Journées d'Informatique Fondamentale de Paris Diderot

April 25, 2013

"Updating Automata Networks"

or "Time & Interaction Systems"

Mathilde Noual

Contents

The (ultimate) object of study

Interaction systems

The (ultimate) object of study

Interaction systems

INFORMAL DEFINITION :

An *interaction system* is any system that can be defined by a set of interacting entities such that all events that are possible in this system

- are caused by interactions between these entities,
- correspond to changes of states of these entities.

The aim.

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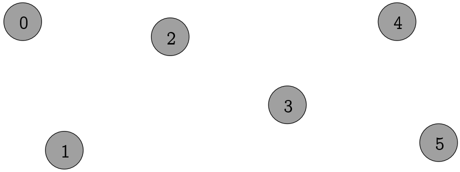
INFORMAL DEFINITION :

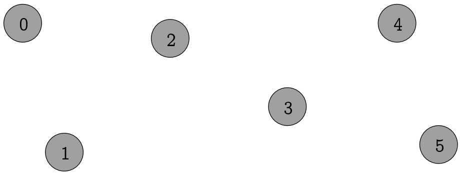
Uncover & understand common, fundamental mechanisms, involved in the processing of information in interaction systems, that can explain properties of their behaviours.

The prototype

(Boolean automata networks)

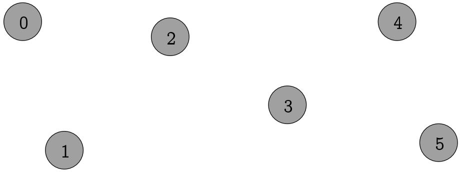


A set V of automata



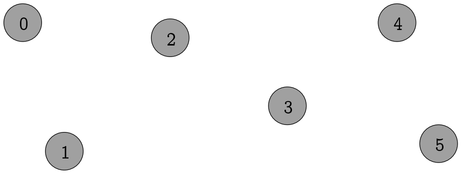
A set V of automata

with variable states x_i, $\forall i \in V$



A set V of automata

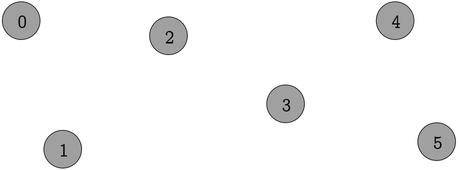
with Boolean states $x_i \in \mathbb{B},\ \forall i \in V$



A set V of automata

with Boolean states $x_i \in \mathbb{B}$, $\forall i \in V$

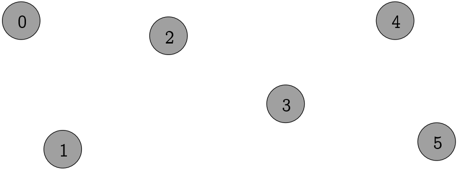
⇒ to concentrate on state changes x_i



A set V of automata

with Boolean states $x_i \in \mathbb{B}, \ \forall i \in V$

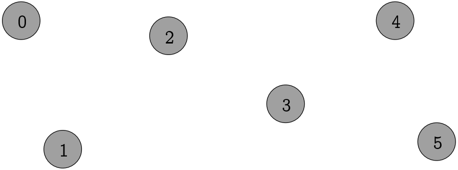
$\Rightarrow$ **to concentrate on state changes** x_i ➥



A set V of automata

with Boolean states $x_i \in \mathbb{B}, \ \forall i \in V$

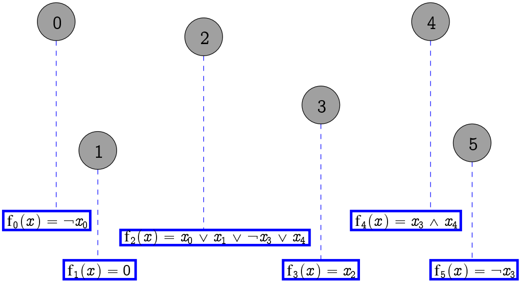
$\Rightarrow$ **to concentrate on state changes** $x_i \hookrightarrow \neg x_i$



state changes $x_i \Rightarrow \neg x_i$

caused by influences of (other) automata on i

Defining the prototype ...



A $\begin{cases} \text{mechanism} \\ \text{local transition function} \\ \text{behaviour rule} \end{cases}$ $\quad f_i : \mathbb{B}^n \mapsto \mathbb{B} \quad \forall i \in V$

⚠ f_i assumed to be locally monotone

In configuration $x \in \mathbb{B}^n$,

i **is updated** $\equiv$ i **obeys to its influences** $\equiv$ $x_i \leftarrow f_i(x)$

In configuration $x \in \mathbb{B}^n$,

$$i \textbf{ changes states} \equiv \boxed{x_i \Rightarrow \mathrm{f}_i(x) = \neg x_i}$$

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$$\equiv \quad i \text{ is updated } \underline{\text{and}}\ i \in \mathcal{U}(x) = \{\, i \in V,\ x_i \neq f_i(x) \,\} \text{ is unstable}$$

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$\equiv$ i **is updated** **and** $i \in \mathcal{U}(\mathbf{x}) = \{\, \mathbf{i} \in \mathbf{V},\ \mathbf{x_i} \neq \mathbf{f_i}(\mathbf{x}) \,\}$ **is unstable**

DEFINITION :

A **transition** is a couple of configurations $(x, y \in \mathbb{B}^n)$ $x \longrightarrow y$ such that $x_i \neq y_i \implies y_i = \mathrm{f}_i(x)$ (and $i \in \mathcal{U}(x)$).

In configuration $x \in \mathbb{B}^n$,

$$i \text{ changes states } \equiv \quad x_i \rightarrow f_i(x) = \neg x_i$$

$$\equiv \quad i \text{ is updated } \underline{\text{and}} \ i \in \mathcal{U}(\mathbf{x}) = \{\, \mathbf{i} \in \mathbf{V},\ \mathbf{x_i} \neq \mathbf{f_i}(\mathbf{x}) \,\} \text{ is unstable}$$

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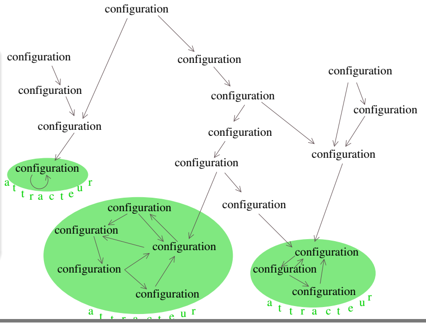
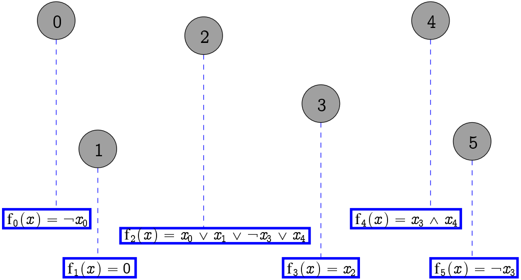
DEFINITION :

A **transition graph** is any digraph of the form

$$\mathcal{T} = (\mathbb{B}^n, T)$$

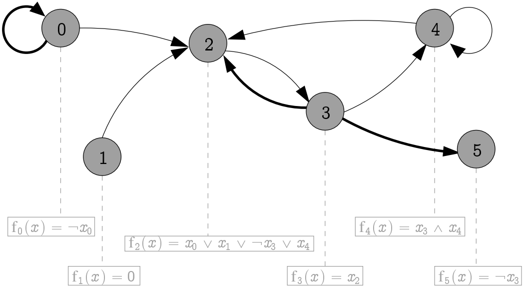
where $T \subset \{x \longrightarrow y\}$.





A network $\mathcal{N} = \{f_i : \mathbb{B}^n \mapsto \mathbb{B} \mid \ i \in V\}$

defined by the set of these functions/mechanisms



$\leadsto$ The $\begin{cases} \text{interaction graph} \\ \text{structure} \end{cases}$ $G = (V, A)$ of the network $\mathcal{N}$

2 questions :

- In what way does the *structure* determine the network behaviour ?
- In what way does the specific *updating* determine the network behaviour ?

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  Influence of the structure under a simple update schedule

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  Influence of the update schedule on simple structures

- In what way does the *structure* determine the network behaviour ?

  Influence of the structure under a simple update schedule

The parallel update schedule

DEFINITION:

The parallel update schedule induces transitions :

$$x \longrightarrow (\mathrm{f}_0(x), \mathrm{f}_1(x), \ldots, \mathrm{f}_{n-1}(x)).$$

i.e. it updates *all* (unstable) automata in each configuration.

