Journées Topologie et Informatique

Taylor expansion,
a round-trip between syntax and semantics.

Christine Tasson
Christine.Tasson@pps.univ-paris-diderot.fr

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1. **Syntactical** Taylor expansion and Resource consumption
2. Taylor expansion in **Semantics**
3. **Semantics vs Syntax :** The full abstraction question

Taylor expansion :

From Mathematics to Computer Science.

In Maths

$$f : \mathbb{R} \to \mathbb{R}$$

Let $x \in \mathbb{R}$,

$$f(x) = \sum_n \frac{1}{n!} f^{(n)}(0) \cdot x^n$$

$x \mapsto \frac{1}{n!} f^{(n)}(0) \cdot x^n$ is the n-linearisation of f.

In Computer Science

$$\mathtt{P} : \mathtt{nat} \to \mathtt{nat}$$

Let $\mathtt{x} : \mathtt{nat}$,

$$\mathtt{P}\ \mathtt{x} = \sum_n \mathtt{P}_\mathtt{n}\ \underbrace{\mathtt{x}\ \cdots\ \mathtt{x}}_{n}$$

$\mathtt{P}_n$, uses exactly n-times $\mathtt{x}$, is the n-linearisation of $\mathtt{P}$.

Programs :

Resource consumption via Taylor expansion

Syntactical Taylor expansion : $\mathtt{P}\ \mathtt{x} = \sum_n \mathtt{P_n}\ \underbrace{\mathtt{x}\ \cdots\ \mathtt{x}}_n$

$$\lambda\text{-calculus} \xrightarrow{\text{Taylor Expansion}} \text{Resource-calculus}$$

λ-calculus : $M, N := x \mid \lambda x\, M \mid (M)N$

$$(\lambda x\, M)N \to M\,[N/x]$$

« Substitute every occurrence of x in M by N. »

`Example:` $(\lambda x\,(x)x)\lambda z\, z \to (\lambda z\, z)\lambda z\, z \to \lambda z\, z.$

Resource calculus : $s, t := x \mid \lambda x\, s \mid \langle s\rangle[t_1 \ldots t_n]$

$$\langle \lambda x\, s\rangle[t_1 \ldots t_n] \to \partial_x(s, t_1 \ldots t_n)$$

« Substitute each occurrence of x in s by one t_i if possible or reduces to 0. »

`Example:` $\langle \lambda x\, \langle x\rangle[x]\rangle[\lambda z\, z, \lambda z\, z] \rightrightarrows \langle \lambda z\, z\rangle[\lambda z\, z] \to \lambda z\, z.$

Syntactical Taylor expansion : $\mathtt{P}\ \mathtt{x} = \sum_n \mathtt{P_n}\ \underbrace{\mathtt{x} \cdots \mathtt{x}}_{n}$

$$\begin{array}{ccc} \lambda\text{-calculus} & \xrightarrow{\text{Taylor Exp.}} & \text{Resource-calculus} \\ M^* & = & \displaystyle\sum_{t \in \mathcal{T}(M)} \frac{1}{m(t)}\ t \end{array}$$

`Example:` $t \in \mathcal{T}(M)$ with $m(t) = 2$.

$$\begin{array}{rcllcllcl} M & = & (\lambda x\,(x)x)\lambda z\,z & \to & (\lambda z\,z)\lambda z\,z & \to & \lambda z\,z \\ t & = & \langle \lambda x\,\langle x\rangle[x]\rangle[\lambda z\,z, \lambda z\,z] & \rightrightarrows & \langle \lambda z\,z\rangle[\lambda z\,z] & \to & \lambda z\,z \end{array}$$

$$[(\lambda x\,(x)x)\lambda z\,z]^* = \sum_{p,q} \frac{1}{p!\,q!} \langle \lambda x\,\langle x\rangle[\underbrace{x, \ldots, x}_{p}]\rangle[\underbrace{\lambda z\,z, \ldots, \lambda z\,z}_{q}]$$

Syntactical Taylor expansion and Resource Consumption

Idea :

$$\mathtt{P}\ \mathtt{x} = \sum_n \mathtt{P_n}\ \underbrace{\mathtt{x}\ \cdots\ \mathtt{x}}_{n}$$

« $\mathtt{P_n}$ is the part of $\mathtt{P}$ that uses $\mathtt{x}$ n-times. »

Proposition :

Let $M \to \bullet$. Then $\exists!\ s \in \mathcal{T}(M)$ such that $\begin{cases} s \not\to 0 \\ s \to m(s)\bullet \end{cases}$.

