Around a problem of Mahler and Mendès France

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One motivation for this talk comes from a number theoretical problem concerning the expansion of algebraic numbers in integer bases. It appears at the end of a paper of Mendès France, but in conversation he attributes the paternity of this problem to Mahler. The problem reads as follows: for any non-eventually periodic binary sequence $(a_n)_{\geq 0}$, prove that at least one of the two real numbers

$$\alpha = \sum_{n\geq 0} \frac{a_n}{2^n} \quad \text{and} \quad \beta = \sum_{n\geq 0} \frac{a_n}{3^n}$$

is transcendental. At first glance, this problem seems contrived, but behind it hides the more fundamental question of the structure of representations of real numbers in two mutliplicatively independent integer bases. Unfortunately, problems of this type are difficult and, up to now, it seems that no progress has been achieved towards this question.

In this talk, I will survey some results related to the positive characteristic and to the p-adic counterparts of the problem of Mahler and Mendès France. Some of these results are obtained in joint works with Y. Bugeaud and J. Bell.