Games with Discrete Resources

Sylvain Schmitz
with Th. Colcombet, J.-B. Courtois, M. Jurdziński, and R. Lazić

LSV, ENS Paris-Saclay & CNRS

IBISC, October 19, 2017

Outline

multi-dimensional energy parity games

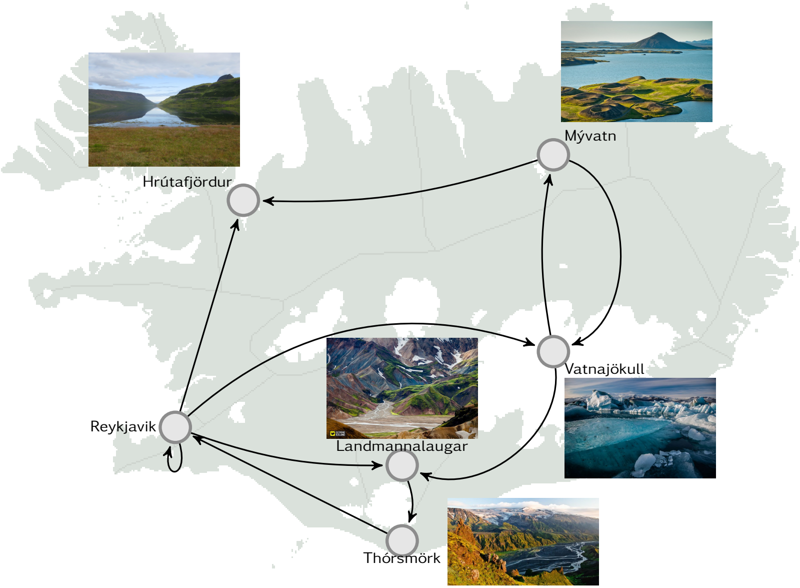
complexity through perfect half-space games
(Colcombet et al., LICS ’17)

related problems:

- multi-dimensional mean-payoff parity games
  (Chatterjee et al., Concur ’12)
- VASS games (too many references!)
- regular VASS simulations (Courtois and S., MFCS ’14)
- $(!,\oplus)$-Horn linear logic (Kanovich, APAL ’95)
- μ-calculus on VASS (Abdulla et al., Concur ’13)
- resource-bounded agent temporal logic RB$\pm$ATL*
  (Alechina et al., RP ’16)

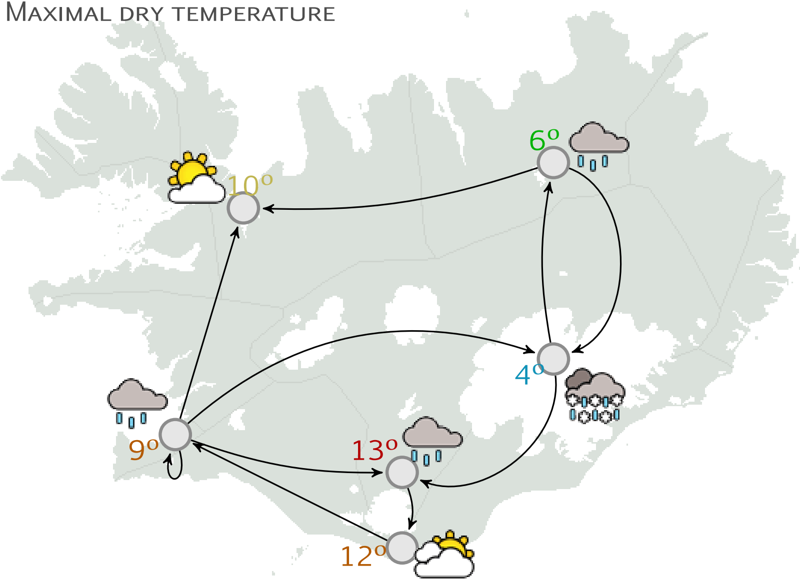
Where to Treck in Iceland?

Where to Treck in Iceland?



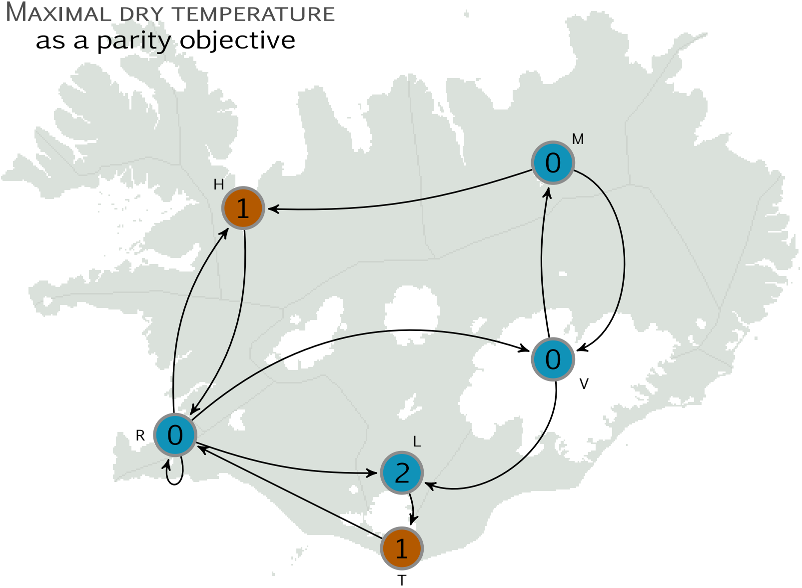
Where to Treck in Iceland?

Maximal dry temperature



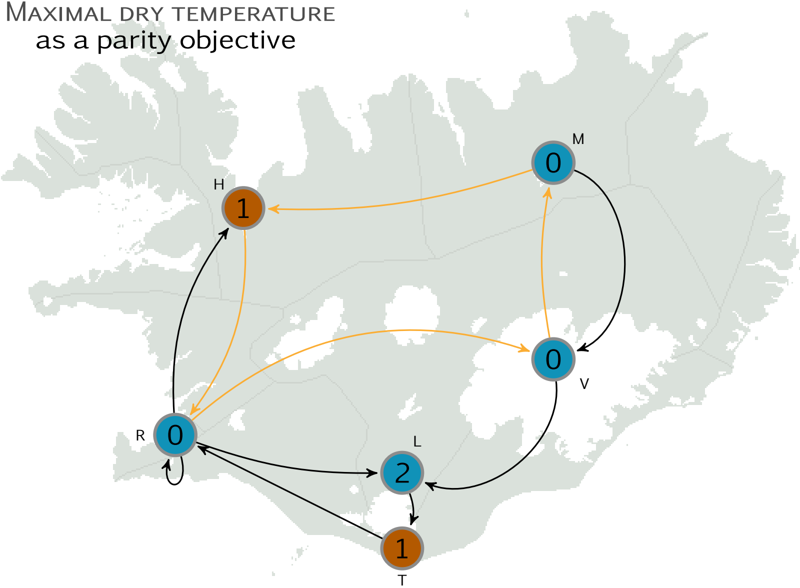
Where to Treck in Iceland?