« s is the version of M with the explicit resources used for computation. »

`Example:` $M = (\lambda x\,(x)x)\lambda z\,z$ and $t = \langle \lambda x\,\langle x\rangle[x]\rangle[\lambda z\,z, \lambda z\,z]$

Conclusion :

$$M \to^* \bullet \iff M^* = \sum_{t \in \mathcal{T}(M)} \frac{1}{m(t)}\ t \to^* \bullet$$

Semantics :

Taylor expansion and derivatives

Type :	σ	$\lvert\sigma\rvert$ is the set of basic values
Data :	$\mathtt{x} : \sigma$	$[\![x]\!]$ part of $[\![\sigma]\!]$
Type :	$\sigma \to \tau$	$\lvert\sigma\rvert \times \lvert\tau\rvert$
Program :	$\mathtt{P} : \sigma \to \tau$	$[\![\mathtt{P}]\!] \subseteq \lvert\sigma\rvert \times \lvert\tau\rvert$ is a relation from input to output values
Interaction :	$\mathtt{P} : \sigma \to \tau$ $\mathtt{Q} : \tau \to \psi$	$[\![\mathtt{P};\mathtt{Q}]\!] = [\![\mathtt{Q}]\!] \circ [\![\mathtt{P}]\!]$ is the composition of relations.

In order to take into account resources, we introduce multisets.

Type :	$\sigma \Rightarrow \tau$	$\mathcal{M}_{\mathsf{fin}}(\lvert\sigma\rvert) \times \lvert\tau\rvert$
Program :	$\mathtt{P} : \sigma \Rightarrow \tau$	$[\![\mathrm{P}]\!] \subset \mathcal{M}_{\mathsf{fin}}(\lvert\sigma\rvert) \times \lvert\tau\rvert$ is a multi relation between inputs and outputs
Interaction :	$\mathtt{P} : \sigma \Rightarrow \tau$ $\mathtt{Q} : \tau \Rightarrow \psi$	$[\![\mathtt{P};\mathtt{Q}]\!] = [\![\mathtt{Q}]\!] \circ^! [\![\mathrm{P}]\!]$ is the composition of multi relations.

Proposition :
Rel is a cartesian closed category cpo-enriched, a model of various functional programing languages.

In order to count the number of non-deterministic reductions, the probability to get a result,... we move to vector spaces.

Type :	σ	$\mathcal{R}^{\|\sigma\|}$ the set of vectors with coefficients in $\mathcal{R}$.
Data :	$\mathtt{x} : \sigma$	$[\![x]\!]$ is a vector
Type :	$\sigma \multimap \tau$	$\mathcal{R}^{\|\sigma\|\times\|\tau\|}$
Program :	$\mathtt{P} : \sigma \multimap \tau$	$[\![\mathtt{P}]\!] \in \mathcal{R}^{\|\sigma\|\times\|\tau\|}$ matrix or $[\![\mathtt{P}]\!] : \mathcal{R}^{\|\sigma\|} \to \mathcal{R}^{\|\tau\|}$ the associated linear map.
Interaction :	$\mathtt{P} : \sigma \multimap \tau$ $\mathtt{Q} : \tau \multimap \psi$	$[\![\mathtt{P};\mathtt{Q}]\!] = [\![\mathtt{Q}]\!] \circ [\![\mathtt{P}]\!]$ is the composition of matrix.

Type : $\sigma \Rightarrow \tau$ $\mathcal{R}^{\mathcal{M}_{\text{fin}}(|\sigma|)\times|\tau|}$

Program : $\texttt{P} : \sigma \Rightarrow \tau$ $[\![\texttt{P}]\!] : \mathcal{R}^{|\sigma|} \to \mathcal{R}^{|\tau|}$ is an entire function.

$$\forall b \in |\tau|\,,\ [\![\texttt{P}]\!](\texttt{x})_{\texttt{b}} = \sum_{\mu \in \mathcal{M}_{\text{fin}}(|\sigma|)} [\![\texttt{P}]\!]_{\mu,\texttt{b}} \cdot \texttt{x}^{\mu}$$

with $x^{\mu} = \prod_{a \in \text{Supp}(\mu)} x_a^{\mu(a)}$

Interaction : $\texttt{P} : \sigma \Rightarrow \tau$, $\texttt{Q} : \tau \Rightarrow \psi$ $[\![\texttt{P};\texttt{Q}]\!] = [\![\texttt{Q}]\!] \circ^{!} [\![\texttt{P}]\!]$ the composition of entire functions.