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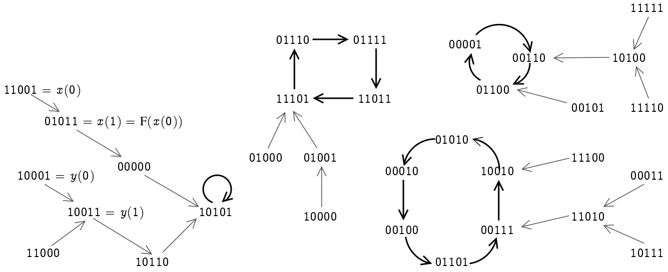
- ⇝ It exploits all local instabilities systematically
- ⇝ It allows to observe the effect of all automata influences as soon as they are effective
- ⇝ It yields the most transparency w.r.t. the network structure


11001 = x(0)
01011 = x(1) = F(x(0))
00000
10001 = y(0)
10011 = y(1)
10101
11000
10110
01110
01111
11101
11011
01000
01001
10000
00001
00110
10100
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00011
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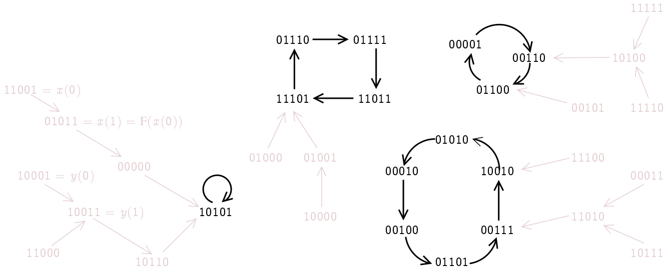
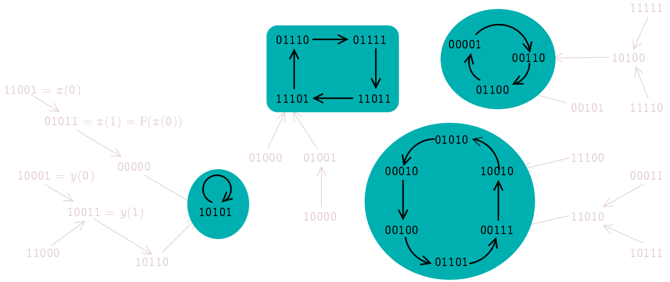
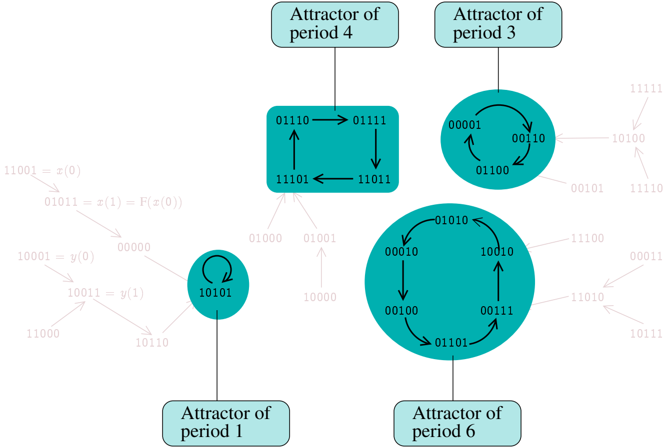
01101
10111



Recurrent/periodic configuration


01110
01111
11101
11011
00001
00110
01100
11111
10100
00101
11110
11001 = $x(0)$
01011 = $x(1) = F(x(0))$
00000
10001 = $y(0)$
10011 = $y(1)$
11000
10110
10101
01000
01001
10000
01010
00010
10010
00100
00111
01101
11100
00011
11010
10111

Attractors


Attractor of period 4
Attractor of period 3
01110
01111
11101
11011
00001
00110
01100
11111
10100
00101
11110
11001 = $x(0)$
01011 = $x(1) = F(x(0))$
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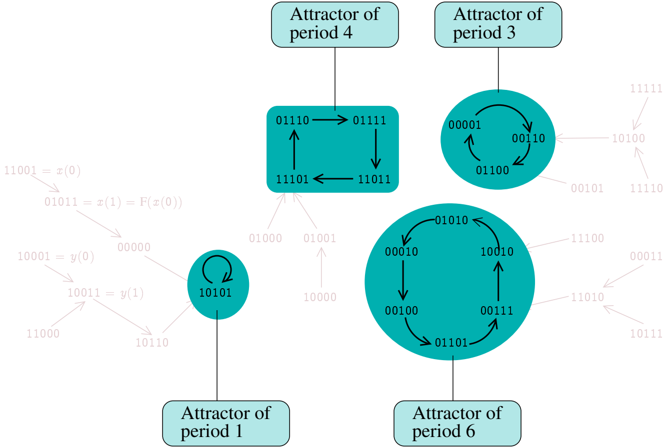
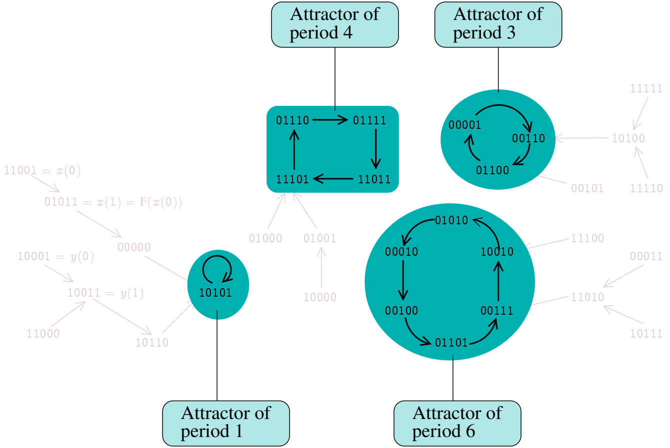
10000
01010
00010
10010
00100
00111
01101
11100
00011
11010
10111
Attractor of period 1
Attractor of period 6

Attractors



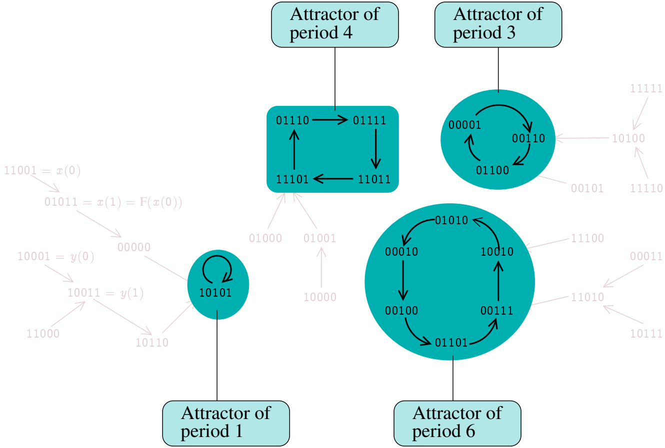
Order

Global instability



Order = lcm of periods

Global instability



Order = lcm of periods

Global instability = number of attractors, T

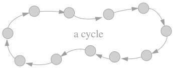
Under the parallel update schedule and for disjunctive networks . . .

PROPOSITION:

Stable configurations depend only on the reduced digraph of the structure.

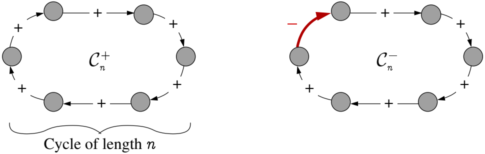
PROPOSITION:

The order of a strongly connected disjunctive network equals the gcd of its cycle lengths.

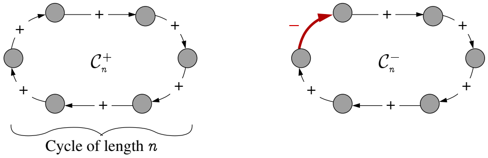


➠ "Global instability is determined outside of non-trivial SCCs & the order is determined inside non-trivial SCCs".

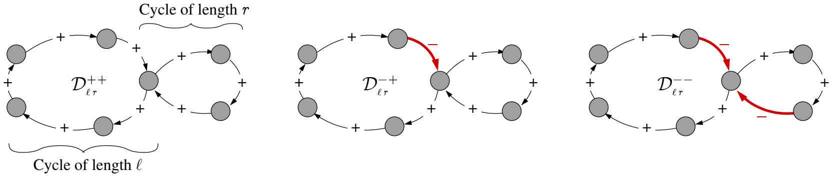
Cycles & cycle interactions



Isolated cycles



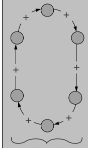
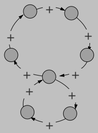
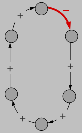
Isolated cycles



Double-cycles

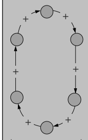
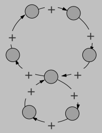
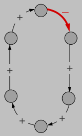
Under the parallel update schedule . . .