Maximal dry temperature
as a parity objective



Where to Treck in Iceland?

Maximal dry temperature
as a parity objective

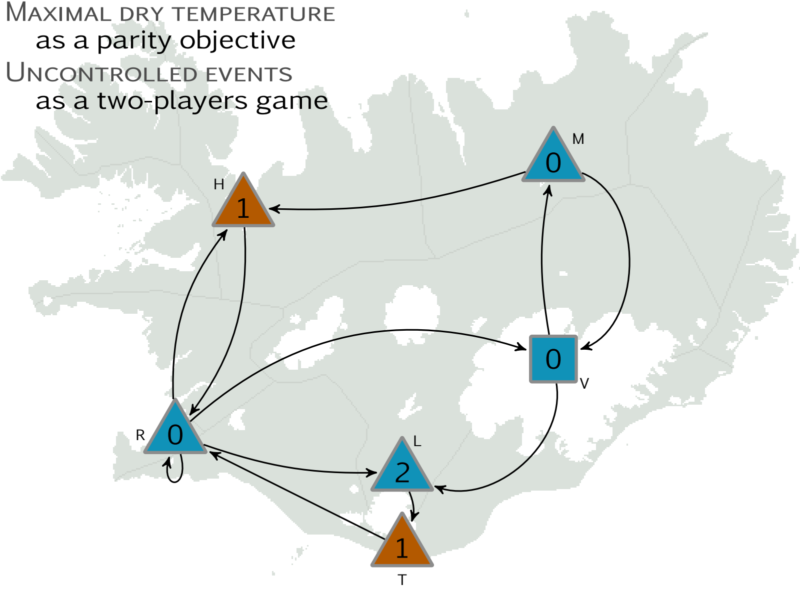


Where to Treck in Iceland?

Uncontrolled events

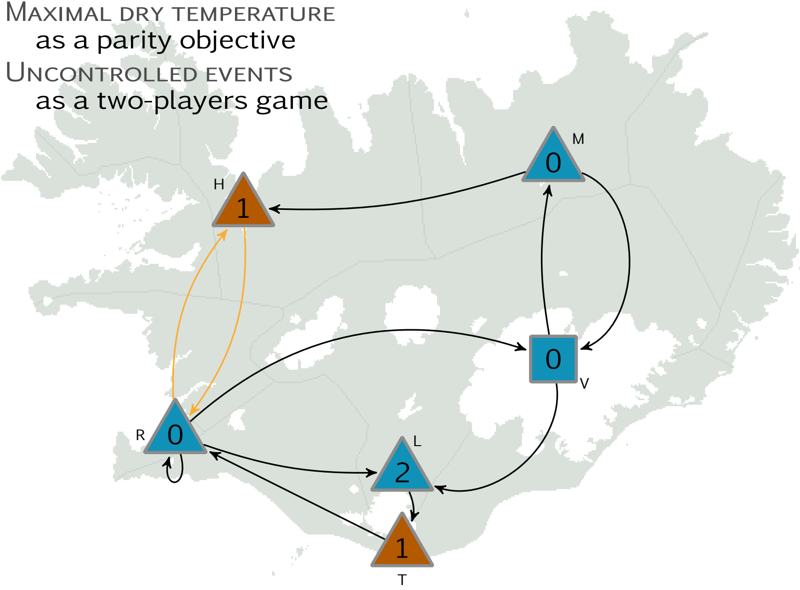
Where to Treck in Iceland?

Maximal dry temperature
as a parity objective
Uncontrolled events
as a two-players game



Where to Treck in Iceland?

Maximal dry temperature
as a parity objective
Uncontrolled events
as a two-players game



Where to Treck in Iceland?

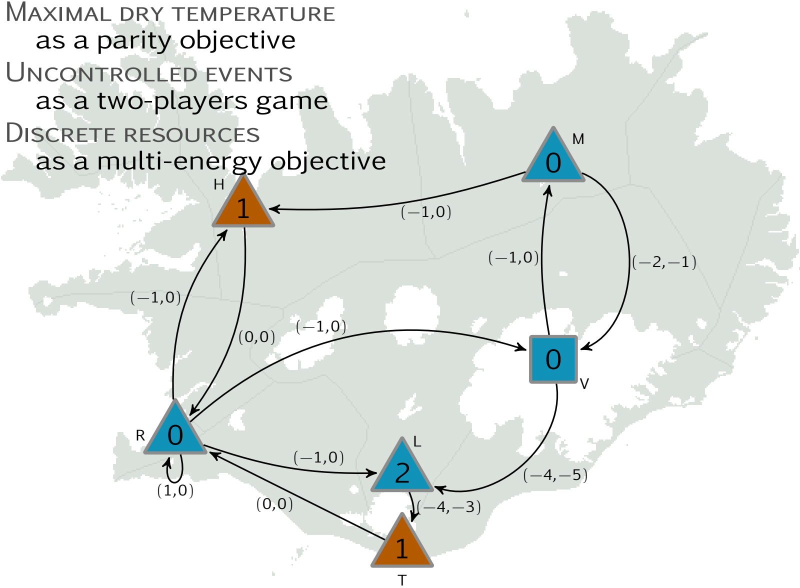
Discrete resources

Where to Treck in Iceland?

Maximal dry temperature
as a parity objective

Uncontrolled events
as a two-players game

Discrete resources
as a multi-energy objective

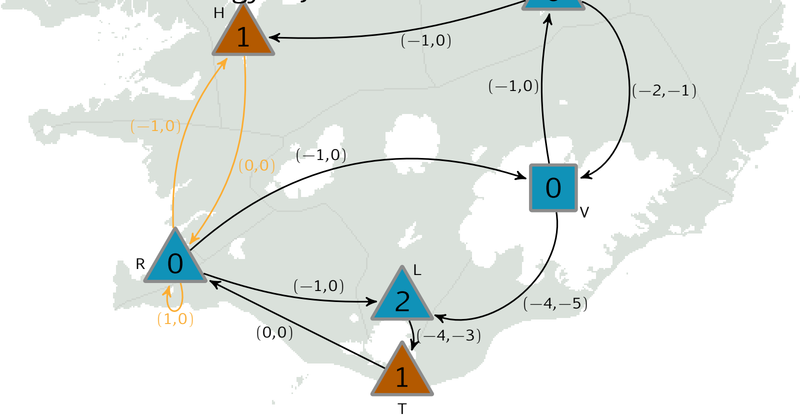


Where to Treck in Iceland?

