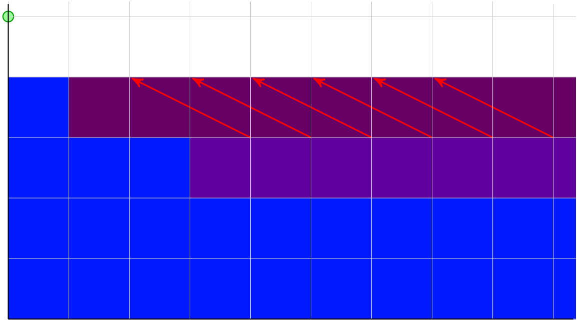

$$D_2 = \downarrow(1,4) \cup \downarrow(3,3) \cup \downarrow(\omega,2)$$

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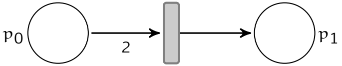

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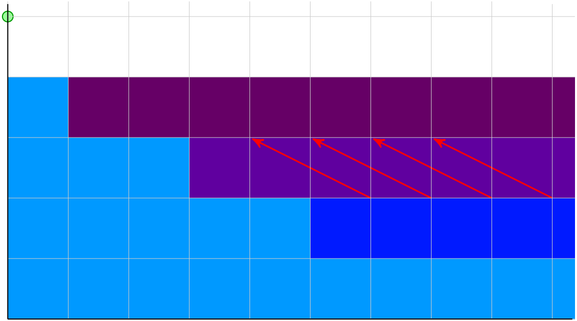

$$D_3 = \downarrow(1,4) \cup \downarrow(3,3) \cup \downarrow(5,2) \cup \downarrow(\omega,1)$$

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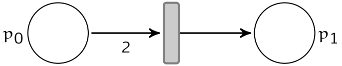

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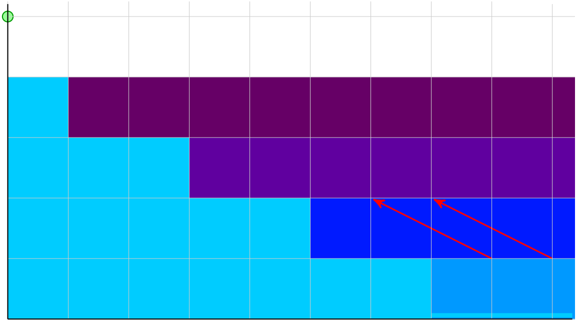

$$D_4 = \downarrow(1,4) \cup \downarrow(3,3) \cup \downarrow(5,2) \cup \downarrow(7,1) \cup \downarrow(\omega,0)$$

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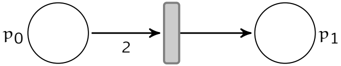

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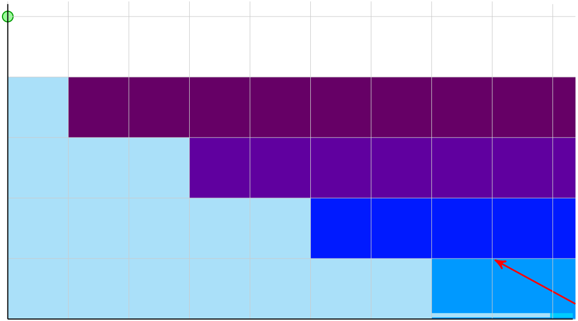

$$D_5 = \downarrow(1,4) \cup \downarrow(3,3) \cup \downarrow(5,2) \cup \downarrow(7,1) \cup \downarrow(9,0) = D_*$$

Controlled Sequences

- consider a norm $\|.\| : X \to \mathbb{N}$ with $\forall n, X_{\leqslant n} \stackrel{\text{def}}{=} \{x \in X \mid \|x\| \leqslant n\}$ finite:

$$\|\mathbf{u}\| \stackrel{\text{def}}{=} \max_{p \in P \mid \mathbf{u}(p) < \omega} \mathbf{u}(p) \qquad \text{for } \mathbf{u} \in \mathbb{N}_\omega^P$$

$$\|B,S\| \stackrel{\text{def}}{=} \max_{\mathbf{u} \in \text{Support}(B), \mathbf{v} \in S} (|B|, \|\mathbf{u}\|, \|\mathbf{v}\|) \qquad \text{for } \downarrow(B,S) \in \text{Idl}((\mathbb{N}^P)^\circledast)$$

$$\|D\| \stackrel{\text{def}}{=} \max_{1 \leqslant i \leqslant n} \|B_i, S_i\| \qquad \text{for } D = \downarrow(B_1,S_1) \cup \cdots \cup \downarrow(B_n,S_n)$$

- consider a control function $g : \mathbb{N} \to \mathbb{N}$ strictly monotone and an initial norm $n \in \mathbb{N}$
- a sequence $x_0, x_1, \ldots$ of elements of X is (g,n)-controlled if $\forall i, \|x_i\| \leqslant g^i(n)$

  strongly (g,n)-controlled if $\|x_0\| \leqslant n$ and $\forall i, \|x_{i+1}\| \leqslant g(\|x_i\|)$

