The opetopic nerve of Operads

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SOCS2020
11/12/20

Part 1: Opetopes in a nutshell
This is an opetope of dimension 3

Many inputs/sources, One output/target
arranged in a pasting diagram

Opetopes are many-to-one shapes

This many-to-one requirement is recursively enforced

These 2-dimensional components are also required
Many sources arranged in a pasting diagram

One target

And (somewhat trivially), these 1-dimensional components also have to be opetopes

Many sources

One target

Definition: An n-dimensional opetope is a pasting diagram of a pasting diagram of a pasting diagram of...
n \{ 
  \begin{align*}
  \text{pasting diagram of} \\
  \text{pasting diagram of} \\
  \text{pasting diagram of} \\
  \text{pasting diagram of} \\
  \text{points}
  \end{align*}
\}

\text{written } \rightarrow

\text{"make pasting diagrams"}

\rightarrow

\text{written } \rightarrow

\begin{align*}
\text{make pasting diagrams}
\end{align*}

\text{(dimension 0)} \quad \text{(dimension 1)}

\begin{align*}
\text{(dimension 2)}
\end{align*}

\begin{align*}
3 & \quad 2 & \quad 0 & \quad 1 & \quad 5 \\
\end{align*}

\begin{align*}
\text{make pasting diagrams}
\end{align*}

\text{(dimension 2)}

\begin{align*}
\text{(dimension 3)}
\end{align*}

\begin{align*}
\Rightarrow \\
\Rightarrow \\
\Rightarrow
\end{align*}
The complexity rapidly explodes, but the process stays the same:

« make more pasting diagrams! »

The term "pasting diagram" can be made precise by considering opetopes as trees.
So really, an $n$-opetope is a tree of tree of tree of ... tree of points

Opetopes naturally assemble into a category $O$

1 - Objects: opetopes
2 - Generating morphisms:
   - source embeddings
3 - Relations: opetopic identities: simple relations that carry the geometrical intuition behind opetopes
Properties

- $\mathcal{C}$ is a Reedy category

- It’s a directed Reedy category

- Presheaves over $\mathcal{C}$ are opetopic sets
Now to part 2 with Chaitanya