Maximal dry temperature
    as a parity objective

Uncontrolled events
    as a two-players game

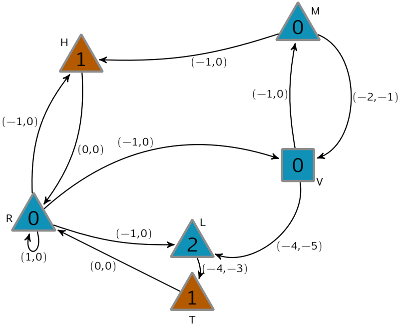
Discrete resources
    as a multi-energy objective



Multi-Dimensional Energy Parity Games

Player 1 wins a play if both

- energy objective: no component goes negative
- parity objective: the maximal priority is odd



Example

$$R(0,0) \xrightarrow{(1,0)} R(1,0) \xrightarrow{(1,0)} R(2,0) \xrightarrow{(-1,0)} H(1,0) \xrightarrow{(0,0)} R(1,0) \rightarrow \cdots$$

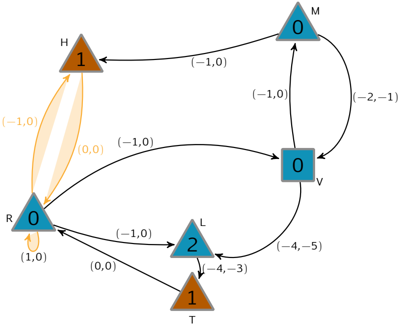
Decision problems: Does Player 1 have a winning strategy

- given initial credit as part of the input
- existential: for some initial credit

Multi-Dimensional Energy Parity Games

Player 1 wins a play if both

- energy objective: no component goes negative
- parity objective: the maximal priority is odd



Example

$$R(0,0) \xrightarrow{(1,0)} R(1,0) \xrightarrow{(1,0)} R(2,0) \xrightarrow{(-1,0)} H(1,0) \xrightarrow{(0,0)} R(1,0) \to \cdots$$

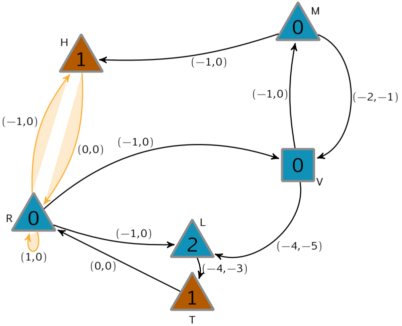
Decision problems: Does Player 1 have a winning strategy

- given initial credit as part of the input
- existential: for some initial credit

Multi-Dimensional Energy Parity Games

Player 1 wins a play if both

- energy objective: no component goes negative
- parity objective: the maximal priority is odd



Example

$$R(0,0) \xrightarrow{(1,0)} R(1,0) \xrightarrow{(1,0)} R(2,0) \xrightarrow{(-1,0)} H(1,0) \xrightarrow{(0,0)} R(1,0) \to \cdots$$

Decision problems: Does Player 1 have a winning strategy

- given initial credit as part of the input
- existential: for some initial credit

Multi-Dimensional Energy Games

Complexity

	lower bound	upper bound
w. initial credit		
∃ initial credit		

Multi-Dimensional Energy Games

Complexity

	lower bound	upper bound
w. initial credit	EXPSPACE (Lasota, IPL '09)	
∃ initial credit		

Multi-Dimensional Energy Games

Complexity

	lower bound	upper bound
w. initial credit	EXPSPACE (Lasota, IPL '09)	TOWER (Brázdil et al., ICALP '10)
$\exists$ initial credit	coNP (Chatterjee et al., FSTTCS '10)	coNP (Chatterjee et al., FSTTCS '10)

Multi-Dimensional Energy Games

Complexity

	lower bound	upper bound
w. initial credit	2-EXP (Courtois and S., MFCS '14)	TOWER (Brázdil et al., ICALP '10)
$\exists$ initial credit	coNP (Chatterjee et al., FSTTCS '10)	coNP (Chatterjee et al., FSTTCS '10)

Multi-Dimensional Energy Games

Complexity

	lower bound	upper bound
w. initial credit	2-EXP (Courtois and S., MFCS '14)	2-EXP (Jurdziński et al., ICALP '15)
$\exists$ initial credit	coNP (Chatterjee et al., FSTTCS '10)	coNP (Chatterjee et al., FSTTCS '10)

Multi-Dimensional Energy Parity Games

Complexity

	lower bound	upper bound
w. initial credit	2-EXP (Courtois and S., MFCS '14)	
∃ initial credit	coNP (Chatterjee et al., Concur '12)	coNP (Chatterjee et al., Concur '12)

Multi-Dimensional Energy Parity Games

Complexity

	lower bound	upper bound
w. initial credit	2-EXP (Courtois and S., MFCS '14)	decidable (Abdulla et al., Concur '13)
$\exists$ initial credit	coNP (Chatterjee et al., Concur '12)	coNP (Chatterjee et al., Concur '12)

Multi-Dimensional Energy Parity Games

Complexity

	lower bound	upper bound
w. initial credit	2-EXP (Courtois and S., MFCS '14)	TOWER (Jančar, RP '15)
$\exists$ initial credit	coNP (Chatterjee et al., Concur '12)	coNP (Chatterjee et al., Concur '12)

Multi-Dimensional Energy Parity Games

Complexity

	lower bound	upper bound
w. initial credit	2-EXP (Courtois and S., MFCS '14)	2-EXP (Colcombet et al., LICS '17)
$\exists$ initial credit	coNP (Chatterjee et al., Concur '12)	coNP (Chatterjee et al., Concur '12)

Fixed-Dimensional Energy Parity Games

Complexity

	lower bound	upper bound
w. initial credit	EXP for $d \geqslant 2$ (Jurdziński et al., LMCS '08)	pseudoP (Colcombet et al., LICS '17)
$\exists$ initial credit		pseudoP (Colcombet et al., LICS '17)

Complexity of Multi-Energy Parity Games

Theorem (Colcombet et al., LICS '17)