Theorem

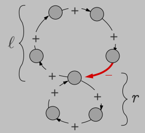
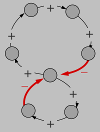
Network $\mathcal{N}$	$\mathcal{C}_n^+$  circuit de taille n	$\mathcal{D}_{\ell\, r}^{++}$  $\text{pgcd}(\ell, r) = n$	$\mathcal{C}_n^-$ 
Order $\omega \in \mathbb{N}$ of $\mathcal{N}$	n		$2n$
Number of configurations of period $p \mid \omega$	2^p		$\neg(p \mid n) \cdot 2^{\frac{p}{2}}$
Number of attractors of period $p \mid \omega$	$\frac{1}{p} \sum_{d \mid p} \mu\left(\frac{p}{d}\right) 2^d$		$\frac{1}{p} \sum_{k \text{ impair } \mid p} \mu(k)\, 2^{\frac{p}{2k}}$
Total number of attractors	$\frac{1}{n} \sum_{d \mid n} \varphi\left(\frac{n}{d}\right) 2^d$		$\frac{1}{2n} \sum_{k \text{ impair } \mid 2n} \varphi(k)\, 2^{\frac{n}{2k}}$

Under the parallel update schedule ...

Theorem

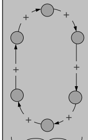
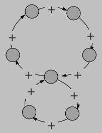
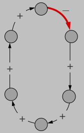
Network $\mathcal{N}$	$\mathcal{C}_n^{+}$  circuit de taille n	$\mathcal{D}_{\ell\,r}^{++}$  $\text{pgcd}(\ell, r) = n$	$\mathcal{C}_n^{-}$ 
Order $\omega \in \mathbb{N}$ of $\mathcal{N}$	n		$2n$
Number of configurations of period $p \mid \omega$	2^p		$\neg(p \mid n) \cdot 2^{\frac{p}{2}}$
Number of attractors of period $p \mid \omega$	$\frac{1}{p}\sum_{d \mid p} \mu\left(\frac{p}{d}\right) 2^d$		$\frac{1}{p}\sum_{k \text{ impair } \mid p} \mu(k)\, 2^{\frac{p}{2k}}$
Total number of attractors	$\frac{1}{n}\sum_{d \mid n} \varphi\left(\frac{n}{d}\right) 2^d$		$\frac{1}{2n}\sum_{k \text{ impair } \mid 2n} \varphi(k)\, 2^{\frac{n}{2k}}$

Theorem

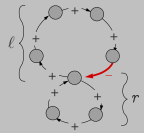
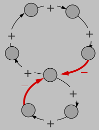
$\mathcal{D}_{\ell\,r}^{-+}$ 	$\mathcal{D}_{\ell\,r}^{--}$ 
r (except exceptions)	$\ell + r$
$\neg(p \mid \ell) \cdot \mathrm{L}\left(\frac{p}{n_p}\right)^{n_p}$	$\neg(p \mid n) \cdot \mathrm{P}\left(\frac{p}{n_p}\right)^{n_p}$
$\frac{1}{p}\sum_{d \mid p, \neg(d \mid \ell)} \mu\left(\frac{p}{d}\right) \mathrm{L}\left(\frac{d}{n_d}\right)^{n_d}$	$\frac{1}{p}\sum_{d \mid p, \neg(d \mid n)} \mu\left(\frac{p}{d}\right) \mathrm{P}\left(\frac{d}{n_d}\right)^{n_d}$
$\frac{1}{r}\sum_{d \mid r, \neg(d \mid \ell)} \varphi\left(\frac{r}{d}\right) \mathrm{L}\left(\frac{d}{n_d}\right)^{n_d}$	$\frac{1}{\ell+r}\sum_{d \mid \ell+r, \neg(d \mid n)} \varphi\left(\frac{\ell+r}{d}\right) \mathrm{P}\left(\frac{d}{n_d}\right)^{n_d}$

Under the parallel update schedule . . .

Theorem

Network $\mathcal{N}$	$\mathcal{C}_n^{+}$	$\mathcal{D}_{\ell\, r}^{++}$	$\mathcal{C}_n^{-}$
	 circuit de taille n	 $\mathrm{pgcd}(\ell, r) = n$	
Order $\omega \in \mathbb{N}$ of $\mathcal{N}$	n		$2n$
Total number of attractors	$T^{+} = T^{++}$		T^{-}

Theorem

$\mathcal{D}_{\ell\, r}^{-+}$	$\mathcal{D}_{\ell\, r}^{--}$
	
r (except exceptions)	$\ell + r$
T^{-+}	T^{--}

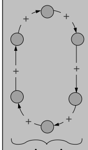
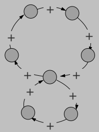
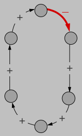
CONJECTURE:

$$T \leqslant 2A_{\omega}$$

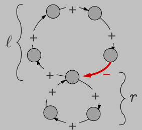
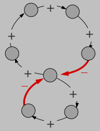
$\Rightarrow$ The mean attractor period is no smaller than $\frac{\omega}{2}$.

Under the parallel update schedule . . .

Theorem

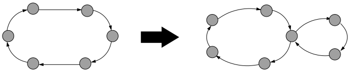
Network $\mathcal{N}$	$\mathcal{C}_n^+$	$\mathcal{D}_{\ell\,r}^{++}$	$\mathcal{C}_n^-$
	 circuit de taille n	 pgcd$(\ell, r) = n$	
Order $\omega \in \mathbb{N}$ of $\mathcal{N}$	n		$2n$
Total number of attractors	$\mathrm{T}^+ = \mathrm{T}^{++}$		$\mathrm{T}^- \leqslant \frac{\mathrm{T}^+}{2^{\omega/2}-1}$

Theorem

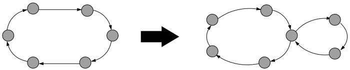
$\mathcal{D}_{\ell\,r}^{-+}$	$\mathcal{D}_{\ell\,r}^{--}$
	
r	$\ell + r$
(except exceptions)	
$\mathrm{T}^{-+} \leqslant \frac{\sqrt{3}^\omega}{2^\omega-1}\mathrm{T}^+$	$\mathrm{T}^{--} \leqslant \frac{3^{\frac{\omega}{3}}}{2^\omega-1}\mathrm{T}^+$

CONSEQUENCE:
Comparing total number of attractors of cycles & double-cycles of same order $\omega \in \mathbb{N}$:

The effect of cycle intersections

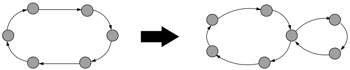


The effect of cycle intersections



THE CONJECTURE

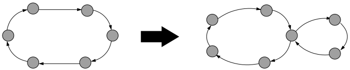
The effect of cycle intersections



The conjecture

↗↗ *Cycle intersections*

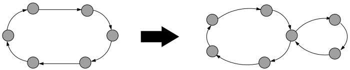
The effect of cycle intersections



The conjecture

↗↗ *Cycle intersections* ⇒ ↘↘ *Global instability*

The effect of cycle intersections

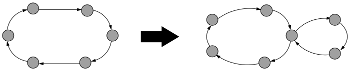


THE CONJECTURE

↗↗ *Cycle intersections* ⇒ ↘↘ *Global instability*

⇒ ↘↘ *Local instabilities*

The effect of cycle intersections



The conjecture

↗↗ *Cycle intersections* ⇒ ↘↘ *Global instability*

⇒ ↘↘ *Local instabilities*

⇒ ↘↘ *Sensitivity to time*

2 questions :

- In what way does the *structure* determine the network behaviour ?
- In what way does the specific *updating* determine the network behaviour ?

Transition

Transition :

(Complete transition graphs of Cycles)

DEFINITION:

A synchronous transition corresponds to the occurence of several possible changes.

DEFINITION:

A transition of the parallel update schedule corresponds to the occurence of all possible changes.

DEFINITION:

The complete transition graph of a network is :

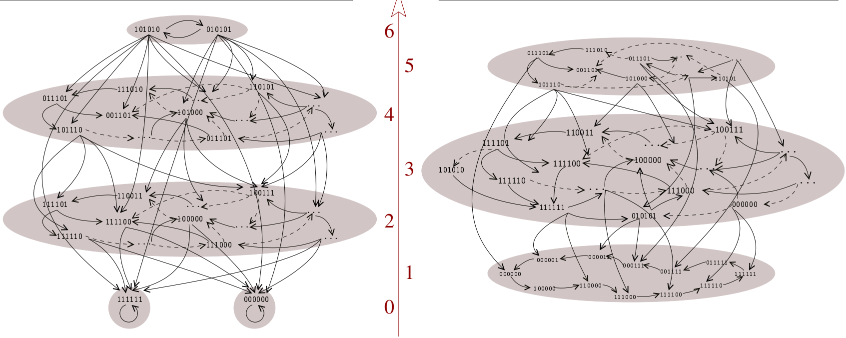
$$(\,\mathbb{B}^n\,,\,\{x \longrightarrow y\}\,)$$

(containing *all* possible transitions).