Proposition : For a suitable $\mathcal{R}$, we can interpret various functional programing languages.

PCF$^{\texttt{or}}$: $L, M, P := x \mid \lambda x\, M \mid (M)P \mid \mathsf{fix}(M) \mid \underline{0} \mid \mathsf{pred}(M) \mid \mathsf{succ}(M)$
$\mid \mathsf{if}\ (M = \underline{0})\ \mathsf{then}\ P\ \mathsf{else}\ L \mid p \cdot M \mid M\ \texttt{or}\ P$

$$p \cdot M \xrightarrow{\texttt{p}} M \qquad M\ \texttt{or}\ P \xrightarrow{1} M \qquad M\ \texttt{or}\ P \xrightarrow{1} P$$

Program Analysis : $M : \texttt{nat}$ a program and $\mathcal{R}$ a semiring.

$\mathcal{B} = \{\mathtt{T}, \mathtt{F}\}, \vee, \wedge, \mathtt{F}, \mathtt{T}, \mathtt{F} < \mathtt{T}$ — $[\![M]\!]^{\mathcal{B}}_n = \mathtt{T} \iff \exists \mathtt{M} \to^* \underline{\mathtt{n}}$.

$\mathcal{N} = \overline{\mathbb{N}}, +, \cdot, 0, 1, \leq$ — $[\![M]\!]^{\mathcal{N}}_n$ number of $M \to^* \underline{n}$.

$\mathcal{R} = \overline{\mathbb{R}^+}, +, \cdot, 0, 1, \leq$ — $[\![M]\!]^{\mathcal{R}}_n$ probability of $M \to^* \underline{n}$.

$\mathcal{T} = \overline{\mathbb{N}}, \mathbf{min}, +, \infty, 0, \geq$ — $[\![M]\!]^{\mathcal{T}}_n$ number of β and fix() redexes induced in $M \to^* \underline{n}$.

Problematics :

- If $\mathcal{R}$ is a Field, then $[\![\sigma]\!] = \mathcal{R}^{|\sigma|}$ is a linear space of infinite dimension.
- What means : $[\![\mathtt{P}]\!](\mathtt{x})_{-} = \sum_{\mu \in \mathcal{M}_{\mathsf{fin}}(|\sigma|)} [\![\mathtt{P}]\!]_{\mu,-} \cdot \mathtt{x}^{\mu}$

Solutions : Choose $\mathcal{R}$ and properties of $[\![\sigma]\!]$ for convergence

- $\mathbb{R}^{+}$ with usual topology — Probabilistic Coherent Spaces
- $\mathbb{R}$ with discrete topology — Finiteness Spaces
- $\mathbb{R}$ with usual topology — Convenient Vector Spaces

Topology :

$$\sum_a x_a \text{ converges in } \mathbb{R}^+ \text{ iff the sum is absolutely convergent.}$$

Orthogonality : $x, y \in \mathbb{R}^{|\sigma|}$.

$$x \perp y \iff \sum_{a \in |\sigma|} x_a y_a \in [0,1].$$

Types :

$$[\![\sigma]\!] \text{ is a probabilistic coherent space, that is } \begin{cases} [\![\sigma]\!] \subseteq \mathbb{R}^{|\sigma|} \\ [\![\sigma]\!]^{\perp\perp} = [\![\sigma]\!] \end{cases}$$

with $[\![\sigma]\!]^{\perp} = \{x \in \mathbb{R}^{|\sigma|} \mid \forall y \in [\![\sigma]\!],\ x \perp y\}$

Proposition : Probabilistic Coherent Spaces interpret $\mathbf{PCF}^{\mathrm{or}}$.

