# Algorithms for Stochastic Games

Hugo Gimbert, CNRS, LaBRI, Bordeaux

March 3, 2009

Algorithms and Games

Solving Simple Stochastic Games

Solving Stochastic Games

Solving Stochastic Games with Signals

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- Undecidable problems: Halting problem. Post Correspondence Problem.

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- Games on graphs: the same algorithm works for games on graphs. States $S = S_1 \cup S_2$ controlled by player 1 or 2.

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- Minimize $\sum_s v(s)$ with constraints:

$$s \in S, \quad 0 \leq v(s) \leq 1$$

$$t \text{ target}, \quad v(t) = 1$$

$$s \in S_1, \quad (s,u) \in E, \quad v(s) \geq v(u)$$

$$s \in S_R, \quad v(s) = \sum_{u \in S} p(s,u) \cdot v(u)$$

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- Generalization: perfect-information payoff games with stationary deterministic optimal strategies.

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- Strategy Improvement Algorithm [Hoffman, Karp, 66] [Rao, Chandrasekaran and Nair, 73]. Compute better and better strategies.

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- Strategy Improvement Algorithm [Hoffman, Karp, 66] [Rao, Chandrasekaran and Nair, 73]. Compute better and better strategies.
- Not exact computation: converge to the value but no guarantee on the number of steps for a given precision. May be efficient in practice.

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- First order theory on reals: well-formed formula with rational constants $\frac{a}{b}$, arithmetic operations $*$ and $+$, variables $x_1, x_2, \ldots, x_n$, comparison $\leq$, quantifiers $\exists$ and $\forall$, boolean operators $\neg, \wedge, \vee$ and parentheses ( and ).
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- Corollary [Chatterjee, 06] : whether player 1 can guarantee payoff $> 0$ is decidable. Exponential time, polynomial space.

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- Fixed stationary strategies $\sigma : S \to \mathcal{D}(I)$ and $\tau : S \to \mathcal{D}(J)$, expected payoff $\text{val}(\sigma, \tau) : S \to \mathbb{R}$ is the unique solution to $\text{val}(s) = r(s) + \lambda \sum_{i,j,u} \sigma(i)\tau(j)p(s,i,j,u)\text{val}(u)$ . $(**)$

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- $\exists \sigma : S \to \mathcal{D}(I), \forall \tau : S \to \mathcal{D}(J), \exists v : S \to [0,1], (\forall s \in S, (**)) \wedge (v(s_0) > 0)$.

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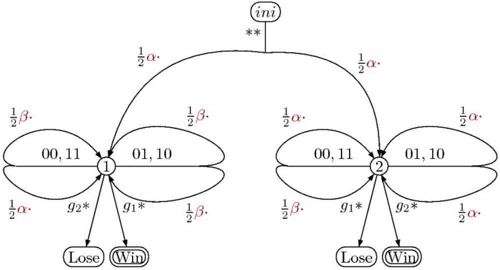


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- Enumeration of finite words $u$?

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- Proof: reduction to Post correspondence problem. Actions in the game = indices of the PCP instance. Reverse binary encoding, strategy wins with probability $\frac{1}{2}u_{i_1}u_{i_2}\cdots u_{i_n} + (1-\frac{1}{2})v_{i_1}v_{i_2}\cdots v_{i_n}$. Strategies win with proba $\frac{1}{2}$ iff $u_{i_1}u_{i_2}\cdots u_{i_n} = v_{i_1}v_{i_2}\cdots v_{i_n}$.

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- Corollary: this decision problem is decidable in doubly-exponential time.
- Remark: the same decision problem is undecidable for stochastic games with Büchi conditions.

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- Avoid reduction to first order logic.
- Finding a polynomial-time algorithm for simple stochastic games.