1. *The given initial credit problem for multi-dimensional energy parity games is in 2-EXP.*
2. *With fixed dimension and number of priorities, it is in pseudo polynomial time.*

- series of reductions using notably perfect half-space games
- fine understanding of Player 2's strategies:
  Player 2 can win by announcing in which perfect half space he will escape

Complexity of Multi-Energy Parity Games

Theorem (Colcombet et al., LICS '17)

1. *The given initial credit problem for multi-dimensional energy parity games is in 2-EXP.*
2. *With fixed dimension and number of priorities, it is in pseudo polynomial time.*

- series of reductions using notably perfect half-space games
- fine understanding of Player 2's strategies:
  Player 2 can win by announcing in which perfect half space he will escape

Reductions and Strategy Transfers

multi-dimensional energy parity games

$\downarrow$ (Jančar, RP '15)

extended multi-dimensional energy games (Brázdil et al., ICALP '10)

$\downarrow$

bounding games (Jurdziński et al., ICALP '15)

$\downarrow$

perfect half space games (Colcombet et al., LICS '17)

$\downarrow$

lexicographic energy games (Colcombet and Niwiński)

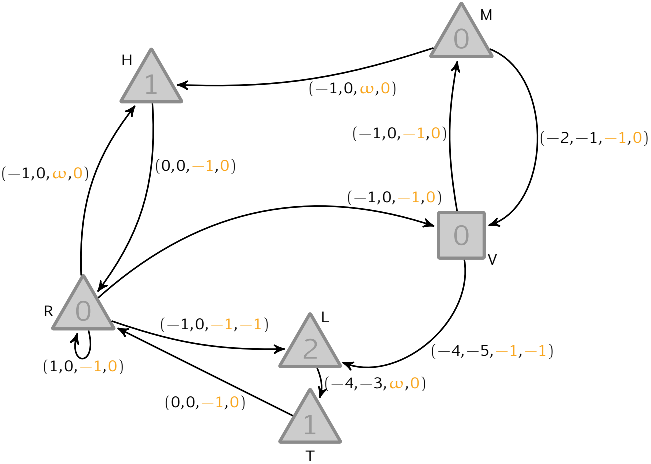
$\downarrow$

mean-payoff games (Comin and Rizzi, Algorithmica '16)

Extended Multi-Dimensional Energy Games

Encode Priorities as Energy (Jančar, RP '15)

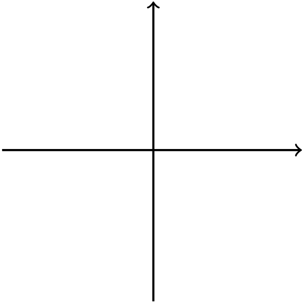
Two new dimensions: tolerance to humid low/high temperature



