Deciding the Word Problem in the Union of Equational Theories Sharing Constructors

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Abstract

The main contribution described in this talk is a new method for combining decision procedures for the word problem in equational theories sharing "constructors." The notion of constructors adopted in this talk has a nice algebraic definition and is more general than a related notion introduced in previous work on the combination problem.

The integration of constraint solvers (that is, specialized decision procedures for restricted classes of problems) into general purpose deductive systems (such as Knuth-Bendix completion procedures, resolution-based theorem provers, or Logic Programming systems) aims at combining the efficiency of the specialized method with the universality of the general one. Many applications of the constraint-based systems obtained by such an integration require a combination of more than one constraint language, and thus a solver for the resulting mixed constraints. The development of general combination methods for constraint solvers tries to avoid the necessity of designing a new specialized decision procedure for each new combination of constraint languages.

For equational theories, one is usually interested in solvers for the following decision problems: the word problem, the matching problem, and the unification problem. In this setting, the research on combination of constraint solvers is mainly concerned with finding conditions under which the following question can be answered affirmatively: given two equational theories E_1 and E_2 with decidable word/matching/unification problems, is the word/matching/unification problem for $E_1 \cup E_2$ also decidable?

A very effective (but also rather strong) restriction is to require that E_1 and E_2 be equational theories over disjoint signatures. Under this restriction, decision procedures for the word problems in E_1 and E_2 can always be combined into a decision procedure for the word problem in $E_1 \cup E_2$ [9, 13, 12, 8, 7]. For the matching and the unification problem, there also exist very general combination results under the disjointness restriction (see [11] for matching, and, e.g., [12, 5, 1] for unification).

*Partially supported by the EC Working Group CCL II.

It is not hard to extend these results to theories sharing constant symbols [10, 7, 2]. The only work we are aware of that presents a general combination approach for the union of equational theories having more than constant symbols in common is [6], where the problem of combining algorithms for the unification, matching, and word problem is investigated for theories sharing so-called "constructors."

In this talk, we restrict our attention to the word problem. The combination result we obtain improves on the corresponding result in [6] in the following respects. Firstly, we introduce a notion of constructors, modeled after the one introduced in [14], which is strictly more general than the one in [6]. Whereas [6] does not allow for nontrivial identities between constructor terms, we only require the constructor theory to be collapse-free. Secondly, the definition of constructors in [6] depends strongly on technical details such as the choice of an appropriate well-founded and monotonic ordering. In contrast, our definition uses only abstract algebraic properties. Finally, the combination procedure described in [6], like the ones for the disjoint case [9, 12, 8, 7], directly transforms the terms for which the word problem is to be decided, by applying collapse equations[1] and abstracting alien subterms. This transformation process must be carried on with a rather strict strategy (in principle, going from the leaves of the terms to their roots) and it is not easy to describe. In contrast, our procedure extends the rule-based combination procedure for the word problem introduced in [2] for the case of shared constants. It works on a set of equations rather than terms, and its transformation rules can be applied in arbitrary order, that is, no strategy is needed. We claim that this difference makes the method more flexible and easier to describe and comprehend.

A paper describing the results presented in this talk has been published in the proceedings of RTA'99 [4]; detailed proofs can be found in [3]

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[1] i.e., equations of the form $x = t$, where x is a variable occurring in the non-variable term t.

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