Complete transition graph of a positive cycle

number of instabilities

Complete transition graph of a negative cycle

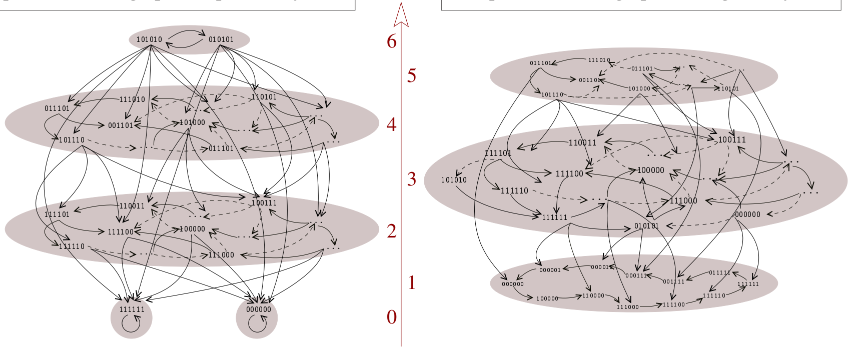


PROPOSITION:

The complete transition graph of a cycle is a layered digraph.

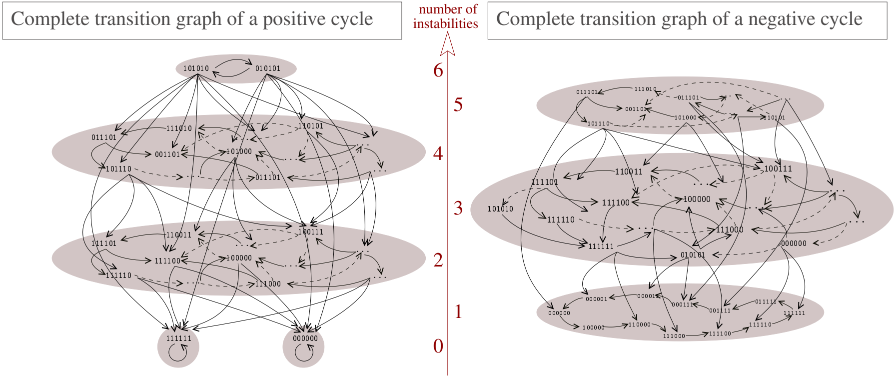
Complete transition graph of a positive cycle

Complete transition graph of a negative cycle



PROPOSITION:

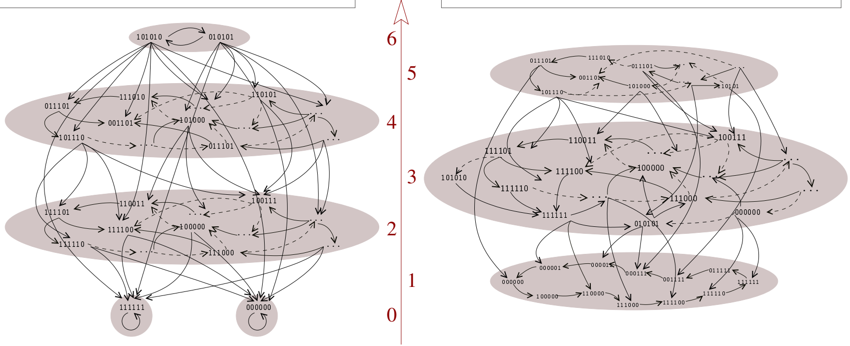
The complete transition graph of a cycle is a layered digraph. **Layer $\mathcal{L}_k$ is a SCC induced by the set of configurations x s.t. $|\mathcal{U}(x)| = k$ (except for layer $\mathcal{L}_0$).**



PROPOSITION:

The complete transition graph of a cycle is a layered digraph. Layer $\mathcal{L}_k$ is a SCC induced by the set of configurations x s.t. $|\mathcal{U}(x)| = k$ (except for layer $\mathcal{L}_k$). **No transition increases $|\mathcal{U}(x)|$.**

Complete transition graph of a positive cycle | Complete transition graph of a negative cycle

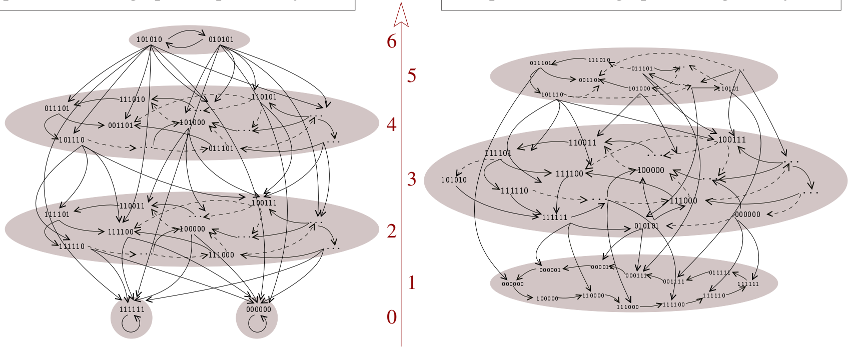


PROPOSITION:

The complete transition graph of a cycle is a layered digraph. Layer $\mathcal{L}_k$ is a SCC induced by the set of configurations x s.t. $|\mathcal{U}(x)| = k$ (except for layer $\mathcal{L}_k$). No transition increases $|\mathcal{U}(x)|$. **To decrease $|\mathcal{U}(x)|$, an effective transition must update only a strict subset of the set $\mathcal{U}(x)$ of unstable automata.**

Complete transition graph of a positive cycle

Complete transition graph of a negative cycle



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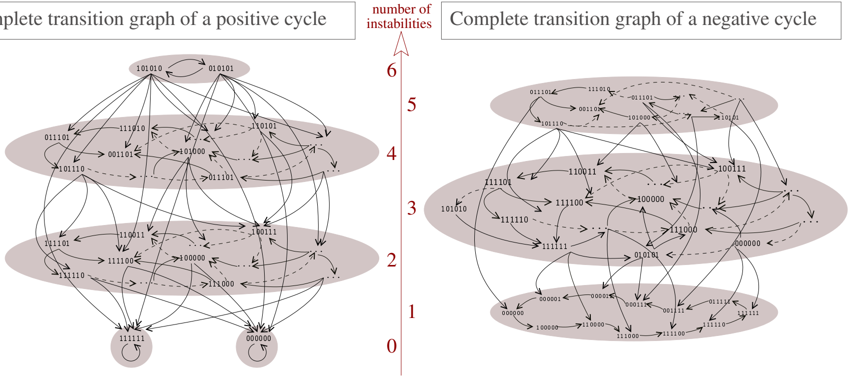
The complete transition graph of a cycle is a layered digraph. Layer $\mathcal{L}_k$ is a SCC induced by the set of configurations x s.t. $|\mathcal{U}(x)| = k$ (except for layer $\mathcal{L}_k$). No transition increases $|\mathcal{U}(x)|$. To decrease $|\mathcal{U}(x)|$, an effective transition must update only a strict subset of the set $\mathcal{U}(x)$ of unstable automata.

Complete transition graph of a positive cycle

Complete transition graph of a negative cycle



QUESTION:

On cycles, sysnchronism tends to maintain local instabilities, asynchronism tends to filter them out and stabilise the overall system. How true is this in general ?

The problem

The problem:

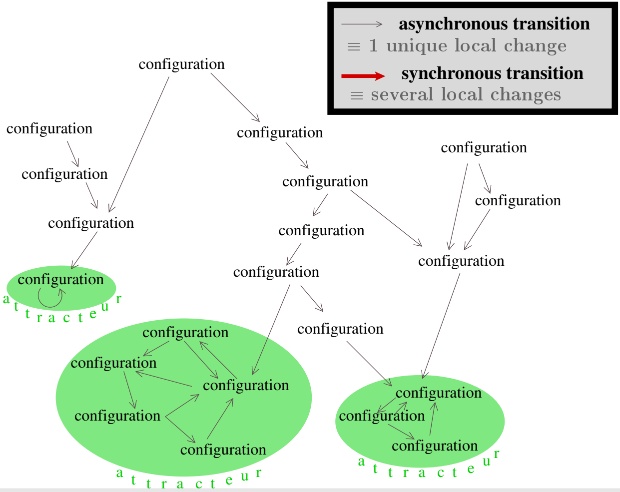
Synchronism *vs* Asynchronism

DEFINITION:

The asynchronous transition graph of a network is :

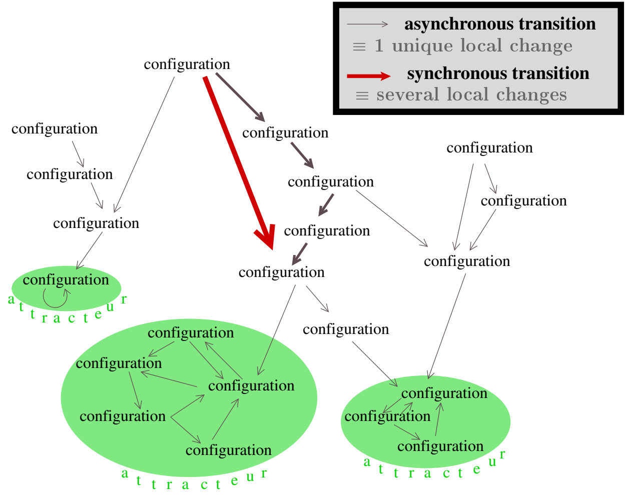
$$(\,\mathbb{B}^n\,,\,\{x \longrightarrow y,\, \mathrm{d}(x\,,y) = 1\}\,)$$

(containing *all* asynchronous transitions transitions corresponding to the update of one unique automaton).