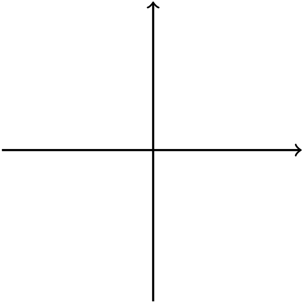
Bounding Games

Player 1's Objective

existential energy



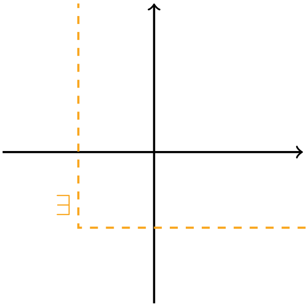
bounding



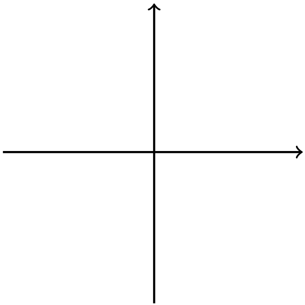
Bounding Games

Player 1's Objective

existential energy

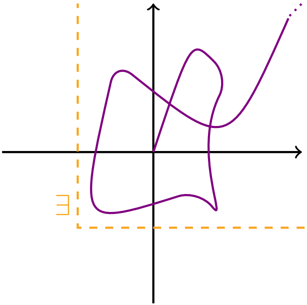


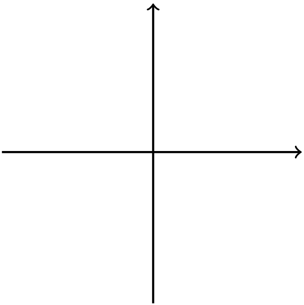
bounding



Bounding Games

Player 1's Objective

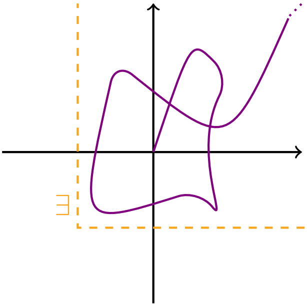




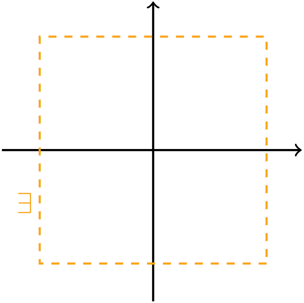
Bounding Games

Player 1's Objective

existential energy



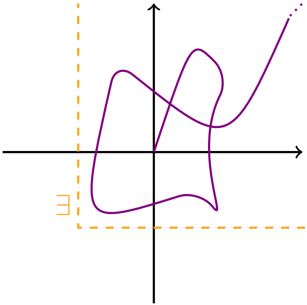
bounding



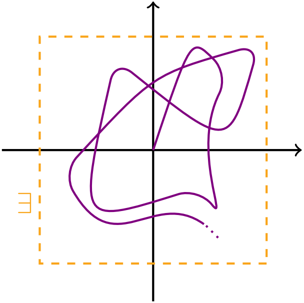
Bounding Games

Player 1's Objective

existential energy



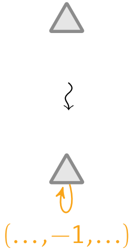
bounding



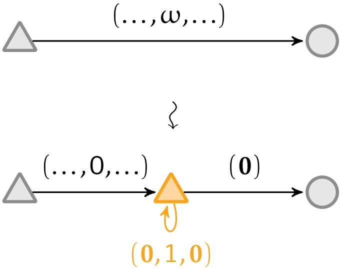
Bounding Games

Encoding Extended Energy Games

Bin excess energy

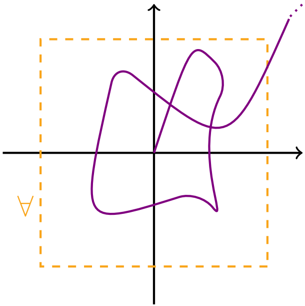


Unbounded replenishing



Perfect Half Space Games

Player 2's Objective in a Bounding Game

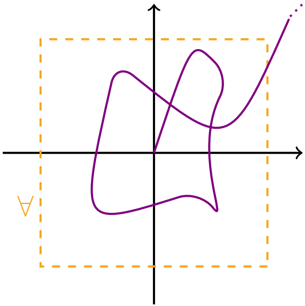


Key Intuition

Player 2 can escape in a perfect half space

Perfect Half Space Games

Player 2's Objective in a Bounding Game

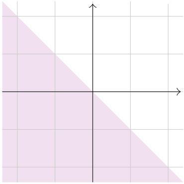


Key Intuition

Player 2 can escape in a perfect half space

Perfect Half Space Games

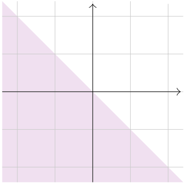
Perfect Half Space



$$\{(x,y) : x + y < 0\}$$

Perfect Half Space Games

Perfect Half Space

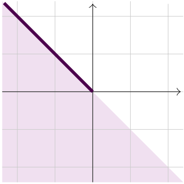


$$\{(x,y) : x+y < 0\}$$

$$\text{boundary: } \{(x,y) : x+y = 0\}$$

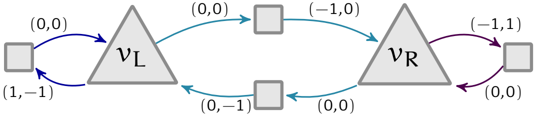
Perfect Half Space Games

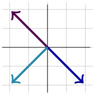
Perfect Half Space



$$\{(x, y) : x + y < 0\}$$
$$\cup\{(x, y) : x + y = 0 \wedge x < 0\}$$

Perfect Half Space Games





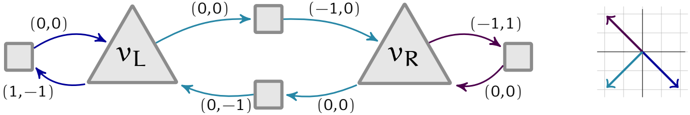
Plays

- pairs of vertices and perfect half spaces:

$$(v_0, \mathbf{H}_0) \xrightarrow{\mathbf{w}_1} (v_1, \mathbf{H}_1) \xrightarrow{\mathbf{w}_2} (v_2, \mathbf{H}_2) \cdots$$

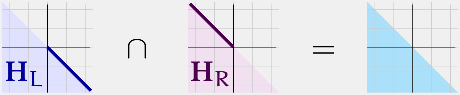
- in his vertices, Player 2 chooses the current perfect half space

Perfect Half Space Games



- Player 2 wins if $\exists i$ s.t. $\sum_{j\geqslant 0} \mathbf{w}_j$ diverges into $\bigcap_{j>i} \mathbf{H}_j$

Example



Solving Perfect Half Space Games

Theorem

Perfect half space games on multi-weighted game graphs (V, E, d) are solvable in $(|V| \cdot \|E\|)^{O(d^3)}$.

Proof Idea

- reduce to a lexicographic energy game (Colcombet and Niwiński)
- $\approx$ perfect half space game with a single fixed $\mathbf{H}$
- itself reduced to a mean-payoff game

Solving Perfect Half Space Games

Theorem

Perfect half space games on multi-weighted game graphs (V, E, d) *are solvable in* $(|V| \cdot \|E\|)^{O(d^3)}$.

Proof Idea

- reduce to a lexicographic energy game (Colcombet and Niwiński)
- $\approx$ perfect half space game with a single fixed $\mathbf{H}$
- itself reduced to a mean-payoff game

Player 2 Strategies

Oblivious Strategy

Player 2 chooses the same $\mathbf{H}_v$ every time it visits vertex v

Theorem

If Player 2 has a winning strategy in a perfect half space game, then it has an oblivious one.

"Counterless" Strategy

Corollary (Brázdil et al., ICALP '10)

If Player 2 has a winning strategy in an existential multi-dimensional energy parity game, then it has a positional one.

Player 2 Strategies

Oblivious Strategy

Player 2 chooses the same $\mathbf{H}_v$ every time it visits vertex v

Theorem

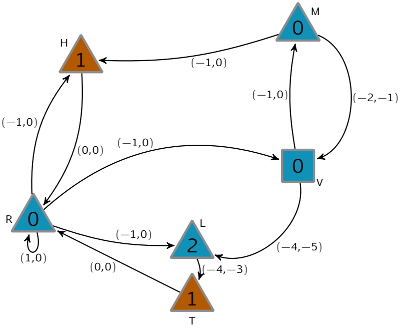
If Player 2 has a winning strategy in a perfect half space game, then it has an oblivious one.

"Counterless" Strategy

Corollary (Brázdil et al., ICALP '10)

If Player 2 has a winning strategy in an existential multi-dimensional energy parity game, then it has a positional one.

VASS Games



multi-energy game configuration arena over $Q \times \mathbb{Z}^d$
+ energy objective

VASS game configuration arena over $Q \times \mathbb{N}^d$

Example

$$R(0,0) \not\rightarrow H(-1,0)$$

Objectives

Monotone objectives:

state reachability given $q_\ell \in Q$, Player 1 wins if any configuration in $\{q_\ell\} \times \mathbb{N}^d$ is visited

non-termination Player 1 wins if the play is infinite

parity given a colouring $c: Q \to \{1, \ldots, k\}$, Player 1 wins if the least colour seen infinitely often is odd

Non-monotone objective:

configuration reachability given $q_\ell \in Q$, Player 1 wins if the configuration $(q_\ell, \mathbf{0})$ is visited

Objectives

Monotone objectives:

state reachability given $q_\ell \in Q$, Player 1 wins if any configuration in $\{q_\ell\} \times \mathbb{N}^d$ is visited

non-termination Player 1 wins if the play is infinite

parity given a colouring $c: Q \to \{1,\ldots,k\}$, Player 1 wins if the least colour seen infinitely often is odd

Non-monotone objective:

configuration reachability given $q_\ell \in Q$, Player 1 wins if the configuration $(q_\ell, \mathbf{0})$ is visited

Objectives

Monotone objectives:

state reachability: given $q_\ell \in Q$, Player 1 wins if any configuration in $\{q_\ell\} \times \mathbb{N}^d$ is visited

non-termination: Player 1 wins if the play is infinite

parity: given a colouring $c : Q \to \{1, \ldots, k\}$, Player 1 wins if the least colour seen infinitely often is odd