Discrete topology :

$$\sum_a x_a \text{ converges in } \mathbb{R} \text{ iff the sum is finite.}$$

Orthogonality : $x, y \in \mathbb{R}^{|\sigma|}$.

$$x \perp y \iff \sum_{a \in |\sigma|} x_a y_a \in \mathbb{R}.$$

Types :

$$[\![\sigma]\!] \text{ is a finiteness space, that is } \begin{cases} [\![\sigma]\!] \subseteq \mathbb{R}^{|\sigma|} \\ [\![\sigma]\!]^{\perp\perp} = [\![\sigma]\!] \end{cases}$$

with $[\![\sigma]\!]^{\perp} = \{x \in \mathbb{R}^{|\sigma|} \mid \forall y \in [\![\sigma]\!],\ x \perp y\}$

Properties :

- $[\![\sigma]\!]$ is a linear space with infinite dimension
- $[\![\sigma]\!]$ is endowed with a linearized topology
- opens and bounded are orthogonal

Linear Program : $\mathtt{P} : \sigma \multimap \tau.$

$[\![\mathrm{P}]\!] : [\![\sigma]\!] \to [\![\tau]\!]$ is a continuous linear map.

Usual Program : $\mathtt{P} : \sigma \Rightarrow \tau.$

$[\![\mathrm{P}]\!] : [\![\sigma]\!] \to [\![\tau]\!]$ is an analytic function :

$$\forall x \in [\![\sigma]\!], \quad \mathtt{P}(\mathtt{x}) = \sum_{\mathtt{k} \leq \mathtt{n}} \mathtt{P}_{\mathtt{k}}(\underbrace{x, \ldots, x}_{\mathtt{k}})$$ with P_k the k^{th} linearization of P;

Proposition :
Finiteness Spaces interpret differential λ-calculus but no fixpoints.

Types : $[\![\sigma]\!]$ is a convenient vector space

- Locally Convex Vector Spaces over $\mathbb{R}$ (usual).
- Duality bounded vs. opens
- Mackey complete

Linear Programs : $\mathtt{P} : \sigma \multimap \tau$

$$[\![\mathrm{P}]\!] \in \mathcal{L}_{\mathsf{c}}([\![\sigma]\!], [\![\tau]\!]) \text{ is linear and continuous.}$$

Usual Programs : $\mathtt{P} : \sigma \Rightarrow \tau$

$$[\![\mathrm{P}]\!] \in \mathcal{C}^{\infty}([\![\sigma]\!], [\![\tau]\!]) \text{ is smooth.}$$

i.e. preserves smooth curves.

Proposition : Convenient Vector Spaces interpret differential λ-calculus without reference to basis and with usual topology.

Semantics vs Syntax :

The full abstraction question

« *Decide what you want to say before you worry how you are going to say it.* » **The Scott-Strachey Approach to Programming Language Theory, preface, Scott (77)**

Denotational semantics :

a program as a function between mathematical spaces

Operational semantics :

a program as a sequence of computation steps

« *Full Abstraction studies connections between denotational and operational semantics.* » **LCF Considered as a Programming Language, Plotkin (77)**

Full Abstraction = Adequacy + Full completeness

FA relates Semantical and Observational equivalences :

$$[\![M]\!] = [\![N]\!] \quad \underset{\text{Full Completeness}}{\overset{\text{Adequacy}}{\Longrightarrow \atop \Longleftarrow}} \quad M \simeq_o N$$

$(\forall C[\,],\ C[M] \to v \iff C[N] \to v)$

How to prove Full Completeness :

1. By contradiction, start with $[\![M]\!] \neq [\![N]\!]$
2. Find testing context : f such that $f[\![M]\!] \neq f[\![N]\!]$
3. Prove definability :
   $\exists C[\cdot],\ \forall M,\ f[\![M]\!] = [\![C[M]]\!]$ and $C[M] \to m$.
4. Conclude :
   $\exists C[\cdot],\ [\![C[M]]\!] \neq [\![C[N]]\!] \Rightarrow m \neq n \Rightarrow M \not\simeq_o N$.