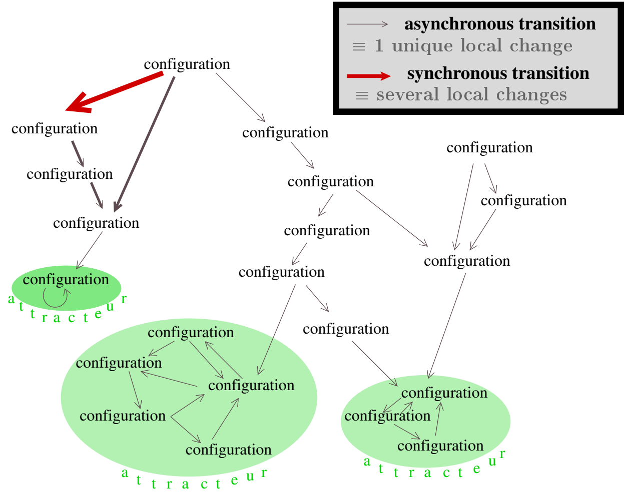


QUESTION:

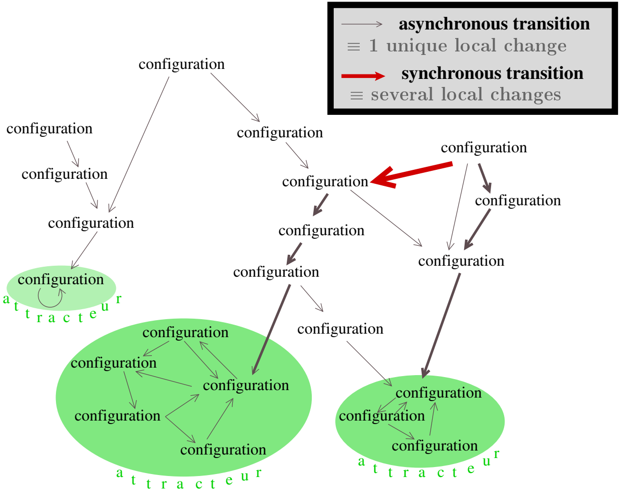
What impact can the addition of a sycnhronous transition have on the possibilities of evolution and asymptotic behaviours of the network ?



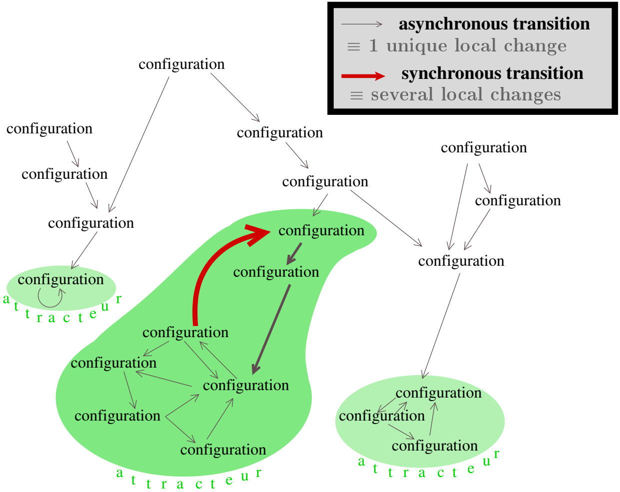
Shortcutting of Asynchronous Trajectories



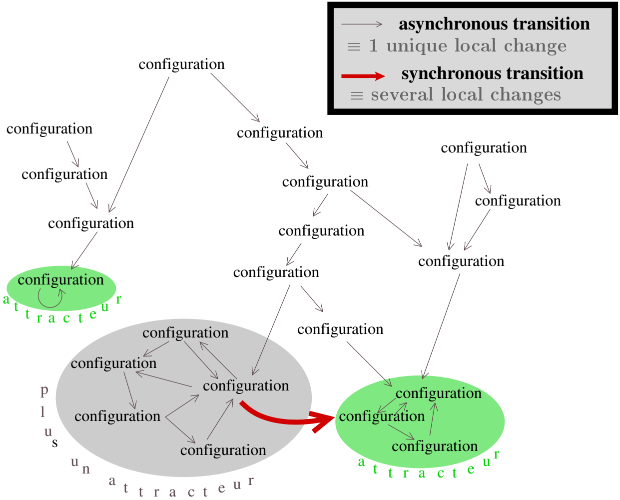
Deviation of Asynchronous Trajectories



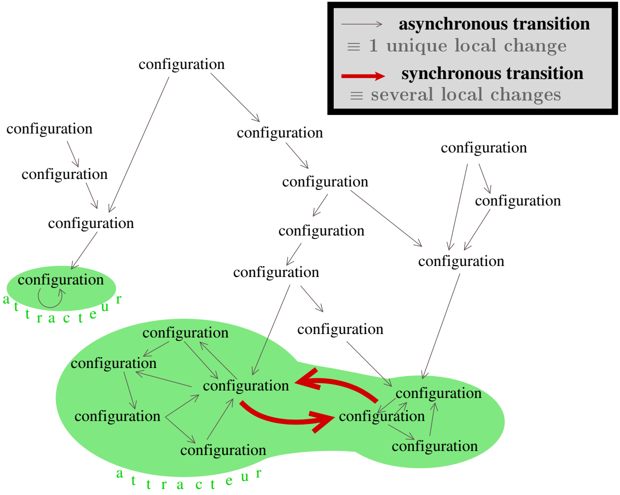
Increase in the Degrees of Freedom of Transient Configurations



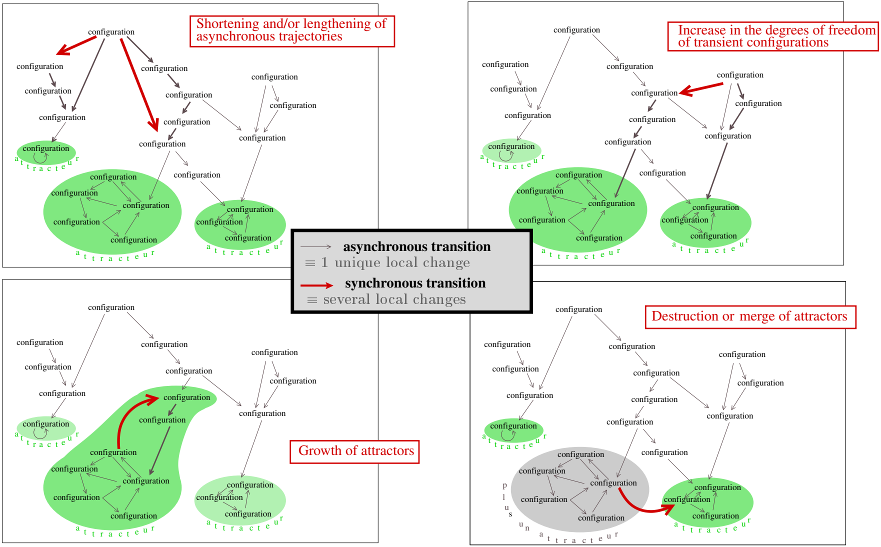
Growth of Attractors



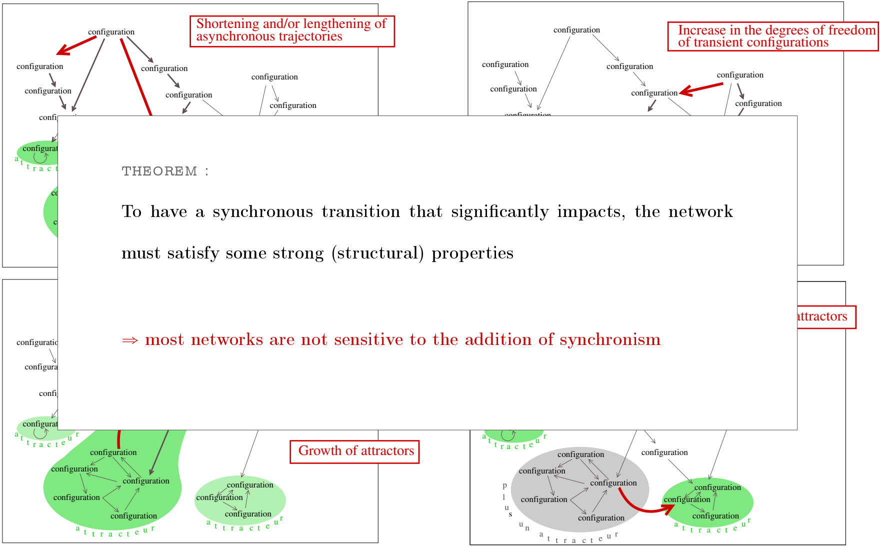
Destruction of Attractors



Merge of Attractors



This list of effects is exhaustive.