Non-monotone objective:

configuration reachability: given $q_\ell \in Q$, Player 1 wins if the configuration $(q_\ell, \mathbf{0})$ is visited

Objectives

Monotone objectives:

state reachability given $q_\ell \in Q$, Player 1 wins if any configuration in $\{q_\ell\} \times \mathbb{N}^d$ is visited

non-termination Player 1 wins if the play is infinite

parity given a colouring $c\colon Q \to \{1,\ldots,k\}$, Player 1 wins if the least colour seen infinitely often is odd

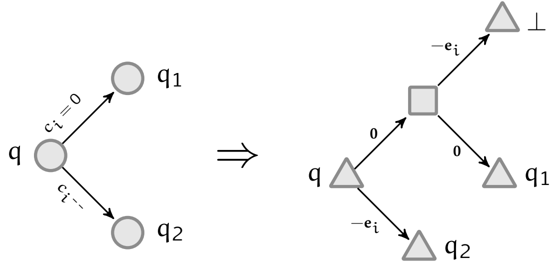
Non-monotone objective:

configuration reachability given $q_\ell \in Q$, Player 1 wins if the configuration $(q_\ell, \mathbf{0})$ is visited

State Reachability VASS Games

Player 2 can enforce zero-tests:

Minsky machine Symmetric VASS Game

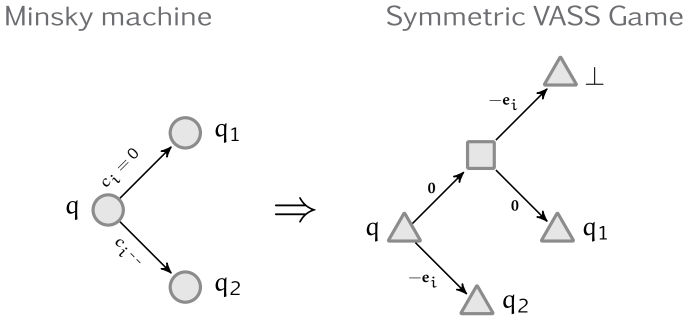


Theorem (Raskin et al., AVoCS '04)

State reachability VASS games with given initial credit are undecidable.

State Reachability VASS Games

Player 2 can enforce zero-tests:



Theorem (Raskin et al., AVoCS '04)
State reachability VASS games with given initial credit are undecidable.

Asymmetric VASS Games

Player 2 moves restricted to use the zero vector.

A frequent assumption

- and-branching VASS (Lincoln et al., APAL '92)
- vector games (Kanovich, APAL '95)
- B-games (Raskin et al., AVoCS '04)
- single-sided games (Abdulla et al., Concur '13)
- alternating VASS (Courtois and S., MFCS '14)

Monotone Objectives (1/2)

Lemma (Monotone Objectives)

If Player 1 wins a monotone AVASS game from a configuration $q,\mathbf{v}$ and $\mathbf{v}' \geqslant \mathbf{v}$, then she also wins from $q,\mathbf{v}'$.

Corollary (by Dickson's Lemma)

- *finite-memory strategies suffice for Player 1*
- *state reachability and non-termination objectives are decidable*

Monotone Objectives (1/2)

Lemma (Monotone Objectives)

If Player 1 wins a monotone AVASS game from a configuration $q, \mathbf{v}$ and $\mathbf{v}' \geqslant \mathbf{v}$, then she also wins from $q, \mathbf{v}'$.

Corollary (by Dickson's Lemma)

- *finite-memory strategies suffice for Player 1*
- *state reachability and non-termination objectives are decidable*

Monotone Objectives (1/2)

Lemma (Monotone Objectives)

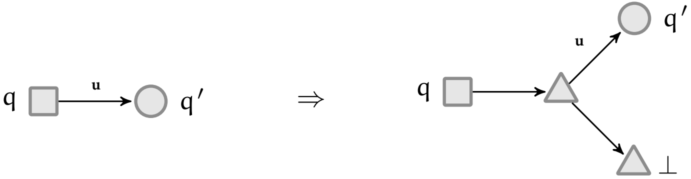
If Player 1 wins a monotone AVASS game from a configuration $\mathrm{q},\mathbf{v}$ and $\mathbf{v}' \geqslant \mathbf{v}$, then she also wins from $\mathrm{q},\mathbf{v}'$.

Corollary (by Dickson's Lemma)

- *finite-memory strategies suffice for Player 1*
- *state reachability and non-termination objectives are decidable*

Monotone Objectives (2/2)

Multi-Energy Games AVASS



Theorem (Abdulla et al., Concur ′13)

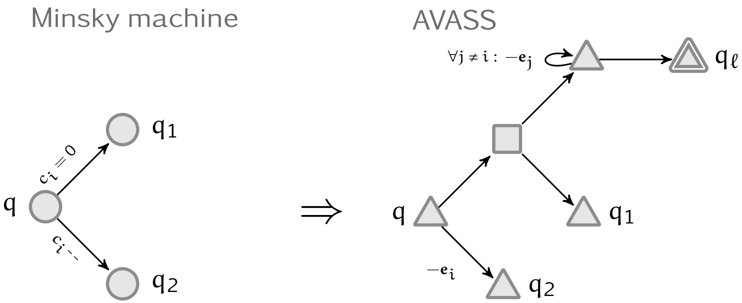
Monotone AVASS games and multi-dimensional energy games are LOGSPACE-equivalent.

Corollary

Monotone AVASS games with given initial credit are 2-EXP-complete.

Configuration Reachability Objective (1/2)

Player 2 can enforce zero-tests using the reachability objective $(q_\ell, \mathbf{0})$:



Theorem (Lincoln et al., APAL '92)

Configuration reachability AVASS games with given initial credit are undecidable.

Configuration Reachability Objective (2/2)

Existential initial credit ≅ gainy game
where $\forall q \in Q. \forall 1 \leqslant i \leqslant d. q \xrightarrow{\mathbf{e}_i} q$

Theorem (Urquhart, JSL '99; Lazić and S., ToCL '15)
Configuration reachability AVASS games with existential initial credit are ACKERMANN-complete.

Model-Checking Resource-Aware Logics

VASS models fragment of the μ-calculus on VASS executions
(Abdulla et al., Concur '13)

resource-bounded concurrent game structures RB±ATL*
(Alechina et al., RP '16)

Both are 2-EXP-complete by reduction to multi-energy parity games / parity AVASS games.

