# TD 4

## 1 Complexity of LTL Model Checking

**Exercise 1** (Model Checking a Path)**.** We want to verify a model with a single run $w$, which is an ultimately periodic word $uv^\omega$ with $u$ in $\Sigma^*$ and $v$ in $\Sigma^+$.

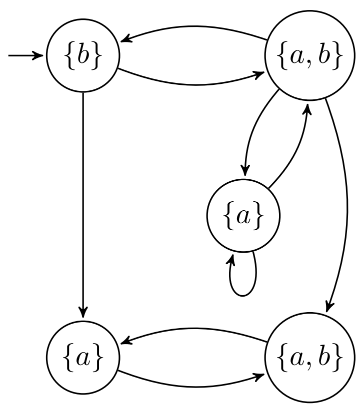
Give an algorithm for checking whether $w, 0 \models \varphi$ holds, where $\varphi$ is a LTL($\mathsf{X}, \mathsf{U}$) formula, in time bounded by $O(|uv| \cdot |\varphi|)$.

## 2 CTL*

**Exercise 2** (Equivalences)**.** Are the following formulæ equivalent?

1. $\mathsf{AXAG}\varphi$ et $\mathsf{AXG}\varphi$
2. $\mathsf{EXEG}\varphi$ et $\mathsf{EXG}\varphi$
3. $\mathsf{A}(\varphi \wedge \psi)$ et $\mathsf{A}\varphi \wedge \mathsf{A}\psi$
4. $\mathsf{E}(\varphi \wedge \psi)$ et $\mathsf{E}\varphi \wedge \mathsf{E}\psi$
5. $\neg\mathsf{A}(\varphi \Rightarrow \psi)$ et $\mathsf{E}(\varphi \wedge \neg\psi)$

**Exercise 3** (Model Checking)**.**




Check whether the above Kripke structure verifies the following CTL* formula:

$$\mathsf{E}(\mathsf{X}(a \wedge \neg b) \wedge \mathsf{XA}(b\,\mathsf{U}(\mathsf{G}a)))$$

# 3 CTL and CTL$^+$

**Exercise 4** (CTL Equivalences)**.**

1. Are the two formulæ $\varphi = \mathsf{AG}(\mathsf{EF}p)$ and $\psi = \mathsf{EF}p$ equivalent? Does one imply the other?

2. Same questions for $\varphi = \mathsf{EG}q \vee (\mathsf{EG}p \wedge \mathsf{EF}q)$ and $\psi = \mathsf{E}(p \,\mathsf{U}\, q)$.

**Exercise 5** (CTL$^+$)**.** CTL$^+$ extends CTL by allowing boolean connectives on path formulæ, according to the following abstract syntax:

$$f ::= \top \mid a \mid f \wedge g \mid \neg f \mid \mathsf{E}\varphi \mid \mathsf{A}\varphi \qquad \text{(state formulæ } f, g)$$
$$\varphi ::= \varphi \wedge \psi \mid \neg\varphi \mid \mathsf{X}f \mid f \,\mathsf{U}\, g \qquad \text{(path formulæ } \varphi, \psi)$$

where $a$ is an atomic proposition. The associated semantics is that of CTL$^*$.

We want to prove that, for any CTL$^+$ formula, there exists an equivalent CTL formula.

1. Give an equivalent CTL formula for
$$\mathsf{E}((a_1 \,\mathsf{U}\, b_1) \wedge (a_2 \,\mathsf{U}\, b_2)) \ .$$

2. Generalize your translation for any formula of form
$$\mathsf{E}(\bigwedge_{i=1,\ldots,n} (\psi_i \,\mathsf{U}\, \psi'_i) \wedge \mathsf{G}\varphi) \ . \qquad (1)$$
What is the complexity of your translation?

3. Give an equivalent CTL formula for the following CTL$^+$ formula:
$$\mathsf{E}(\mathsf{X}a \wedge (b \,\mathsf{U}\, c)) \ .$$

4. Using subformulæ of form (1) and $\mathsf{EX}$ modalities, give an equivalent CTL formula to
$$\mathsf{E}(\mathsf{X}\varphi \wedge \bigwedge_{i=1,\ldots,n} (\psi_i \,\mathsf{U}\, \psi'_i) \wedge \mathsf{G}\varphi') \ . \qquad (2)$$
What is the complexity of your translation?

5. We only have to transform any CTL$^+$ formula into (nested) disjuncts of form (2). Detail this translation for the following formula:
$$\mathsf{A}((\mathsf{F}a \vee \mathsf{X}a \vee \mathsf{X}\neg b \vee \mathsf{F}\neg d) \wedge (d \,\mathsf{U}\, \neg c)) \ .$$

**Exercise 6** (Fair CTL)**.** We consider *strong* fairness constraints, which are conjunctions of formulæ of form

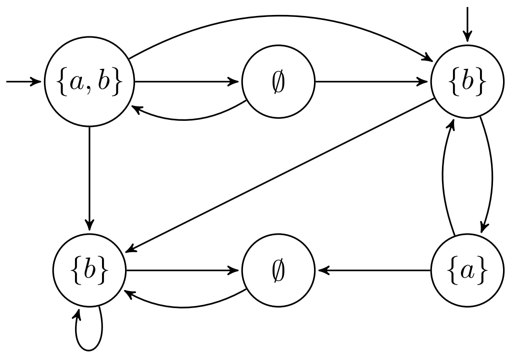
$$\mathsf{GF}\psi_1 \Rightarrow \mathsf{GF}\psi_2 \;.$$

We want to check whether the following Kripke structure fairly verifies

$$\varphi = \mathsf{AGAF}a$$

under the fairness requirement $e$ defined by

$$\begin{aligned} \psi_1 &= b \wedge \neg a \\ \psi_2 &= \mathsf{E}(b \,\mathsf{U}(a \wedge \neg b)) \\ e &= \mathsf{GF}\psi_1 \Rightarrow \mathsf{GF}\psi_2 \;. \end{aligned}$$




1. Compute $[\![\psi_1]\!]$ et $[\![\psi_2]\!]$.
2. Compute $[\![\mathsf{EG}\top]\!]_e$.
3. Compute $[\![\varphi]\!]_e$.