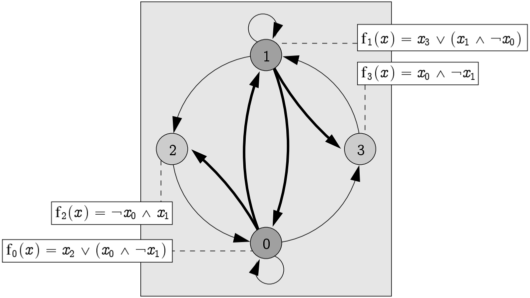


THEOREM :

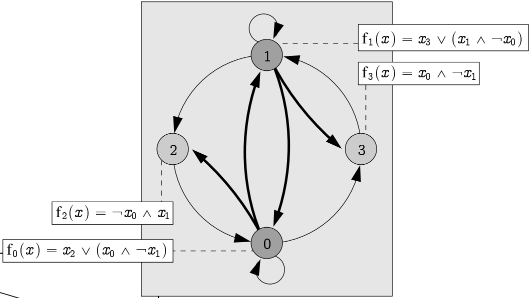
To have a synchronous transition that significantly impacts, the network must satisfy some strong (structural) properties

⇒ most networks are not sensitive to the addition of synchronism

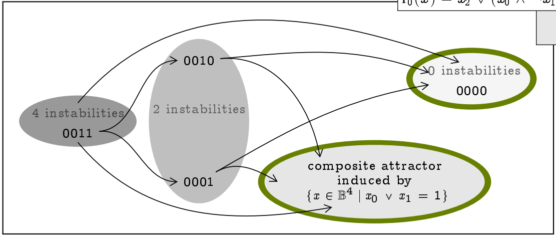
This list of effects is exhaustive.



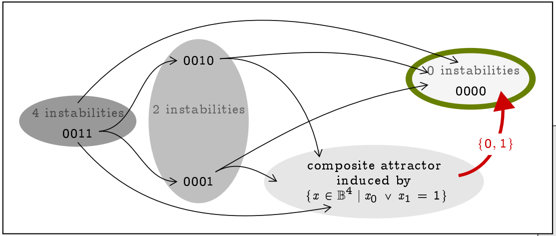
1
2
3
0
$f_1(x) = x_3 \vee (x_1 \wedge \neg x_0)$
$f_3(x) = x_0 \wedge \neg x_1$
$f_2(x) = \neg x_0 \wedge x_1$
$f_0(x) = x_2 \vee (x_0 \wedge \neg x_1)$



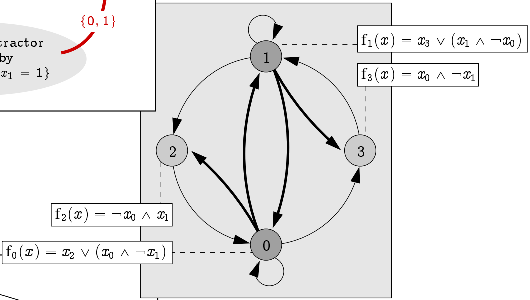
⇧ Network



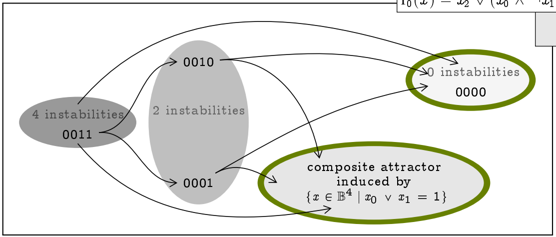
⇦ Reduced asynchronous TG



⇦ Reduced complete TG



⇧ Network



⇦ Reduced asynchronous TG

2 questions :

- In what way does the *structure* determine the network behaviour ?
- In what way does the specific *updating* determine the network behaviour ?

Outline

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OBSERVATION :
The relative organisation of local events in time impacts.

≠ asynchronism / synchronism ?

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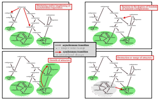
The relative organisation of local events in time impacts.

≠ asynchronism / synchronism ?

all asynchronous ⊕ some synchronism



OBSERVATION :
The relative organisation of local events in time impacts.

all synchronous
(*i. e.* parallel) $\oplus$
some asynchronism

$\neq$ asynchronism / synchronism ?

all asynchronous $\oplus$
some synchronism

OBSERVATION :
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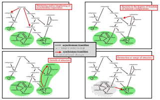

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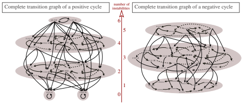
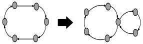
effect of a punctual addition of asynchronism





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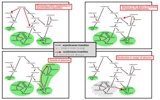

OBSERVATION :
The relative organisation of local events in time impacts.

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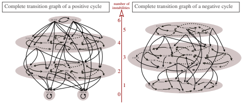
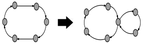
effect of a particular sequentialisation

effect of a punctual addition of asynchronism





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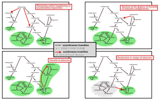

OBSERVATION :
The relative organisation of local events in time impacts.

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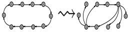
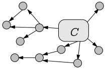
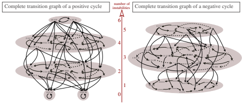
effect of a particular sequentialisation

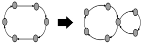
without redundance





effect of a punctual addition of asynchronism





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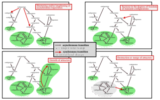

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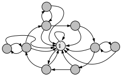
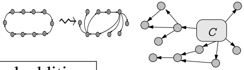
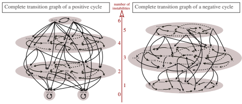
with redundance

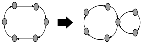


without redundance



effect of a punctual addition of asynchronism





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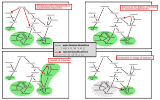

the end

(backup slides)

OBSERVATION :
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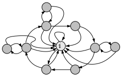
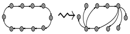
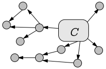
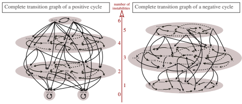


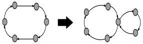
without redundance





effect of a punctual addition of asynchronism





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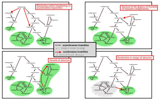
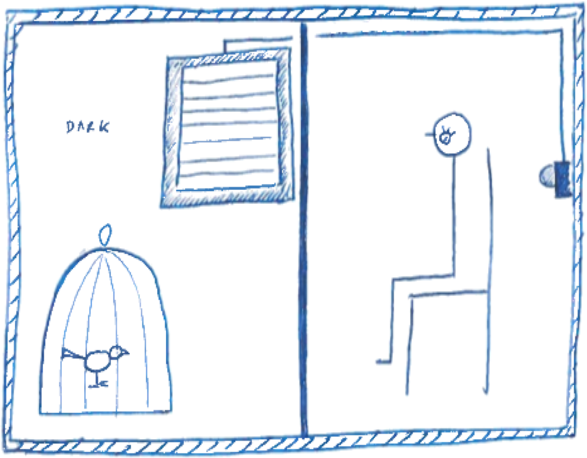


