# The opetopic nerve of Operads

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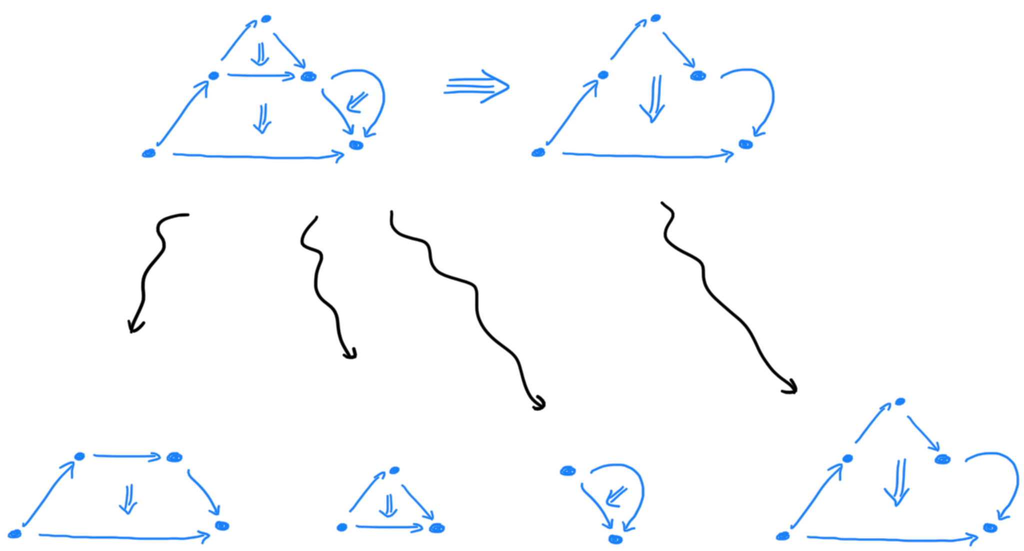
## Part 1: Opetopes in a nutshell

This is an opetope of dimension 3

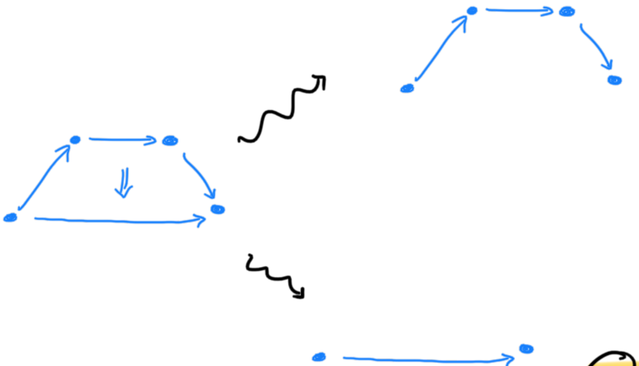
Many inputs / sources, One output / target
arranged in a pasting diagram

Opetopes are many-to-one shapes

This many-to-one requirement is recursively enforced



These 2-dimensional components are also required to be opetopes!



Many sources arranged in a pasting diagram

One target

And (somewhat trivially), these 1-dimensional components also have to be opetopes

Many sources

One target

Definition An n-dimensional opetope is a

pasting diagram of
pasting diagram of
pasting diagram of

n {
pasting diagram of
⋮
pasting diagram of
pasting diagram of
pasting diagram of
pasting diagram of
}

points

(dimension 0)

written

" make pasting diagrams "

(dimension 1)

written

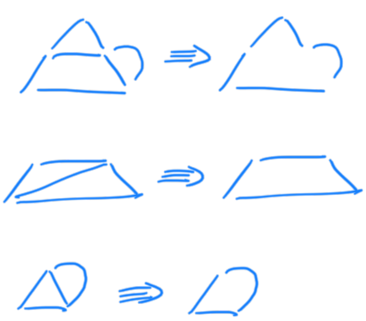
make pasting diagrams

(dimension 2)