Controlled Sequences

- consider a norm $\|.\| : X \to \mathbb{N}$ with $\forall n, X_{\leqslant n} \stackrel{\text{def}}{=} \{x \in X \mid \|x\| \leqslant n\}$ finite:

$$\|\mathbf{u}\| \stackrel{\text{def}}{=} \max_{p \in P \mid \mathbf{u}(p) < \omega} \mathbf{u}(p) \qquad \text{for } \mathbf{u} \in \mathbb{N}_\omega^P$$

$$\|B,S\| \stackrel{\text{def}}{=} \max_{\mathbf{u} \in \text{Support}(B), \mathbf{v} \in S} (|B|, \|\mathbf{u}\|, \|\mathbf{v}\|) \qquad \text{for } {\downarrow}(B,S) \in \text{Idl}((\mathbb{N}^P)^\circledast)$$

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Length Function Theorems (1/3)

(Figueira et al., 2011; S. and Schnoebelen, 2012)

Fact (Length Function Theorem for Bad Sequences in $\mathbb{N}_\omega^P$)

Let $n > 0$. *Any* (g,n)*-controlled bad sequence* $\mathbf{e}_0, \mathbf{e}_1, \ldots, \mathbf{e}_\ell$ *of extended markings in* $(\mathbb{N}_\omega^P, \leqslant)$ *has length at most "Ackermannian in"* $g(\max(n, |P|))$.

Length Function Theorems (2/3)

(Lazić and S., 2015)

- consider a descending chain $D_0 \supsetneq D_1 \supsetneq \cdots \supsetneq D_\ell$
- extract at each step $0 \leqslant k < \ell$ a proper ideal I_k from the canonical decomposition of D_k, s.t. $I_k \not\subseteq D_{k+1}$
- bad sequence of proper ideals $I_0, I_1, \ldots, I_{\ell-1}$
- in particular, for descending chains ${\downarrow}S_0 \supsetneq {\downarrow}S_1 \supsetneq \cdots \supsetneq {\downarrow}S_\ell$ of antichains

Corollary (Length Function Theorem for Hoare-Descending Chains over $\mathbb{N}_\omega^P$)

Let $n > 0$. Any (g, n)-controlled descending chain ${\downarrow}S_0 \supsetneq {\downarrow}S_1 \supsetneq \cdots \supsetneq {\downarrow}S_\ell$ of antichains of $(\mathbb{N}_\omega^P, \leqslant)$ has length at most "Ackermannian in" $g(\max(n, |P|))$.

Length Function Theorems (2/3)

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Length Function Theorems (3/3)

- a descending chain $D_0 \supsetneq D_1 \supsetneq \cdots \supsetneq D_\ell$ over $(\mathbb{N}^P)^\circledast$ is star-monotone if $\forall 0 \leqslant k < \ell - 1, \forall I_{k+1} = {\downarrow}(B_{k+1}, S_{k+1})$ proper ideal from the canonical decomposition of D_{k+1}, $\exists I_k = {\downarrow}(B_k, S_k)$ proper ideal from the canonical decomposition of D_k s.t. ${\downarrow}S_{k+1} \subseteq {\downarrow}S_k$

Theorem (Length Function Theorem for Star-Monotone Descending Chains over $(\mathbb{N}_\omega^P)^\circledast$)

Let $n > 0$. Any strongly (g, n)-controlled star-monotone descending chain $D_0 \supsetneq D_1 \supsetneq \cdots \supsetneq D_\ell$ of configurations in $(\mathbb{N}_\omega^P)^\circledast$ has length at most “double Ackermannian in” $g(\max(n, |P|))$.

Wrapping Up

Lemma (Strong Control for νPNs)

The descending chain computed by the backward algorithm for a νPN N *and target* tgt *is strongly* (g, n)*-controlled for* $g(x) \stackrel{def}{=} x + |N|$ *and* $n \stackrel{def}{=} \|\mathrm{tgt}\|$.

Lemma (νPN Descending Chains are Star-Monotone)

The descending chains computed by the backward coverability algorithm for νPNs are star-monotone.

Theorem (Upper Bound)

The coverability problem for νPNs is in $\mathbf{F}_{\omega\cdot 2}$.

Concluding Remarks

- first "natural" decision problem complete for $\mathbf{F}_{\omega\cdot 2}$
- ideals and downwards-closed sets as algorithmic tools
  - here, backward analysis (Lazić and S., 2015)
  - forward analysis (Finkel and Goubault-Larrecq, 2009, 2012)
  - reachability in Petri nets (Leroux and S., 2015)
  - formal languages (Zetzsche, 2015; Hague et al., 2016)
  - invariant inference (Padon et al., 2016)
  - piecewise testable separability (Goubault-Larrecq and S., 2016)

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