Intelligent backtracking techniques in resolution decision procedures

Lilia Georgieva

Department of Computer Science, Heriot-Watt University
Edinburgh EH14 10AS, United Kingdom, lilia@macs.hw.ac.uk

Abstract. We study the usability of intelligent backtracking techniques in resolution based decision procedures for modal logics, description logics, and fragments of first-order clause logic. We analyse the computational effects of the techniques and compare the size of the search space in: (i) decision procedures which use intelligent backtracking, (ii) decision procedures which use splitting and backtracking, and (iii) decision procedures where all clauses are transformed in Horn form and consequently all inferences are unit resulting and splitting is avoided.

1 Introduction

Resolution-based decision procedures for modal and description logics, for fragments of first-order logic and for syntactically defined clausal classes [2, 4, 7–9, 5, 10, 11] are often influenced by tableau algorithms for description logics [6, 7]. Essential for the tableaux algorithms is the splitting rule which allows for dealing with disjunction in the tableaux algorithms for modal logics and description logics [3, 7]. The rule supports explicit case analysis and originates from the β rule in the tableaux-based theorem proving. The splitting rule does not modify the processed clausal set by adding new clauses or deleting redundant clauses but instead divides the clausal set into two sets which are presented by different branches of the derivation tree and are considered as different cases by the inference rules. The cases are processed separately. The splitting rule combined with the resolution inference results in a complete inference system for a refutation search. However, the integration of the splitting rule in a saturation-based theorem prover is non-trivial. The difficulty of the implementation of the rule is that additional information about the split level of each clause has to be maintained [13]. In resolution-based decision procedures splitting is combined with backtracking. When backtracking, clauses already generated on the currently expanded branch of the derivation tree are deleted up to the stage where the derivation has split. Some of these clauses may need to be regenerated.

Splitting can be avoided by Horn clause transformation. We compare the theoretical properties of three types of decision procedures: (i) decision procedures which use Horn clause transformation; (ii) decision procedure where clauses are not transformed in Horn form and which use use splitting and backtracking and (iii) decision procedures where clauses are not transformed in Horn form and which use intelligent backtracking, i.e. the variants of backtracking including backjumping, dependency-directed backtracking and dynamic backtracking.

Our investigation is in the framework of resolution [1, 12]. We show that the space complexity and the memory consumption of the procedures is dependent on the backtracking mechanisms and the use of non-standard backtracking techniques leads to increase of space complexity.

We discuss the effects on the search space of the parameters that guide intelligent backtracking algorithms. These parameters include: (i) deciding when to backtrack; (ii) identifying the reasons because of which the branch is closed or the derivation cannot proceed (ii) deciding which literals to resolve and which clauses in the input clause set have he highest number of partner clauses; (iii) remembering the reason for the failure and avoiding repeated derivation of the literals involved in the generation of the empty clause.

References

1. L. Bachmair and H. Ganzinger. Resolution theorem proving. In A. Robinson and A. Voronkov, editors, *Handbook of Automated Reasoning*, pp. 19–99. Elsevier, 2001.
2. H. de Nivelle, R. A. Schmidt, and U. Hustadt. Resolution-based methods for modal logics. *Logic J. IGPL*, 8(3):265–292, 2000.
3. F. M. Donini, M. Lenzerini, D. Nardi, and A. Schaerf. Reasoning in description logics. In G. Brewka, editor, *Principles in Knowledge Representation*, Studies in Logic, Language and Information, pp. 191–236. CSLI Publications, Stanford, 1996.
4. L. Georgieva, U. Hustadt, and R. A. Schmidt. A new clausal class decidable by hyperresolution. In A. Voronkov, editor, *Proceedings of Conference on Automated Deduction*, vol. 2392 of *Lecture Notes in Artificial Intelligence*, pp. 260–274. Springer, 2002.
5. L Georgieva, U. Hustadt, and R.A. Schmidt. Hyperresolution for guarded formulae. *J. Symbolic Computat.*, 36:163–192, 2003.
6. R. Goré. Tableau methods for modal and temporal logics. In M. D'Agostino, D. Gabbay, R. Hähnle, and J. Posegga, editors, *Handbook of Tableau Methods*, pp. 297–396. Kluwer, 1999.
7. U. Hustadt and R. A. Schmidt. On the relation of resolution and tableaux proof systems for description logics. In T. Dean, editor, *Proceedings of International Joint Conference Artificial Intelligence*, pp. 110–115. Morgan Kaufmann, 1999.
8. U. Hustadt and R. A. Schmidt. Issues of decidability for description logics in the framework of resolution. In *Automated Deduction in Classical and Non-Classical Logics*, vol. 1761 of *LNAI*, pp. 191–205. Springer, 2000.
9. U. Hustadt and R. A. Schmidt. Using resolution for testing modal satisfiability and building models. In *SAT2000: Highlights of Satisfiability Research in the Year 2000.* IOS Press, 2000.
10. A. Leitsch. Deciding clause classes by semantic clash resolution. *Fundamenta Informatica*, 18:163–182, 1993.
11. A. Leitsch. Decision procedures and model building, or how to improve logical information in automated deduction. In *Automated Deduction in Classical and Non-Classical Logics, Selected Papers*, vol. 1761 of *Lecture Notes in Computer Science*, pp. 62–79. Springer, 1998.

12. J. A. Robinson. Automatic deduction with hyper-resolution. *International Journal of Computer Mathematics*, 1(3):227–234, 1965.
13. C. Weidenbach. Combining superposition, sorts and splitting. In A. Robinson and A. Voronkov, editors, *Handbook of Automated Reasoning*, pp. 1967–2015. Elsevier, 2001.