3 2 0 1 5

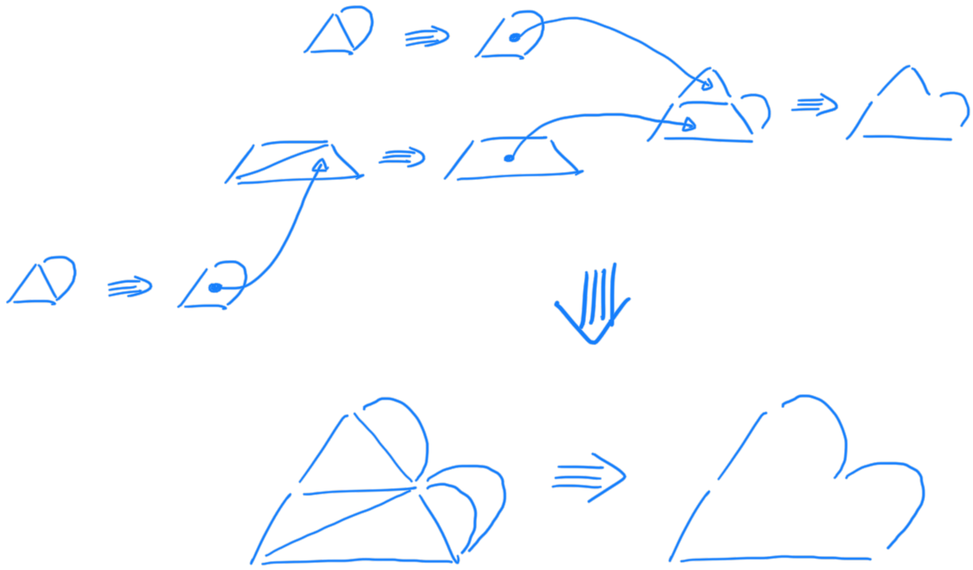
make pasting diagrams

(dimension 3)



make pasting diagrams

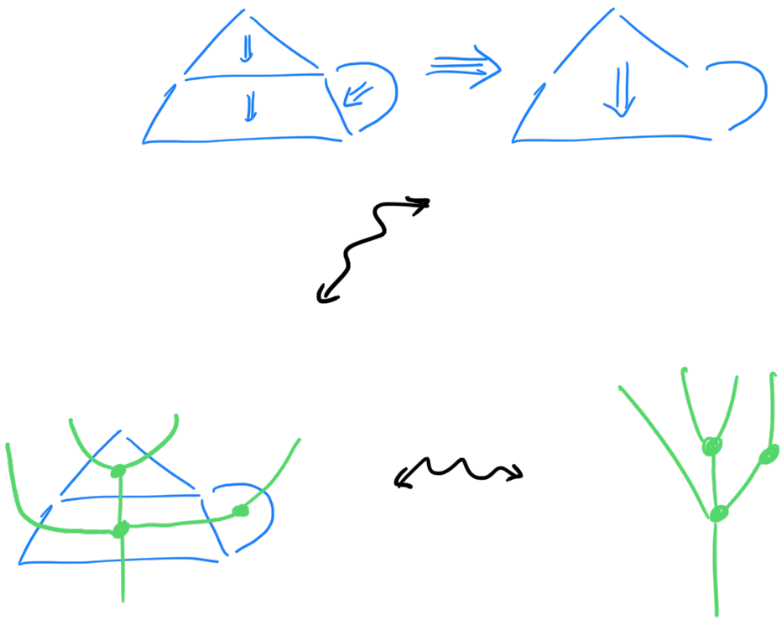
(dimension 4)



The complexity rapidly explodes, but the process stays the same:

« make more pasting diagrams! »

The term "pasting diagram" can be made precise by considering opetopes as **trees**



So really, an n-opetope is a

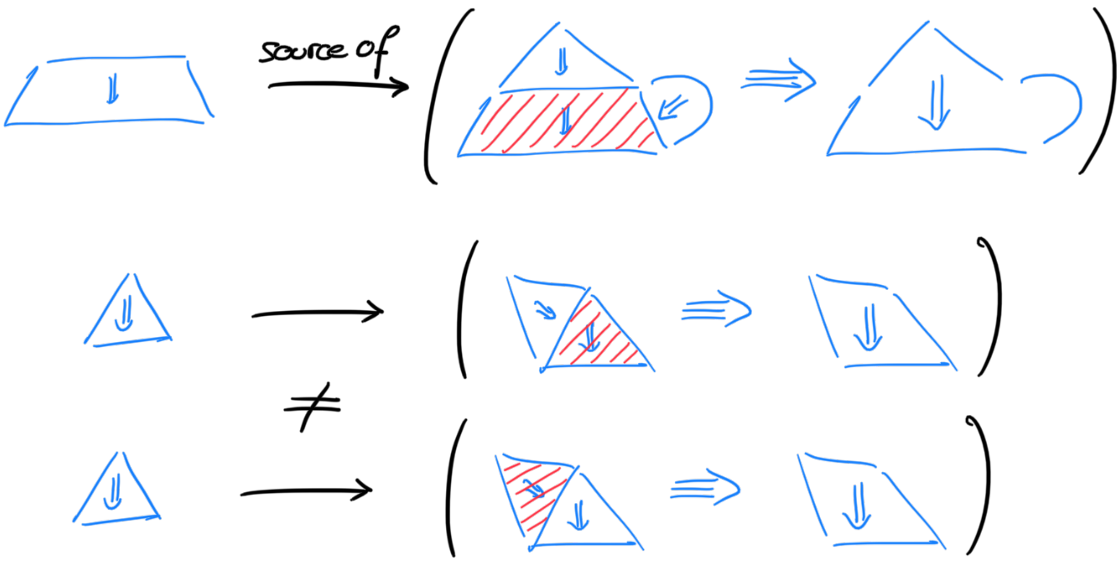
$\underbrace{\text{tree of tree of tree of} \dots \text{tree of}}_{n}$ points