Model-Checking Resource-Aware Logics

VASS models fragment of the μ-calculus on VASS executions
(Abdulla et al., Concur '13)

resource-bounded concurrent game structures RB$\pm$ATL*
(Alechina et al., RP '16)

Both are 2-EXP-complete by reduction to multi-energy parity games / parity AVASS games.

Propositional (Intuitionistic) Linear Logic

$$\frac{}{A \vdash A}(\mathsf{I}) \qquad \frac{\Gamma, !A, !A \vdash B}{\Gamma, !A \vdash B}(\mathsf{C!}) \qquad \frac{\Gamma, A \vdash B}{\Gamma, !A \vdash B}(\mathsf{L!})$$

$$\frac{\Gamma \vdash A \quad \Delta, B \vdash C}{\Gamma, \Delta, A \multimap B \vdash C}(\mathsf{L}_{\multimap}) \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}(\mathsf{R}_{\multimap})$$

$$\frac{\Gamma, A \vdash C}{\Gamma, A \& B \vdash C} \quad \frac{\Gamma, B \vdash C}{\Gamma, A \& B \vdash C}(\mathsf{L}_{\&}) \qquad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \& B}(\mathsf{R}_{\&})$$

$$\frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \oplus B \vdash C}(\mathsf{L}_{\oplus}) \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \oplus B} \quad \frac{\Gamma \vdash B}{\Gamma \vdash A \oplus B}(\mathsf{R}_{\oplus})$$

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C}(\mathsf{L}_{\otimes}) \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}(\mathsf{R}_{\otimes})$$

$$\ldots$$

Propositional (Intuitionistic) Linear Logic

$$\frac{}{A \vdash A}(\mathsf{I}) \qquad \frac{\Gamma, !A, !A \vdash B}{\Gamma, !A \vdash B}(\mathsf{C!}) \qquad \frac{\Gamma, A \vdash B}{\Gamma, !A \vdash B}(\mathsf{L!})$$

$$\frac{\Gamma \vdash A \quad \Delta, B \vdash C}{\Gamma, \Delta, A \multimap B \vdash C}(\mathsf{L}_{\multimap}) \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}(\mathsf{R}_{\multimap})$$

$$\frac{\Gamma, A \vdash C}{\Gamma, A \& B \vdash C} \quad \frac{\Gamma, B \vdash C}{\Gamma, A \& B \vdash C}(\mathsf{L}_{\&}) \qquad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \& B}(\mathsf{R}_{\&})$$

$$\frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \oplus B \vdash C}(\mathsf{L}_{\oplus}) \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \oplus B} \quad \frac{\Gamma \vdash B}{\Gamma \vdash A \oplus B}(\mathsf{R}_{\oplus})$$

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C}(\mathsf{L}_{\otimes}) \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}(\mathsf{R}_{\otimes})$$

$$\ldots$$

Propositional (Intuitionistic) Linear Logic

$$\frac{}{A \vdash A}(\mathsf{I}) \qquad \frac{\Gamma, !A, !A \vdash B}{\Gamma, !A \vdash B}(\mathsf{C!}) \qquad \frac{\Gamma, A \vdash B}{\Gamma, !A \vdash B}(\mathsf{L!})$$

$$\frac{\Gamma \vdash A \quad \Delta, B \vdash C}{\Gamma, \Delta, A \multimap B \vdash C}(\mathsf{L}_{\multimap}) \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}(\mathsf{R}_{\multimap})$$

$$\frac{\Gamma, A \vdash C}{\Gamma, A \& B \vdash C} \quad \frac{\Gamma, B \vdash C}{\Gamma, A \& B \vdash C}(\mathsf{L}_{\&}) \qquad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \& B}(\mathsf{R}_{\&})$$

$$\frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \oplus B \vdash C}(\mathsf{L}_{\oplus}) \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \oplus B} \quad \frac{\Gamma \vdash B}{\Gamma \vdash A \oplus B}(\mathsf{R}_{\oplus})$$

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C}(\mathsf{L}_{\otimes}) \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}(\mathsf{R}_{\otimes})$$

$$\ldots$$

$(!,\oplus)$-Horn Programs (1/3)

connectives $\{\otimes, \multimap, \oplus, !\}$

simple products $W, X, Y, Z ::= p_1 \otimes p_2 \otimes \cdots \otimes p_m$ for atomic p_i's

Horn implications $X \multimap Y$

$\oplus$-Horn implications $X \multimap (Y_1 \oplus \cdots \oplus Y_n)$

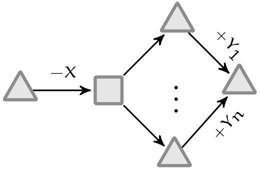
$(!,\oplus)$-Horn sequents $W, !\Gamma \vdash Z$ where Γ contains Horn and $\oplus$-Horn implications

$(!, \oplus)$-Horn Programs (2/3)

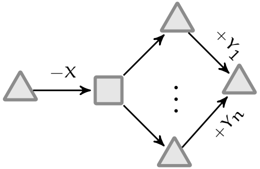
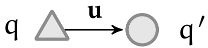
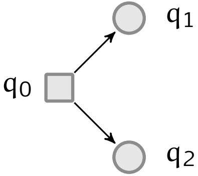
Horn programs | AVASS

$X \multimap Y \quad \Rightarrow$

$X \multimap (Y_1 \oplus \cdots \oplus Y_n) \quad \Rightarrow$



$(!, \oplus)$-Horn Programs (2/3)

Horn programs		AVASS
$X \multimap Y$	$\Rightarrow$	
$X \multimap (Y_1 \oplus \cdots \oplus Y_n)$	$\Rightarrow$	
$q \otimes \mathbf{u}^- \multimap q' \otimes \mathbf{u}^+$	$\Leftarrow$	
$q_0 \multimap (q_1 \oplus q_2)$	$\Leftarrow$	

$(!, \oplus)$-Horn Programs (3/3)

Theorem (Kanovich, APAL '95)

Provability of $(!, \oplus)$-Horn sequents and configuration reachability AVASS games are PSPACE equivalent.

Corollary (Lincoln et al., APAL '92)

Provability in propositional linear logic is undecidable.

Corollary (Courtois and S., MFCS '14; Lazić and S., ToCL '15)

- *Provability of affine $(!, \oplus)$-Horn sequents is 2-EXP-complete.*
- *Provability of contractive $(!, \oplus)$-Horn sequents is ACKERMANN-complete.*

$(!,\oplus)$-Horn Programs (3/3)

Theorem (Kanovich, APAL '95)

Provability of $(!,\oplus)$-Horn sequents and configuration reachability AVASS games are PSPACE equivalent.