Syntax vs Semantics :

Probabilistic PCF

A Typed Probabilistic Functional Programing Language

Integers :

$\underline{n} : \mathtt{nat}$

$\mathrm{pred}(\underline{k+1}) \xrightarrow{1} \underline{k}$

$\mathrm{succ}(\underline{k}) \xrightarrow{1} \underline{k+1}$

Functions and Composition :

$$(\lambda \underset{\sigma \Rightarrow \tau}{x^\sigma M}) \underset{\sigma}{N} \xrightarrow{1} M \underset{\tau}{[N/x]}$$

Fixpoints :

$$\mathrm{fix}(M) \xrightarrow{1} (M)\mathrm{fix}(M)$$

Case Zero :

$\text{if } (\underline{0} = \underline{0}) \text{ then } P_1 \text{ else } P_2 \xrightarrow{1} P_1$

$\text{if } (\underline{k+1} = \underline{0}) \text{ then } P_1 \text{ else } P_2 \xrightarrow{1} P_2$

+ Context Rules

Probabilities : for $p + q \leq 1$

$p \cdot M + q \cdot N \xrightarrow{\mathtt{p}} M$

$p \cdot M + q \cdot N \xrightarrow{\mathtt{q}} N$

where $M \xrightarrow{\rho} M'$ means that :
M reduces to M' with probability ρ

Probabilistic Coherent Spaces

Definition and Adequacy

Types :

$$\llbracket \sigma \rrbracket \subseteq (\mathbb{R}^+)^{|\sigma|}$$

Example :

$|\texttt{nat}|$ is the set $\mathbb{N}$ of natural numbers

$\llbracket \texttt{nat} \rrbracket$ is the set of subprobability distributions over $\mathbb{N}$.

Programs :

For $M : \sigma$, $\llbracket M \rrbracket \in \llbracket \sigma \rrbracket$

Example :
$\frac{1}{2} \cdot \underline{n} + \frac{1}{3} \cdot \underline{m}$ is interpreted by $(0, \ldots, 0, \underbrace{\tfrac{1}{2}}_{n}, 0, \ldots, 0, \underbrace{\tfrac{1}{3}}_{m}, 0, \ldots)$

Adequacy Lemma :

Let $M : \texttt{nat}$ be a closed program. Then for all n,

$$\mathrm{Proba}(M \xrightarrow{*} \underline{n}) = \llbracket M \rrbracket_n.$$

PCoh : Probabilistic Coherent Spaces

Types : $$\llbracket \sigma \Rightarrow \tau \rrbracket \subseteq (\mathbb{R}^+)^{\mathcal{M}_{\text{fin}}(|\sigma|) \times |\tau|}$$

Example :
$\llbracket \texttt{nat} \Rightarrow \texttt{nat} \rrbracket$ set of functions preserving subprobability distributions.

Programs : For $M : \sigma \Rightarrow \tau$, $\llbracket M \rrbracket : (\mathbb{R}^+)^{|\sigma|} \to (\mathbb{R}^+)^{|\tau|}$

$$\llbracket M \rrbracket (x)_- = \sum_{\mu \in \mathcal{M}_{\text{fin}}(|\sigma|)} \llbracket M \rrbracket_{\mu,-} \cdot x^\mu$$

- $x \in (\mathbb{R}^+)^{|\sigma|}$
- $\llbracket M \rrbracket_{\mu,-}$ coefficients
- $x^\mu = \prod_{a \in \text{Supp}(x)} x_a^{\mu(a)}$

Compositionality :

For $P : \sigma \Rightarrow \tau, M : \sigma$, $$\llbracket (P) M \rrbracket_- = \sum_{\mu \in \mathcal{M}_{\text{fin}}(|\sigma|)} \llbracket P \rrbracket_{\mu,-} \llbracket M \rrbracket^\mu$$

Probabilistic Full Abstraction :

The completeness theorem

FA relates Semantical and Observational equivalences :

Let $M, N : \sigma$ $\quad \forall \alpha \in |\sigma|, \ [\![M]\!]_\alpha = [\![N]\!]_\alpha$

Adequacy $\Downarrow$ $\Uparrow$ Full Completeness

$$\forall P : \sigma \Rightarrow \mathtt{nat}, \ \forall n \in |\mathtt{nat}|,$$
$$\mathrm{Proba}((P)M \xrightarrow{*} n) = \mathrm{Proba}((P)N \xrightarrow{*} n))$$

How to prove Adequacy :

1. Apply Adequacy Lemma :

$$\forall n, \ \mathrm{Proba}((P)M \xrightarrow{*} \underline{n}) = [\![(P)M]\!]_n.$$