Example of a network with 2 Boolean automata



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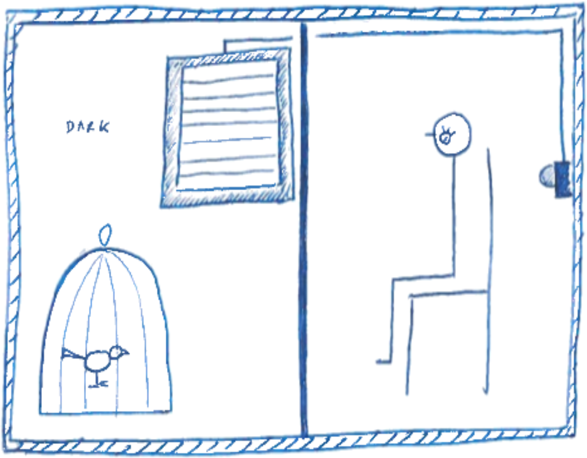
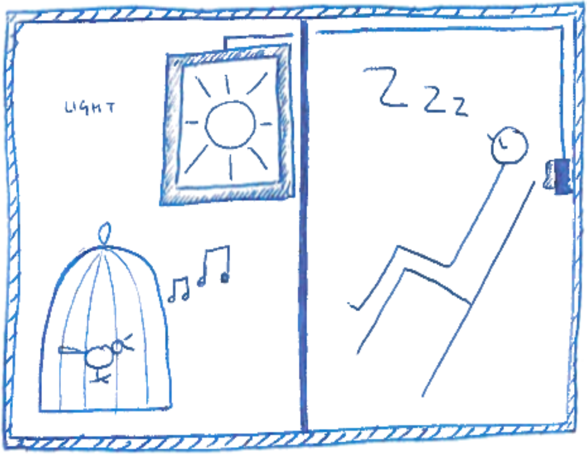


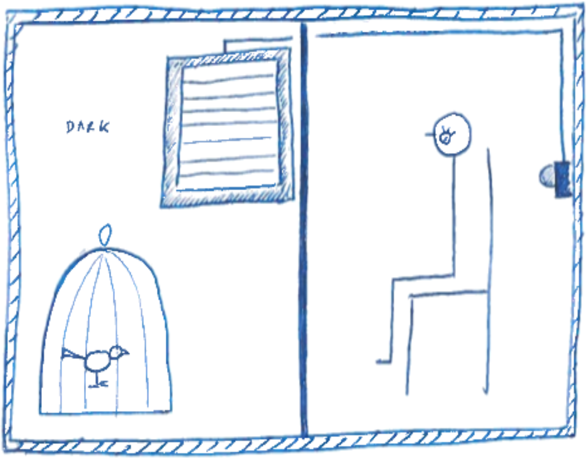
STATE 1
OF AUTOMATON 1
(MAN)

Example of a network with 2 Boolean automata

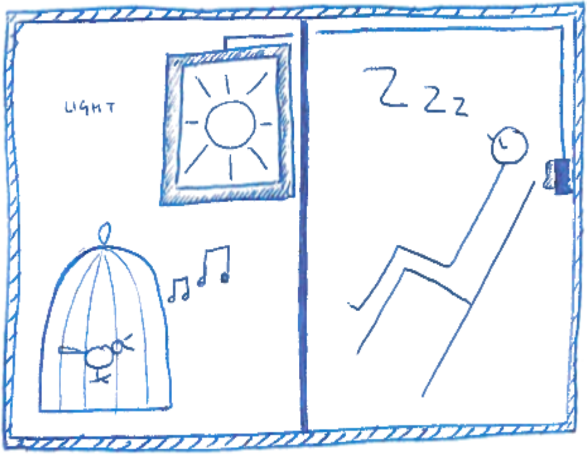


STATE 0
OF AUTOMATON 1
(MAN)

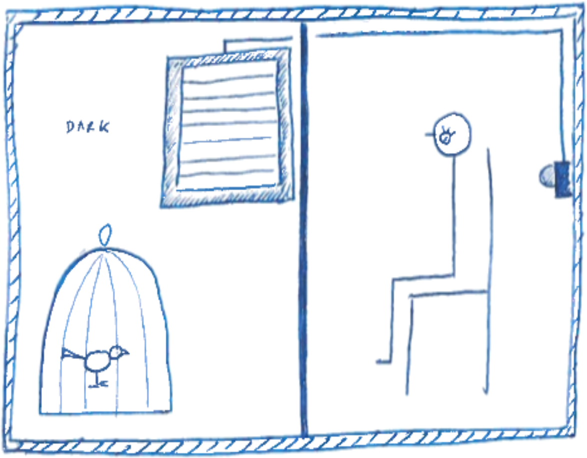
Example of a network with 2 Boolean automata



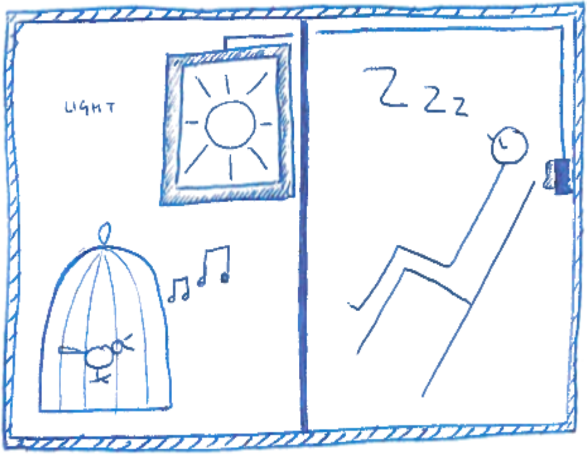
STATE 0
OF AUTOMATON 2
(BIRD)



STATE 1
OF AUTOMATON 2
(BIRD)

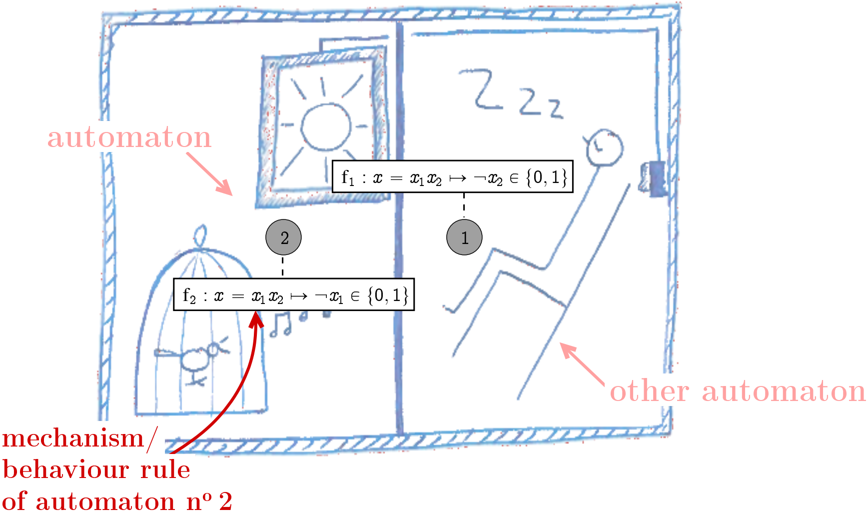


CONFIGURATION 01

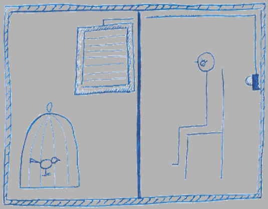


CONFIGURATION 10

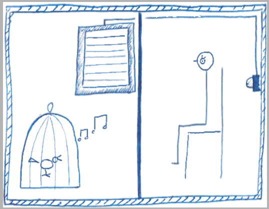
Example of a network with 2 Boolean automata



Example of network's evolution possibilities



CONFIGURATION 01



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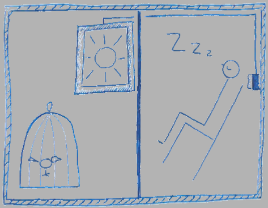

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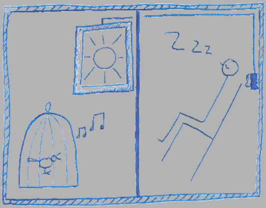