Corollary (Lincoln et al., APAL '92)

Provability in propositional linear logic is undecidable.

Corollary (Courtois and S., MFCS '14; Lazić and S., ToCL '15)

- *Provability of affine $(!,\oplus)$-Horn sequents is 2-EXP-complete.*
- *Provability of contractive $(!,\oplus)$-Horn sequents is ACKERMANN-complete.*

Concluding Remarks

- tight 2-EXP bounds for multi-energy parity games
- impacts numerous problems
  - affine $(\oplus,!)$-Horn linear logic
    (Kanovich, APAL '95)
  - (weak) simulation of finite-state systems by Petri nets
    (Abdulla et al., Concur '13)
  - model-checking Petri nets with a fragment of μ-calculus
    (Abdulla et al., Concur '13)
  - resource-bounded agent temporal logic RB$\pm$ATL*
    (Alechina et al., RP '16)
- fine understanding of Player 2's strategies:
  Player 2 can win by announcing in which perfect half space he will escape

Concluding Remarks

- tight 2-EXP bounds for multi-energy parity games
- impacts numerous problems
  - affine $(\oplus,!)$-Horn linear logic
    (Kanovich, APAL '95)
  - (weak) simulation of finite-state systems by Petri nets
    (Abdulla et al., Concur '13)
  - model-checking Petri nets with a fragment of μ-calculus
    (Abdulla et al., Concur '13)
  - resource-bounded agent temporal logic RB$\pm$ATL*
    (Alechina et al., RP '16)
- fine understanding of Player 2's strategies:
  Player 2 can win by announcing in which perfect half space he will escape

Concluding Remarks

- tight 2-EXP bounds for multi-energy parity games
- impacts numerous problems
  - affine $(\oplus,!)$-Horn linear logic
    (Kanovich, APAL '95)
  - (weak) simulation of finite-state systems by Petri nets
    (Abdulla et al., Concur '13)
  - model-checking Petri nets with a fragment of μ-calculus
    (Abdulla et al., Concur '13)
  - resource-bounded agent temporal logic RB$\pm$ATL*
    (Alechina et al., RP '16)
- fine understanding of Player 2's strategies:
  Player 2 can win by announcing in which perfect half space he will escape

References (1/2)

Abdulla, P.A., Mayr, R., Sangnier, A., and Sproston, J., 2013. Solving parity games on integer vectors. In *Concur 2013*, volume 8052 of *LNCS*, pages 106–120. Springer. doi:10.1007/978-3-642-40184-8_9.

Alechina, N., Bulling, N., Demri, S., and Logan, B., 2016. On the complexity of resource-bounded logics. In *RP 2016*, volume 9899 of *LNCS*, pages 36–50. Springer. doi:10.1007/978-3-319-45994-3_3.

Alechina, N., Bulling, N., Logan, B., and Nguyen, H.N., 2017. The virtues of idleness: A decidable fragment of resource agent logic. *Artif. Intell.* doi:10.1016/j.artint.2016.12.005. to appear.

Brázdil, T., Jančar, P., and Kučera, A., 2010. Reachability games on extended vector addition systems with states. In *ICALP 2010*, volume 6199 of *LNCS*, pages 478–489. Springer. doi:10.1007/978-3-642-14162-1_40. arXiv version available from http://arxiv.org/abs/1002.2557.

Chatterjee, K., Randour, M., and Raskin, J.F., 2014. Strategy synthesis for multi-dimensional quantitative objectives. *Acta Inf.*, 51(3–4):129–163. doi:10.1007/s00236-013-0182-6.

Colcombet, T. and Niwiński, D., 2017. Lexicographic energy games. Manuscript.

Colcombet, Th., Jurdziński, M., Lazić, R., and Schmitz, S., 2017. Perfect half-space games. In *LICS 2017*. IEEE. doi:10.1109/LICS.2017.8005105.

Comin, C. and Rizzi, R., 2016. Improved pseudo-polynomial bound for the value problem and optimal strategy synthesis in mean payoff games. *Algorithmica*. doi:10.1007/s00453-016-0123-1. To appear.

Courtois, J. and Schmitz, S., 2014. Alternating vector addition systems with states. In *MFCS 2014*, volume 8634 of *LNCS*, pages 220–231. Springer. doi:10.1007/978-3-662-44522-8_19.

References (2/2)

Jurdziński, M., Sproston, J., and Laroussinie, F., 2008. Model checking probabilistic timed automata with one or two clocks. *Logic. Meth. in Comput. Sci.*, 4(3):12:1–12:38. doi:10.2168/LMCS-4(3:12)2008.

Jurdziński, M., Lazić, R., and Schmitz, S., 2015. Fixed-dimensional energy games are in pseudo-polynomial time. In *ICALP 2015*, volume 9135 of *LNCS*, pages 260–272. Springer. doi:10.1007/978-3-662-47666-6_21. arXiv version available from https://arxiv.org/abs/1502.06875.

Kanovich, M.I., 1995. Petri nets, Horn programs, linear logic and vector games. *Ann. Pure App. Logic*, 75(1–2): 107–135. doi:10.1016/0168-0072(94)00060-G.

Lasota, S., 2009. EXPSPACE lower bounds for the simulation preorder between a communication-free Petri net and a finite-state system. *Information Processing Letters*, 109(15):850–855. doi:10.1016/j.ipl.2009.04.003.

Lazić, R. and Schmitz, S., 2015. Non-elementary complexities for branching VASS, MELL, and extensions. *ACM Transactions on Computational Logic*, 16(3):20:1–20:30. doi:10.1145/2733375.

Lincoln, P., Mitchell, J., Scedrov, A., and Shankar, N., 1992. Decision problems for propositional linear logic. *Ann. Pure App. Logic*, 56(1–3):239–311. doi:10.1016/0168-0072(92)90075-B.

Raskin, J.F., Samuelides, M., and Begin, L.V., 2005. Games for counting abstractions. In *AVoCS 2004*, volume 128(6) of *Elec. Notes in Theor. Comput. Sci.*, pages 69–85. Elsevier. doi:10.1016/j.entcs.2005.04.005.

Urquhart, A., 1999. The complexity of decision procedures in relevance logic II. *J. Symb. Log.*, 64(4):1774–1802. doi:10.2307/2586811.

Velner, Y., Chatterjee, K., Doyen, L., Henzinger, T.A., Rabinovich, A., and Raskin, J.F., 2015. The complexity of multi-mean-payoff and multi-energy games. *Inform. and Comput.*, 241:177–196. doi:10.1016/j.ic.2015.03.001.