2. Apply Compositionality :

$$\forall n, \ [\![(P)M]\!]_n = \sum_{\mu \in \mathcal{M}_{\mathrm{fin}}(|\sigma|)} [\![P]\!]_{\mu,n} \prod_{\alpha \in \mu} [\![M]\!]_\alpha^{\mu(\alpha)}$$

FA relates Semantical and Observational equivalences :

Let $M, N : \sigma$ $\qquad \forall \alpha \in |\sigma|,\ [\![M]\!]_\alpha = [\![N]\!]_\alpha$

Adequacy $\Downarrow$ $\Uparrow$ Full Completeness

$$\forall P : \sigma \Rightarrow \mathtt{nat},\ \forall n \in |\mathtt{nat}|,$$
$$\mathrm{Proba}((P)M \xrightarrow{*} n) = \mathrm{Proba}((P)N \xrightarrow{*} n))$$

How to prove Full Completeness :

1. By contradiction : $\exists \alpha \in |\sigma|\ ,\ [\![M]\!]_\alpha \neq [\![N]\!]_\alpha$
2. Find testing context : P_α such that $[\![(P_\alpha)M]\!]_0 \neq [\![(P_\alpha)N]\!]_0$
3. Prove definability : $P_\alpha \in PPCF$
4. Apply Adequacy : $\mathrm{Proba}((P_\alpha)M \xrightarrow{*} 0) \neq \mathrm{Proba}((P_\alpha)N \xrightarrow{*} 0)$.

How to prove Full Completeness :

1. By contradiction : $\exists \alpha \in |\sigma|$, $[\![M]\!]_\alpha \neq [\![N]\!]_\alpha$
2. Find testing context : P_α such that $[\![(P_\alpha)M]\!]_0 \neq [\![(P_\alpha)N]\!]_0$
    - Base case : $\sigma = \texttt{nat}$, $\alpha = n$, take

      If $P_n = \lambda x^\iota$ if $(x = \underline{n})$ then $\underline{0}$ Then $[\![(P_n)M]\!]_0 = [\![M]\!]_n$

    - Induction case : by Compositionality,

$$[\![(P_\alpha(\vec{X}))M]\!]_0 = \sum_{\mu \in \mathcal{M}_{\text{fin}}(|\sigma|)} [\![P_\alpha(\vec{X})]\!]_{\mu,0} \prod_{\delta \in \mu} [\![M]\!]_\delta^{\mu(\delta)}$$

If $[\![P_\alpha(\vec{X})]\!]_{\mu,0}$ is

- a power series in $\vec{X}$
- with coeff of $\prod \vec{X} \neq 0 \iff \mu = [\alpha]$

Then $[\![(P_\alpha(\vec{X}))M]\!]_0$ is

- a power series in $\vec{X}$
- with coeff of $\prod \vec{X}$ proportional to $[\![M]\!]_\alpha$.

$[\![(P_\alpha(\vec{X}))M]\!]_0$ and $[\![(P_\alpha(\vec{X}))N]\!]_0$ are different power series

2. Find testing context : $\forall \vec{\lambda} \in [0,1]^{\mathbb{N}},\ P_\alpha(\vec{\lambda}) \in PPCF$ and the series $[\![(P_\alpha(\vec{\lambda}))M]\!]_0$ and $[\![(P_\alpha(\vec{\lambda}))N]\!]_0$ converge absolutely with different coefficients.

3. Prove definability :

$$\exists \vec{\lambda} \in [0,1]^{\mathbb{N}},\ [\![(P_\alpha(\vec{\lambda}))M]\!]_0 \neq [\![(P_\alpha(\vec{\lambda}))N]\!]_0.$$

By contradiction :

- If they were equal, their derivatives near zero would be equal.
- Coefficients of power series are computed by derivation at zero.

⚠ PCoh is NOT a model of differential lambda-calculus.

4. Apply Adequacy : $\exists \vec{\lambda} \in [0,1]^{\mathbb{N}},\ P_\alpha(\vec{\lambda}) \in PPCF$

$$\mathrm{Proba}((P_\alpha(\vec{\lambda}))M \xrightarrow{*} 0) \neq \mathrm{Proba}((P_\alpha(\vec{\lambda}))N \xrightarrow{*} 0).$$

Conclusion : Quantitative semantics

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2. Taylor expansion in **Semantics**

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