Opetopes naturally assemble into a category $\mathbb{O}$

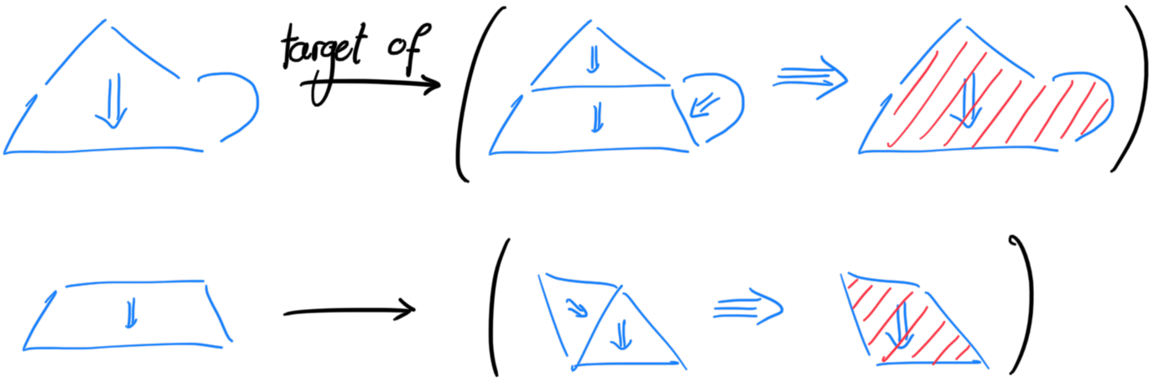
1 – Objects : opetopes

2 – Generating morphisms :

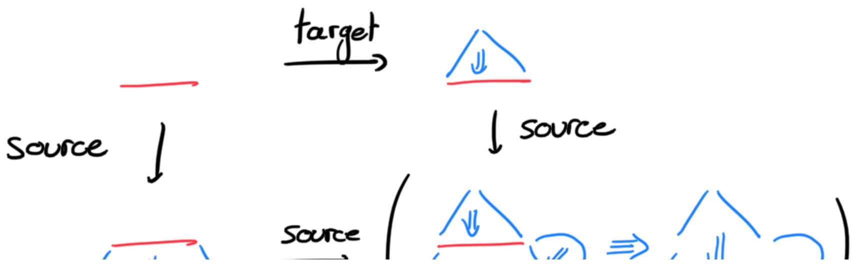
- source embeddings



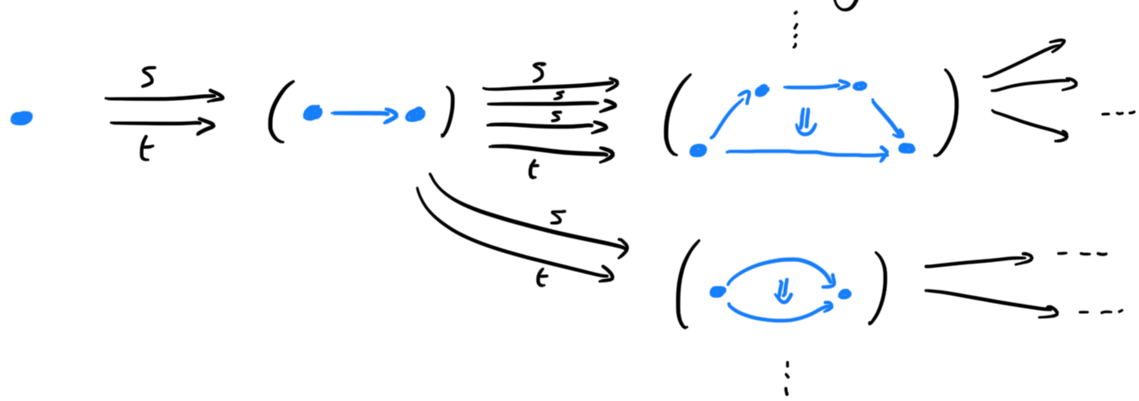

- *target embeddings*




3 – Relations : opetopic identities : simple relations that carry the geometrical intuition behind opetopes



<u>Properties</u> – $\mathbb{O}$ is a Reedy category




- It's a <u>directed</u> Reedy category
- Presheaves over $\mathbb{O}$ are ==opetopic sets==

Now to part 2 with Chaitanya