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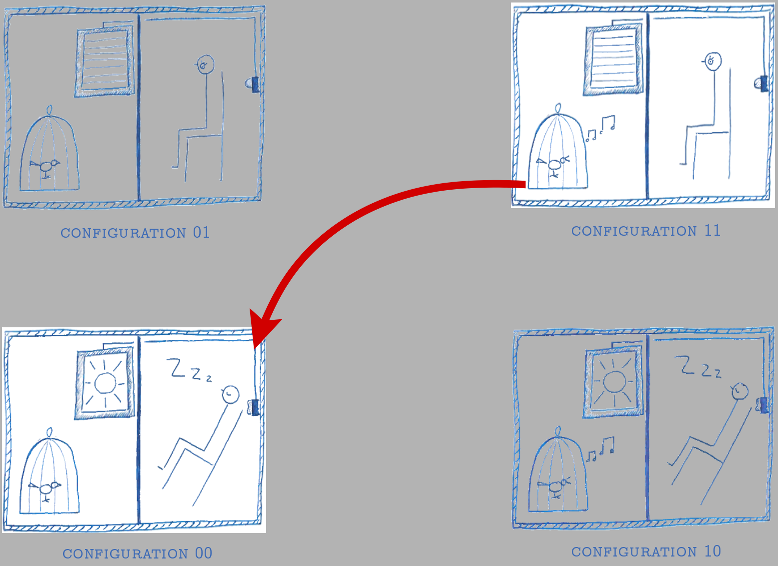
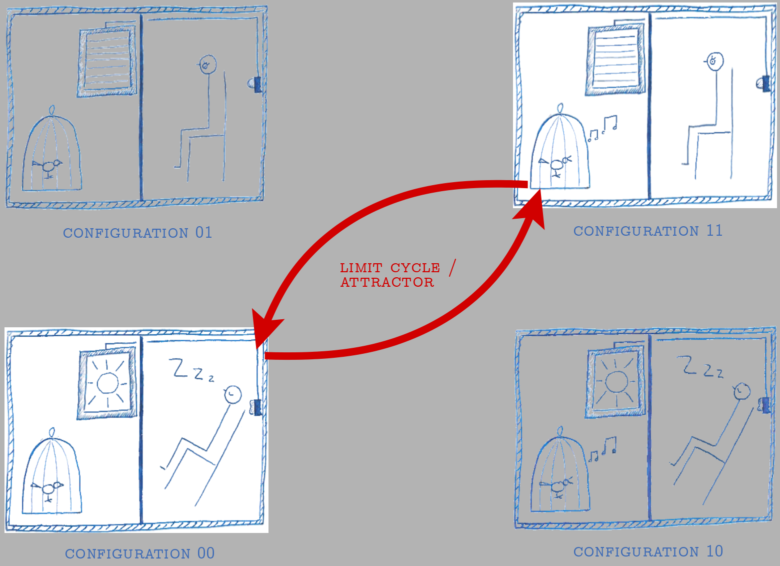
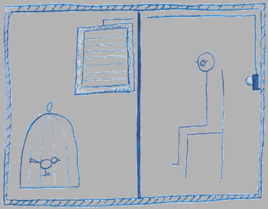

CONFIGURATION 01
CONFIGURATION 11
CONFIGURATION 00
CONFIGURATION 10

Example of network's evolution possibilities



Example of network's evolution possibilities



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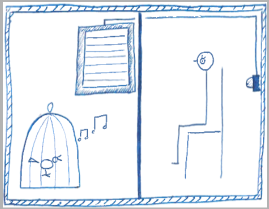

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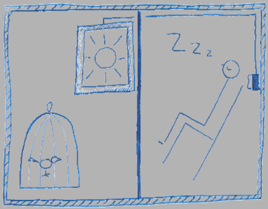
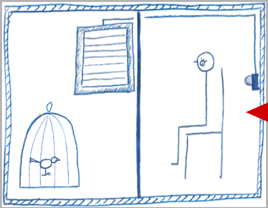


CONFIGURATION 00

The occurences of different changes may differ in relative speed or duration.



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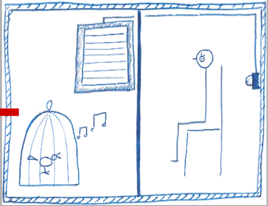

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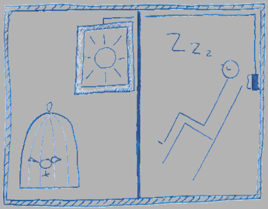

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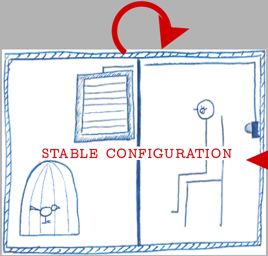

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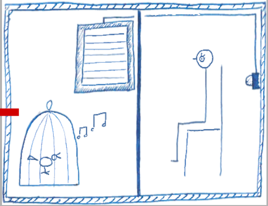

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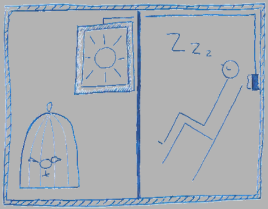
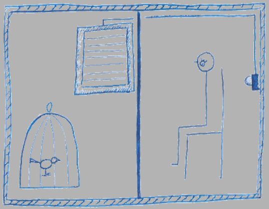


CONFIGURATION 00

The occurences of different changes may differ in relative speed or duration.

Example of network's evolution possibilities



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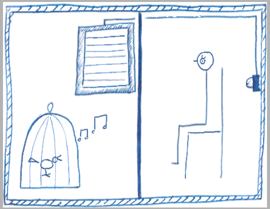

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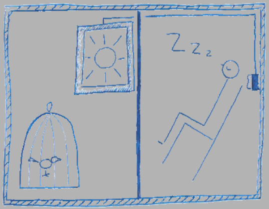
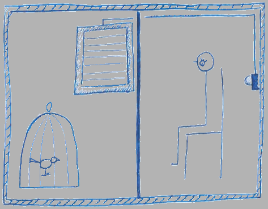


CONFIGURATION 00

The processing of the different informations runing between automata may differ in relative speed or duration.



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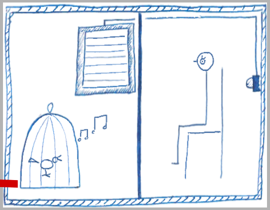

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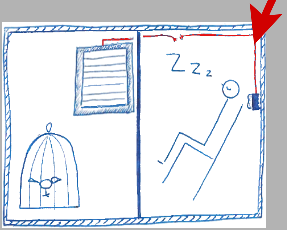

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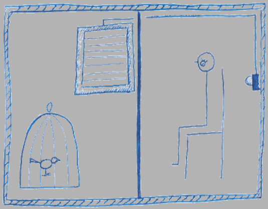

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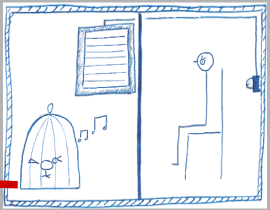

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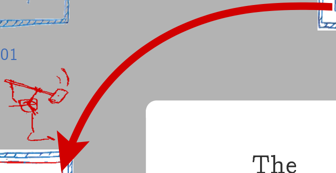
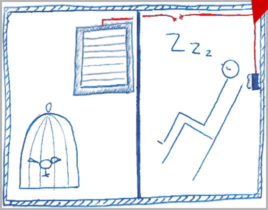
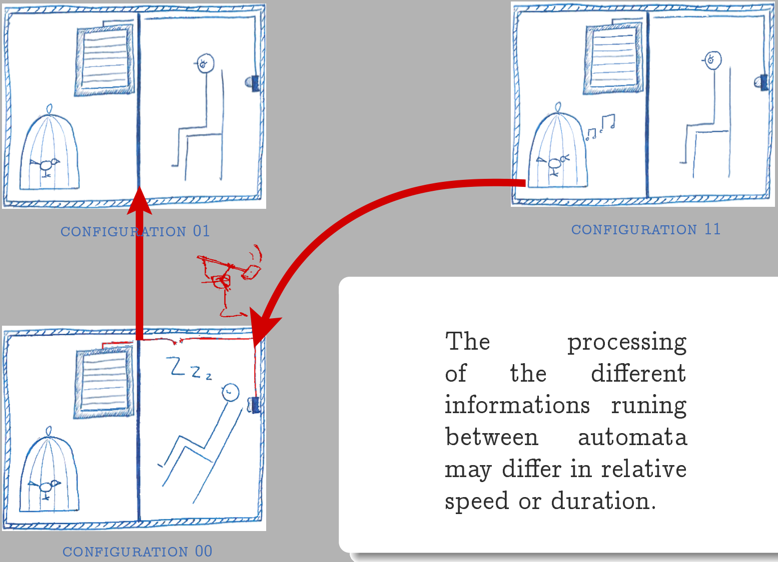




CONFIGURATION 00

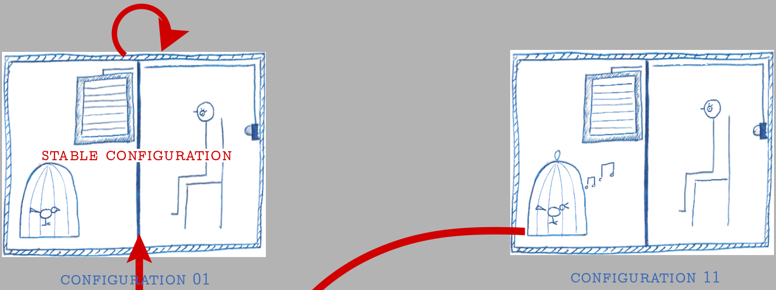
The processing of the different informations runing between automata may differ in relative speed or duration.

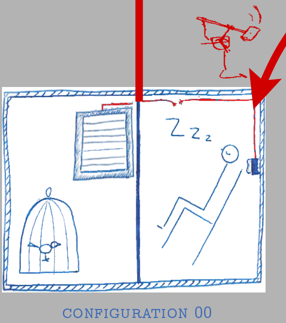
Example of network's evolution possibilities



The processing of the different informations runing between automata may differ in relative speed or duration.

Example of network's evolution possibilities





The processing of the different informations runing between automata may differ in relative speed or duration.

Example of network's evolution possibilities